Causal Reversibility Implies Time Reversibility

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Reversibility

- *What does (ir)reversibility mean in computing?*
- Well established concept in mathematics, physics, chemistry, biology: inverse function, inverse operation, inverse element, . . .
- Much more recent in informatics: seminal papers by Landauer in 1961 and Bennett in 1973 on IBM Journal of Research and Development.
- Landauer principle states that any manipulation of information that is *irreversible* – i.e., causing information loss – such as:
  - erasure/overwriting of bits
  - merging of computation paths

  must be accompanied by a corresponding *entropy increase*.
- Minimal *heat generation* is necessary to standardize signals and making them independent of their history, so that it becomes *impossible to determine the input from the output*.

- Landauer principle establishes that logical irreversibility of a function implies physical irreversibility of computing and its dissipative effects.
- Experimentally verified by Bérut et al in 2012 and revisited in terms of its physical foundations by Frank in 2018.
- Every reversible computation, where no information is lost instead, may be potentially carried out without dissipating further heat.
- Lower energy consumption could therefore be achieved by resorting to reversible computing.
- There are many other applications of reversible computing:
  - Biochemical reaction modeling.
  - Parallel discrete-event simulation.
  - Robotics and control theory.
  - Fault tolerant computing systems.
  - Concurrent program debugging.

- Two directions of computation in a reversible system:
  - Forward: coincides with the normal way of computing.
  - Backward: the effects of the forward one are undone when needed.
- *How to proceed backward? Same path as the forward direction?*
- Not necessarily, especially in the case of a concurrent system.
- Different notions of reversibility in different settings:
  - Causal reversibility is the capability of going back to a past state in a way that is *consistent with the computational history* of the system (easy for sequential systems, hard for concurrent and distributed ones).
  - Time reversibility refers to the conditions under which the stochastic behavior remains the same when the *direction of time* is reversed (quantitative system models, efficient performance evaluation).
- Any relationship between causal reversibility and time reversibility?

Causal Reversibility

- Undoing actions in reverse order starting from the last performed one.
- Sequential systems: just follow the total order over executed actions.
- Concurrent systems: the last performed action may not be unique!
- Causal reversibility by Danos & Krivine [CONCUR 2004, ToTA 2023].
- An action can be undone provided that its consequences, if any, have been undone beforehand.
- Correctness: no previously inaccessible state can be encountered when going backward (causality preservation).
- Flexibility: it is not required to follow the same path undertaken in the forward direction (causality vs. history).
- Independent actions can be undone in any order.

- Formalized through a model inspired by [LanesePU20] and [SNW96].
- Reversible labeled transition system with independence (RLTSI) is a tuple $(S, A, \longrightarrow, \dashrightarrow, \iota)$ where:
  - $S \neq \emptyset$ is an at most countable set of states.
  - $A \neq \emptyset$ is a countable set of actions.
  - $\longrightarrow \subseteq S \times A \times S$ is a *forward transition relation*.
  - $\dashrightarrow \subseteq S \times A \times S$ is a *backward transition relation*.
  - Loop property: $(s, a, s') \in \longrightarrow$ iff $(s', a, s) \in \dashrightarrow$ for all s, s', a.
  - $\iota \subseteq \longmapsto \times \longmapsto$ is an irreflexive and symmetric *independence relation* over transitions, where $\longmapsto = \longrightarrow \dot{\cup} \dashrightarrow$ (disjoint union).
- The loop property is a necessary condition for reversibility [DK04]:
  - Every executed action can be undone.
  - Every undone action can be redone.

