Types and effects seen through linear logic

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Outline

1. Lambda-calculus with regions
   - Memory
   - Types and effects

2. Translating into Proof Nets
   - The target
   - The translation

Intro

- Side effects:
  all that is not functional, i.e. not the mere input-output relation.
- e.g. memory, I/O, exceptions, messages, continuations,...
- Memory is challenging for static analysis: behaviour depends on runtime external behaviour.

```
function f(x)
  return x + 1;
```

```
function g(x)
  y := y + 1;
  return x + y;
```

Types and effects

- One approach is types and effects systems.
- One adds an abstract description of side effects to usual types: e.g. $A \xrightarrow{e} B$ types procedures having effects e.
- Memory access: locations are divided in regions (r, s, . . .), effects are sets of regions.
- $A \xrightarrow{\{r_1,\ldots,r_k\}} B$: reads, writes, allocates or frees locations in r_i (finer distinctions possible).
- Analysis can be used to parallelize evaluation, or make safe garbage collection.

J. M. Lucassen and D. K. Gifford.
Polymorphic effect systems.
In *POPL '88*, pages 47–57, New York, NY, USA, 1988. ACM.

A toy language

We will work on Λ_{reg}, a call-by-value calculus with two basic memory access ops (`set` and `get`), where regions *are* locations.

$$\texttt{set}(r, M) \qquad \texttt{get}(r).$$

Roberto M. Amadio.
On stratified regions.
In Zhenjiang Hu, editor, *APLAS*, volume 5904 of *Lecture Notes in Computer Science*, pages 210–225. Springer, 2009.

The syntax of Λ_{reg}

Functions are values:

$$U, V ::= x \mid \langle\rangle \mid \lambda x.M$$

Terms can also be memory ops:

$$M, N ::= V \mid MN \mid \mathtt{set}(r, M) \mid \mathtt{get}(r)$$

Call-by-value order enforced via evaluation contexts:

$$E, F ::= [\,] \mid EM \mid VE \mid \mathtt{set}(r, E)$$

Memory represented by stores:

$$S, T ::= r_1 \Leftarrow V_1, \ldots, r_k \Leftarrow V_k$$

Evaluation

Call-by-value beta-reduction:

$$E[(\lambda x.M)V], S \to E[M\{V/x\}], S$$

Reading from memory:

$$E[\mathtt{get}(r)], r \Leftarrow V, S \to E[V], r \Leftarrow V, S$$

Writing to memory:

$$E[\mathtt{set}(r, V)], r \Leftarrow U, S \to E[\langle\rangle], r \Leftarrow V, S$$

(notice we do not do allocation and garbage collection here)

An example

```
function pow(n, m)
    r := 1;
    for i := 1 to m
        r := n * r;
    return r;
```

$$\begin{aligned}&\mathtt{pow} := \lambda n, m.\\&\quad \mathtt{set}(r, \underline{1});\\&\quad m\\&\qquad (\lambda d.\, \mathtt{set}(r, \mathtt{mult}\; n\; \mathtt{get}(r)))\,\langle\rangle\,;\\&\quad \mathtt{get}(r)\end{aligned}$$

(notation: $M; N := (\lambda d.N)M$)

$$\mathtt{pow}\,\underline{3}\,\underline{2},\ r \Leftarrow \underline{0}$$

$$\to \mathtt{set}(r, \underline{1}); \underline{2}(\lambda d.\, \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r))\,\langle\rangle\,; \mathtt{get}(r),\ r \Leftarrow \underline{0}$$

$$\xrightarrow{*} \underline{2}(\lambda d.\, \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r))\,\langle\rangle\,; \mathtt{get}(r),\ r \Leftarrow \underline{1}$$

$$\xrightarrow{*} \langle\rangle\,; \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r)); \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r)); \mathtt{get}(r),\ r \Leftarrow \underline{1}$$

$$\xrightarrow{*} \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \underline{1}); \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r)); \mathtt{get}(r),\ r \Leftarrow \underline{1}$$

$$\xrightarrow{*} \mathtt{set}(r, \mathtt{mult}\, \underline{3}\, \mathtt{get}(r)); \mathtt{get}(r),\ r \Leftarrow \underline{3}$$

$$\xrightarrow{*} \mathtt{get}(r),\ r \Leftarrow \underline{9} \to \underline{9},\ r \Leftarrow \underline{9}$$

Types and effects

- Types: $A ::= 1 \mid A \xrightarrow{e} B$, e set of accessed regions.
- $R = r_1 : A_1, \ldots, r_k : A_k$ is a region context: r_i contains values of type A_i.
- Typing judgments $R; \Gamma \vdash M : A, e$: means M accesses e.

