SOME COMMENTARIES ON THE EXERCISES

- ABOUT RANDOM WALKS:

→ THE EXPECTED NUMBER OF STEPS TO TERMINATION, CALL IT N, SATISFIES THE EQUATION $N = p \cdot 1 + (1-p) \cdot 2N$. THEN

$$N = p \cdot 1 + (1-p) \cdot 2 \cdot N \iff p = (1-p) \cdot 2N + N$$
$$\iff p = N(1 - 2 + 2p)$$
$$\iff p = N(2p - 1)$$

THEN p MUST BE $> 1/2$ FOR ALL THIS TO MAKE SENSE.

→ THE PROBABILITY OF CONVERGENCE, CALL IT q, SATISFIES THIS EQUATION WHEN $p = 1/2$.

$$q = \frac{1}{2} + \frac{1}{2} q^2 \iff \frac{1}{2} q^2 - q + \frac{1}{2} = 0$$
$$\iff q^2 - 2q + 1 = 0$$
$$\iff (q-1)^2 = 0$$

SO, THE ONLY POSSIBLE VALUE q CAN HAVE IS 1

- ABOUT THE COUPON COLLECTOR PROBLEM, LET US STUDY THE EXPECTED NUMBER OF STEPS TO TERMINATION, CALL IT T.

$$T = \sum_{i=1}^{n} t_i$$

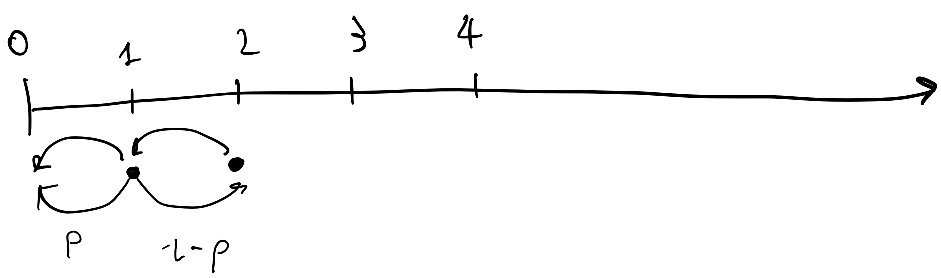
THE PROBABILITY TO GET THE i-th COUPON WHEN YOU HAVE ALL THE PREVIOUS ONES IS

$$p_i = \frac{n-i+1}{n}$$

BASIC RESULTS FROM PROBABILITY THEORY, THE GEOMETRIC DISTRIBUTION TELL YOU THAT $t_i = 1/p_i$. THEN

$$T = n \cdot \underbrace{\left(1 + \frac{1}{2} + \ldots + \frac{1}{n}\right)}_{H_n} = n \cdot H_n$$

$$= \Theta(n \lg n).$$



EXAMPLES OF PCF TERMS

· THEOREM

PCF IS TURING-POWERFUL

PROOF

- THE SET OF COMPUTABLE FUNCTIONS CAN BE CHARACTERIZED AS THE SMALLEST SET PR OF FUNCTIONS $f : \mathbb{N}^k \to \mathbb{N}$ SUCH THAT
  - zero, succ, $\pi^n_i \in PR$
  - PR IS CLOSED BY COMPOSITION, PRIMITIVE RECURSION AND MINIMIZATION

COMPOSITION

$$h(\overline{x}) = f(g_1(\overline{x}), \dots g_n(\overline{x}))$$

PRIMITIVE RECURSION

$$h(0, \overline{x}) = f(\overline{x})$$
$$h(n+1, \overline{x}) = g(n, h(n, \overline{x}), \overline{x})$$

MINIMIZATION

$$h(\overline{x}) = \mu y . f(y, \overline{x}) = 0$$

- LET US ENCODE COMPOSITION AND PRIMITIVE RECURSION IN PCF.

$$\begin{aligned} M V_1, \dots V_n \equiv\ & \text{let } M = x \text{ in} \\ & \text{let } x V_1 = y_1 \text{ in} \\ & \vdots \\ & \text{let } y_{n-1} V_n = y_n \text{ in} \\ & y_n \end{aligned}$$

COMPOSITION

SUPPOSE YOU HAVE TERMS M_f AND $M_{g_1}, \dots M_{g_n}$ EACH OF THEM IMPLEMENTING THE CORRESPONDING FUNCTION. THEN M_h CAN BE THE FOLLOWING TERM

$$\lambda x_1, \dots \lambda x_m . \text{ let } M_{g_1} \overline{x} = y_1 \text{ in}$$
$$\vdots$$

$$\begin{aligned}&\text{let } M_{g_n}\overline{x} = y_n \text{ in}\\ &M_f\,\overline{y}\end{aligned}$$

PRIMITIVE RECURSION

M_h CAN BE OBTAINED FROM M_g AND M_f AS FOLLOWS.

$$\begin{aligned}M_h = \text{fix}\ h.\lambda x_0.\lambda x_1 \ldots \lambda x_k\,.\ &\text{if } x_0 \text{ then } M_f\, x_1 \ldots x_k\\ &\text{else let } \text{pred}(x_0) = z \text{ in}\\ &\qquad \text{let } h\, z\, x_1 \ldots x_k = y \text{ in}\\ &\qquad M_g\, z\, y\, x_1 \ldots x_k\end{aligned}$$

MINIMIZATION

EXERCISE!

EXERCISES

- WRITE A TERM COMPUTING THE MINIMIZATION OF THE FUNCTION f ASSUMING f CAN BE ITSELF COMPUTED IN PCF
- PROVE THE FOLLOWING LEMMA: IF $M \Rightarrow_n \mathcal{D}$, THEN $|\text{SUPP}(\mathcal{D})| < +\infty$
- WRITE A TERM IN $PCF_\oplus$ SIMULATING THE FAIR RANDOM WALK.