- An RLTSI meets causal consistency (CC) iff every two coinitial and cofinal paths ω_1, ω_2 satisfy $\omega_1 \asymp \omega_2$.
- Causal equivalence is the smallest equivalence relation $\asymp$ over paths that satisfies the following for all paths $\omega_1, \omega_2, \omega_3, \omega_4$ composable with the transitions mentioned in the two operations below:
  - For all transitions t, $\omega_1 \, t \, \underline{t} \, \omega_2 \asymp \omega_1 \, \omega_2$ and $\omega_3 \, \underline{t} \, t \, \omega_4 \asymp \omega_3 \, \omega_4$ (cancel).
  - For all commuting squares t, u, u', t', $\omega_1 \, t \, u' \, \omega_2 \asymp \omega_1 \, u \, t' \, \omega_2$ (swap).
- Transitions t, u, u', t' form a commuting square iff:
  - $t : s \xmapsto{a_1} s'_1$, $u : s \xmapsto{a_2} s'_2$,
    $u' : s'_1 \xmapsto{a_2} s''$, $t' : s'_2 \xmapsto{a_1} s''$.
  - $(t, u) \in \iota$.
- Whenever an RLTSI meets CC, then:
  - If $s \xmapsto{a_1} s'$ and $s \xmapsto{a_2} s'$, then $a_1 = a_2$ (uniqueness of pairs (UP)).
  - For all s there is no a such that $s \xmapsto{a} s$ (absence of self-loops (AS)).

Time Reversibility

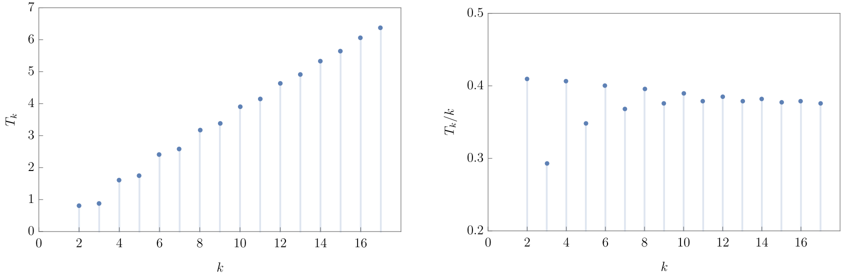
- A CTMC $X(t)$ is time reversible (TR) iff $(X(t_i))_{1\leq i\leq n}$ has the same joint distribution as $(X(t'-t_i))_{1\leq i\leq n}$ for all $n\in\mathbb{N}_{\geq 1}$ and $t_1<\cdots<t_n, t'\in\mathbb{R}_{\geq 0}$.
- In this case $X(t)$ and its reversed version $X^{\mathrm{r}}(t)=X(-t)$, $t\in\mathbb{R}_{\geq 0}$, are stochastically identical: both stationary, same $\boldsymbol{\pi}$.
- For a *stationary* CTMC $X(t)$ the following statements are equivalent [Kelly 1979]:
    - $X(t)$ is time reversible.
    - For all distinct $s,s'\in S$, it holds that $\pi(s)\cdot q_{s,s'} = \pi(s')\cdot q_{s',s}$ (partial/detailed balance equations).
    - For all distinct $s_1,\ldots,s_n\in S$, $n\geq 2$, (on cycles) it holds that $q_{s_1,s_2}\cdot\ldots\cdot q_{s_{n-1},s_n}\cdot q_{s_n,s_1} = q_{s_1,s_n}\cdot q_{s_n,s_{n-1}}\cdot\ldots\cdot q_{s_2,s_1}$.
- Example: birth-death processes and their tree-like variants.

Causal Reversibility Implies Time Reversibility

- Reversible Markovian labeled transition system with independence (RMLTSI) is a tuple $(S, A \times \mathbb{R}_{>0}, \longrightarrow, \dashrightarrow, \iota)$ where:
  - $S \neq \emptyset$ is an at most countable set of states.
  - $A \neq \emptyset$ is a countable set of actions.
  - $\longrightarrow \subseteq S \times (A \times \mathbb{R}_{>0}) \times S$ is a forward transition relation.
  - $\dashrightarrow \subseteq S \times (A \times \mathbb{R}_{>0}) \times S$ is a backward transition relation.
  - Rate loop property: for all $s, s' \in S$ and $a \in A$ there exists $\lambda \in \mathbb{R}_{>0}$ such that $(s, a, \lambda, s') \in \longrightarrow$ iff there exists $\mu \in \mathbb{R}_{>0}$ such that $(s', a, \mu, s) \in \dashrightarrow$.
  - $\iota \subseteq \longmapsto \times \longmapsto$ is an irreflexive and symmetric independence relation over transitions, where $\longmapsto \, = \, \longrightarrow \dot{\cup} \dashrightarrow$ (disjoint union).
- CC, $\asymp$, commuting square, UP, AS, TR extend to RMLTSI.
- Underlying CTMC obtained by removing actions from transitions (UP and AS hold under CC).