Typing rules

$$\frac{}{R;\Gamma, x : A \vdash x : A, \emptyset} \qquad \frac{}{R;\Gamma \vdash \langle\rangle : 1, \emptyset}$$

$$\frac{R;\Gamma, x : A \vdash M : B, e}{R : \Gamma \vdash \lambda x.M : A \xrightarrow{e} B, \emptyset} \qquad \frac{R;\Gamma \vdash M : A \xrightarrow{e_3} B, e_1 \qquad R;\Gamma \vdash N : A, e_2}{R : \Gamma \vdash MN : B, e_1 \cup e_2 \cup e_3}$$

$$\frac{R, r : A;\Gamma \vdash M : A, e}{R, r : A;\Gamma \vdash \mathtt{set}(r, M) : 1, e \cup \{r\}} \qquad \frac{}{R, r : A;\Gamma \vdash \mathtt{get}(r) : A, \{r\}}$$

$$\frac{R;\Gamma \vdash M : A, e \qquad e \subsetneq f \subseteq \mathrm{dom}(R)}{R;\Gamma \vdash M : A, f}$$

Typing rules

$$\frac{}{R;\Gamma, x : A \vdash x : A, \emptyset} \qquad \frac{}{R;\Gamma \vdash \langle\rangle : 1, \emptyset}$$

Regular axioms, no effects

$$\frac{R;\Gamma, x : A \vdash M : B, e}{R : \Gamma \vdash \lambda x.M : A \xrightarrow{e} B, \emptyset} \qquad \frac{R;\Gamma \vdash M : A \xrightarrow{e_3} B, e_1 \qquad R;\Gamma \vdash N : A, e_2}{R : \Gamma \vdash MN : B, e_1 \cup e_2 \cup e_3}$$

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Effects annotate arrow type and are reset

$$\frac{R, r : A;\Gamma \vdash M : A, e}{R, r : A;\Gamma \vdash \mathtt{set}(r, M) : 1, e \cup \{r\}} \qquad \frac{}{R, r : A;\Gamma \vdash \mathtt{get}(r) : A, \{r\}}$$

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Effects are merged, annotated ones are "extracted"

$$\frac{R,r:A;\Gamma\vdash M:A,e}{R,r:A;\Gamma\vdash \mathtt{set}(r,M):1,e\cup\{r\}}\qquad\frac{}{R,r:A;\Gamma\vdash\mathtt{get}(r):A,\{r\}}$$

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$$\frac{R, r : A; \Gamma \vdash M : A, e}{R, r : A; \Gamma \vdash \mathtt{set}(r, M) : 1, e \cup \{r\}} \qquad \frac{}{R, r : A; \Gamma \vdash \mathtt{get}(r) : A, \{r\}}$$

Accessed regions are noted

$$\frac{R; \Gamma \vdash M : A, e \qquad e \subsetneq f \subseteq \mathrm{dom}(R)}{R; \Gamma \vdash M : A, f}$$

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Dummy effects can be added

$$\frac{R;\Gamma \vdash M : A, e \qquad e \subsetneq f \subseteq \mathrm{dom}(R)}{R;\Gamma \vdash M : A, f}$$

Typed fixpoints

$$r : 1 \xrightarrow{\{r\}} A; \vdash Y = \lambda f.\, \mathtt{set}(r \Leftarrow \lambda x.f(\mathtt{get}(r)x)); \mathtt{get}(r)\,\langle\rangle \; : (A \to A) \to A$$

$$\begin{aligned} YF, r \Leftarrow I &\to \mathtt{set}(r, \lambda x.F(\mathtt{get}(r)x); \mathtt{get}(r)\,\langle\rangle\,, r \Leftarrow I \\ &\to \mathtt{get}(r)\,\langle\rangle\,, r \Leftarrow \lambda x.F(\mathtt{get}(r)x) \\ &\to (\lambda x.F(\mathtt{get}(r)x))\,\langle\rangle\,, r \Leftarrow \lambda x.F(\mathtt{get}(r)x) \\ &\to F(\mathtt{get}(r)\,\langle\rangle), r \Leftarrow \lambda x.F(\mathtt{get}(r)x) \end{aligned}$$

- In particular, $Y(\lambda z.z)$ loops.
- Typing avoids self-application, but not self-reference.

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Stratification

- Intuition: stratify regions, so that "lower" regions cannot reference "higher" ones (in particular themselves).

$$\frac{}{\emptyset \vdash} \qquad \frac{R \vdash A \quad r \notin \mathrm{dom}(R)}{R, r : A \vdash}$$

$$\frac{R \vdash}{R \vdash 1} \qquad \frac{R \vdash A \quad R \vdash B \quad e \subseteq \mathrm{dom}(R)}{R \vdash A \xrightarrow{e} B}$$

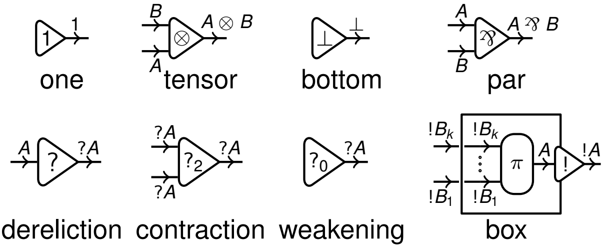
Gérard Boudol.
Fair cooperative multithreading.
In *CONCUR*, volume 4703 of *LNCS*, pages 272–286. Springer, 2007.

Roberto M. Amadio.
On stratified regions.
In *APLAS*, volume 5904 of *LNCS*, pages 210–225. Springer, 2009.

- For example: ~~$r : 1 \xrightarrow{\{r\}} A \vdash$~~ as $1 \xrightarrow{\{r\}} A$ not definable from $\emptyset$.
- $R \vdash$ and $R; \vdash M : A, \emptyset \implies M$ terminates.

The target

- Proof nets are the parallel representation of linear logic proofs.
- Types: $X \mid X^{\perp} \mid 1 \mid \perp \mid A \otimes B \mid A \parr B \mid !A \mid ?A$ with duality $A^{\perp}$, linear arrow $A \multimap B = A^{\perp} \parr B$, systems of equations $X_i \doteq A_i$.
- Cells:



- Proof nets formed matching wires and enforcing a correctness criterion.

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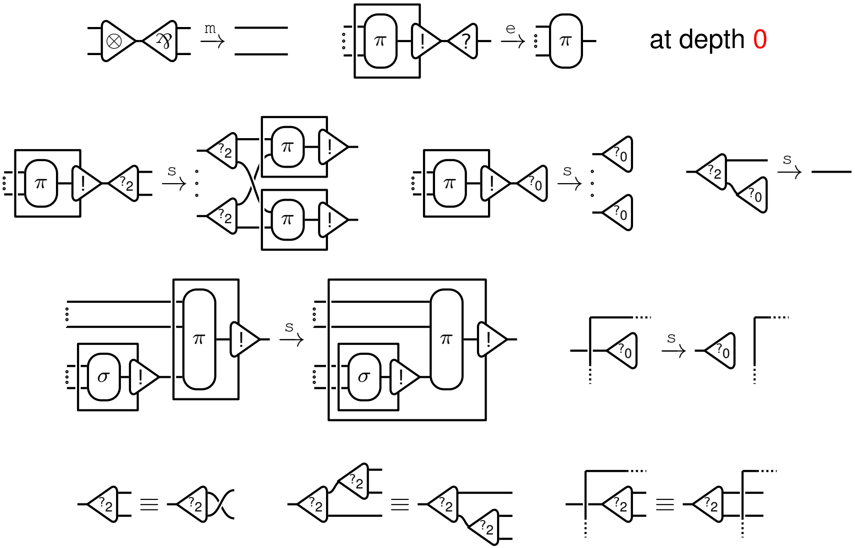
Surface reduction