- Need for a condition on rates in addition to CC.
- An RMLTSI meets product preservation along squares (PPS) iff, in all commuting squares t, u, u', t', $\mathit{rate}(t) \cdot \mathit{rate}(u') = \mathit{rate}(u) \cdot \mathit{rate}(t')$ and $\mathit{rate}(\underline{u'}) \cdot \mathit{rate}(\underline{t}) = \mathit{rate}(\underline{t'}) \cdot \mathit{rate}(\underline{u})$.
- TR follows from CC and PPS via the related necessary and sufficient condition based on rate products along cycles.
- If an RMLTSI meets CC and PPS, then for all coinitial and cofinal ω, ω' $\mathit{rateprod}(\omega) = \mathit{rateprod}(\underline{\omega})$ iff $\mathit{rateprod}(\omega') = \mathit{rateprod}(\underline{\omega'})$.
- If an RMLTSI meets CC and PPS, then for all cycles ω we have $\mathit{rateprod}(\omega) = \mathit{rateprod}(\underline{\omega})$.
- If an RMLTSI meets CC and PPS and its underlying CTMC is stationary, then the RMLTSI meets TR too.
- Proof verified with the theorem prover Matita [ARSacerdotiCoen14].

- Applied to a reversible variant of dining philosophers (no deadlock).
- The investigation of commuting squares in the RMLTSI model reveals that CC and PPS hold and hence so does TR.
- The partial balance equations arising from TR yield a product-form expression for π from which we compute total/normalized throughput:



- Impact of the concurrency level on the throughput, which is negative when k is odd but this tends to vanish as k increases.

Generality and Usefulness of the Result

- The reversible Markovian process calculus of [BernardoMezzina23] thus satisfies not only causal reversibility but also time reversibility (conjecture) and its sufficient conditions for TR are superseded.
- Hence the method for reversing process calculi of [PU07] is robust not only w.r.t. causal reversibility, but also w.r.t. time reversibility.
- PPS does not require rates on opposite sides of a commuting square to be equal (this is what we get when executing concurrent actions).
- LTSI has been considered within a classification of concurrency models [SNW96] addressing behavioral vs. system models, interleaving vs. truly concurrent models, or linear-time vs. branching-time models.
- RMLTSI generalizes LTSI in such a way that our result applies to concurrent and distributed models, calculi, and languages that include performance aspects: Petri nets, process algebras, Erlang, . . .

- Concurrent variants of birth-death processes satisfying PPS turn out to be time reversible.
- Proving TR via CC and PPS is simpler than proving TR based on the necessary and sufficient condition on rate products along cycles:
  - The latter requires identifying all the cycles in the RMLTSI model, while the former works with commuting squares.
  - CC can be proven by showing that the RMLTSI meets [LanesePU20]:
    - *Square property*: whenever two coinitial transitions are independent, then there exist other two transitions respectively composable with the previous two such that a commuting square is obtained.
    - *Backward transitions independence*: every two coinitial backward transitions are independent.
    - *Well foundedness*: there are no infinite backward paths.
  - If the sufficient condition above for CC does not apply to the RMLTSI, it can nevertheless provide some diagnostic information, in the form of which of the three properties fails and for which transitions, that could be useful for TR as well.

Future Work

- Adding nondeterminism? Non-Markovian models?
- Are there constraints less demanding than PPS under which causal reversibility still implies time reversibility?
- Opposite direction: can time reversibility imply causal reversibility?
- Not trivial, even considering birth-death processes:
  - There is an order over states such that causality can never be violated when going backward, but a notion of independence over transitions departing from the same state is lacking because every backward transition retracts the premises (causes) for the forward transition.
  - In the case of circular variants of birth-death process (which may not be necessarily time reversible if backward rates are not equal to their corresponding forward rates), the number of performed backward transitions may exceed the number of performed forward transitions, which amounts to a violation of well foundedness exploited to prove causal reversibility in [LanesePU20].