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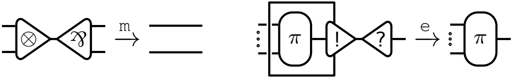
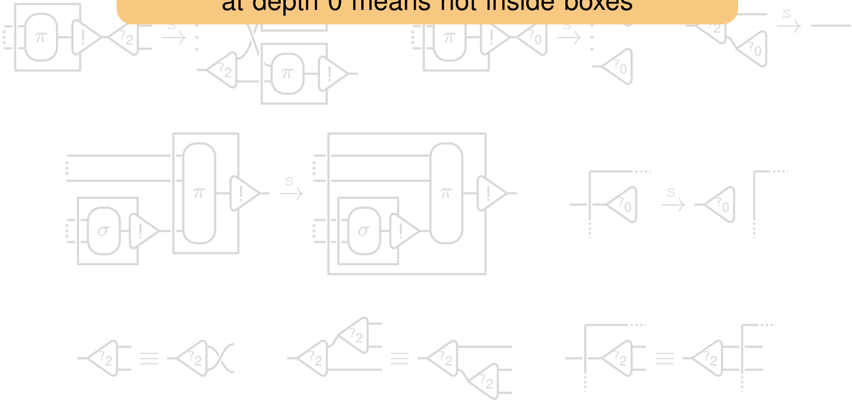


at depth 0

logical reductions (multiplicative and exponential)
at depth 0 means not inside boxes



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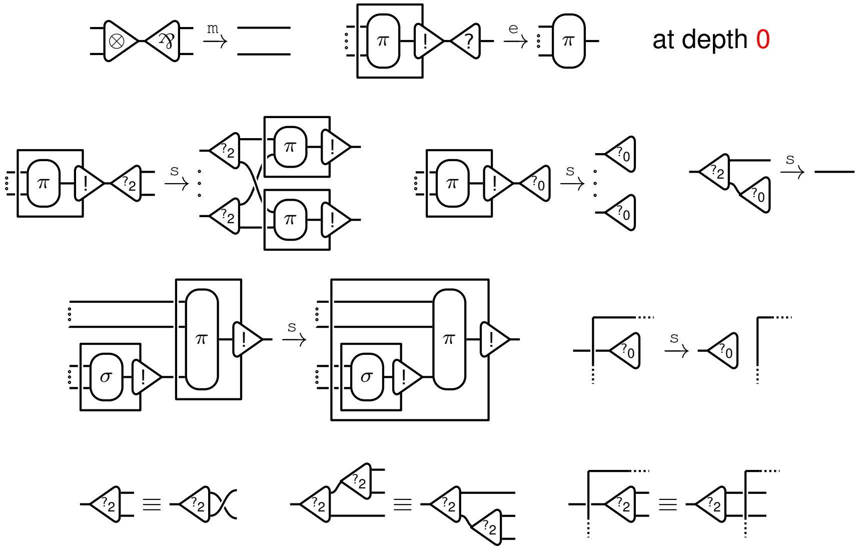

The results

We present a translation $(M, S)^\bullet$ from typed Λ_{reg} programs to resursively typed proof nets, generalizing Girard's call-by-value translation (i.e. $(A \to B)^\bullet = !(A^\bullet \multimap B^\bullet)$).

Theorem

If $M, S \to N, T$ then $(M, S)^\bullet \stackrel{\mathtt{e}}{\to}\stackrel{\mathtt{m*}}{\longrightarrow}\stackrel{\mathtt{s*}}{\longrightarrow} (N, T)^\bullet$.

Theorem

$(M, S)^\bullet$ normalizes by surface reduction to π iff $\pi = (V, T)^\bullet$ and $M, S \stackrel{*}{\to} V, T$.

Recursive equations come from a translation of region contexts $R^\bullet$.

Theorem

R is stratified iff $R^\bullet$ is solvable (i.e. no real recursive types!)

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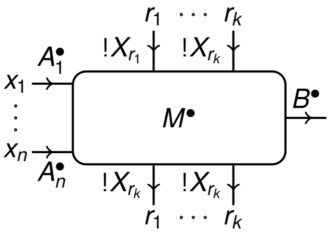
General form of the translation

- $R; x_1 : A_1, \ldots, x_n : A_n \vdash M : B, \{r_1, \ldots, r_k\}$ gets translated to a net

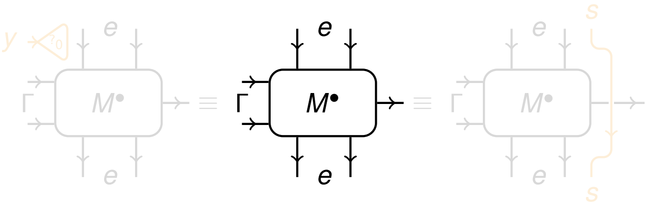


(we will show the translation of types and effects along the way)

- It is useful to visualize programs as processing streams of regions going top to bottom.

Dummy variables and dummy effects

We consider translations up to dummy variables and dummy effects.



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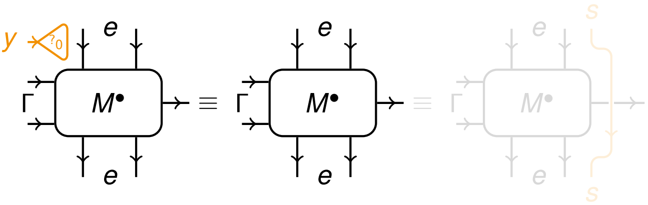

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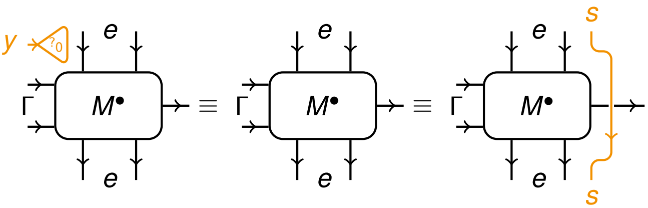
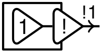


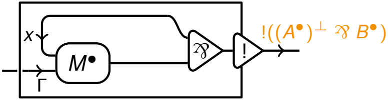
The translation: variable and unit

$x^\bullet =$ 

$\langle\rangle^\bullet =$ 

Types: $1^\bullet = !1$.

The translation: abstraction

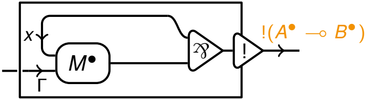
$(\lambda x.M)^\bullet =$ 

Usual call-by-value translation extended by encapsulating the effects.

Types: $e^\bullet = \bigotimes_{r \in e} !X_r, \quad (A \xrightarrow{e} B)^\bullet = !(A^\bullet \multimap e^\bullet \multimap (e^\bullet \otimes B^\bullet))$.

(anybody sees anything familiar? Some tidbits on state monads later...)

The translation: abstraction

$(\lambda x.M)^\bullet =$ 

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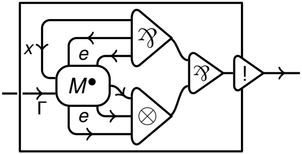
$(\lambda x.M)^\bullet =$ 

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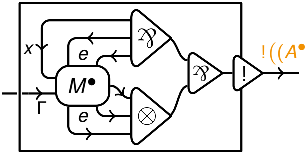
(anybody sees anything familiar? Some tidbits on state monads later...)

The translation: abstraction

$(\lambda x.M)^\bullet =$



$!((A^\bullet)^\perp \parr \bigparr_{r\in e} ?X_r^\perp \parr (\bigotimes_{r\in e} !X_r \otimes B^\bullet))$

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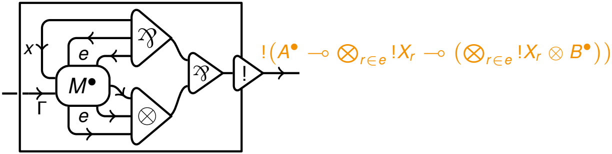
$(\lambda x.M)^\bullet =$ 

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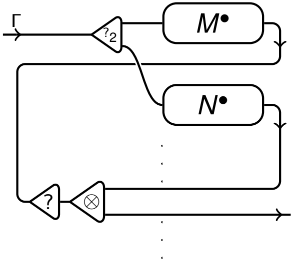
The translation: application

Suppose $M : A \to B, \emptyset$ and $N : A, \emptyset$.

$(MN)^{\bullet} =$

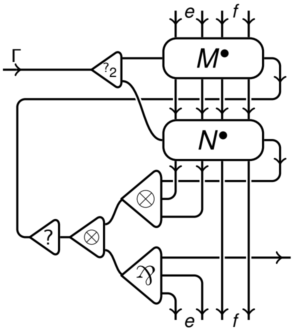


Usual translation extended by extracting effects and linking in evaluation order.

The translation: application

Suppose $M : A \xrightarrow{e} B, e + f$ and $N : A, e + f$.

$(MN)^\bullet =$



Usual translation extended by extracting effects and linking in evaluation order.

The translation of memory operations: get(r) and set(r, M).

$(\texttt{get}(r))^\bullet =$

$(\texttt{set}(r, M))^\bullet =$

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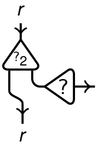
$(\texttt{get}(r))^\bullet =$ 

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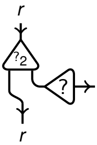
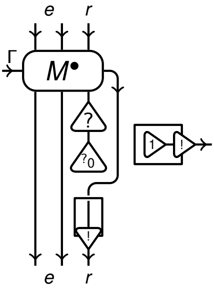
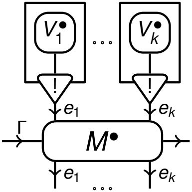
$(\texttt{get}(r))^\bullet =$ 

$(\texttt{set}(r, M))^\bullet =$ 

The translation: stores

$$(M, r_1 \Leftarrow V_1, \ldots, r_k \Leftarrow V_k)^\bullet =$$



(for proofnet geeks like me:
adding garbage collection by weakenings $\rightsquigarrow$ switching connectedness)

The translation: summing up

- Sets of regions: $e^\bullet = \bigotimes_{r \in e} !X_r$.
- Types: $1^\bullet = !1 \qquad (A \xrightarrow{e} B)^\bullet = !(A^\bullet \multimap e^\bullet \multimap (e^\bullet \otimes B^\bullet))$
  (we consider $(A \xrightarrow{\emptyset} B)^\bullet = !(A^\bullet \multimap B^\bullet))$
- Region contexts: $(r_1 : A_1, \ldots, r_k : A_k)^\bullet = (X_{r_1} \doteq A_1^\bullet, \ldots, X_{r_k} \doteq A_k^\bullet)$.

Theorem

R is stratified iff $R^\bullet$ is solvable (i.e. $(M, S)^\bullet$ simply typed!).

Theorem

If $M, S \to N, T$ then $(M, S)^\bullet \xrightarrow{\mathtt{e}}\xrightarrow{\mathtt{m*}}\xrightarrow{\mathtt{s*}} (N, T)^\bullet$.

Theorem

$(M, S)^\bullet$ normalizes by surface reduction to π iff $\pi = (V, T)^\bullet$ and $M, S \xrightarrow{*} V, T$.

Proof nets as parallel evaluators

- Proof nets instantiate as connections the dependencies described by effects.
- E.g. $M : A, \{s\}$, $N : B, \{r\}$, and $\mathtt{set}(r, V); M; N$. After unfolding the seq. composition. . .
- N can be safely evaluated before or at the same time of M.
- The third result

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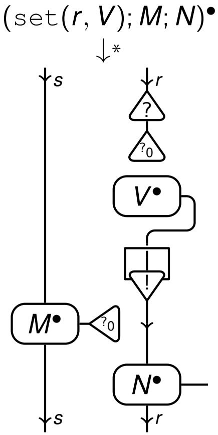
ensures sequential semantics is preserved.



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Theorem

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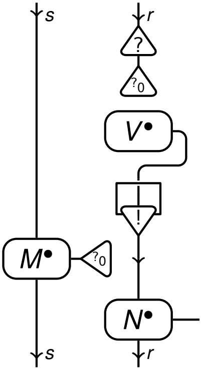
$(\mathtt{set}(r, V); M; N)^\bullet$

$\downarrow^*$



What was left out of this talk

- In fact the translation can be carried out completely in λ-calculus, using localized state monads $T_e A = S_e \to (S_e \times A)$ with $S_e = \prod_{r \in e} X_r$ and corresponding return and bind operators. (though the graphical parallel evaluation intuition is lost)
- Multithreading and differential nets (from state passing style to lock passing style)
- Allocation/garbage collection operations (stuff like $\nu r \Leftarrow V.M$)

Thanks!

Questions?