A Characterization of Hypercoherent Correctness in MALL

Paolo Tranquilli

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Computer Science Logic 2008 – 17/09/2008

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* Multiplicative Additive Linear Logic

Outline

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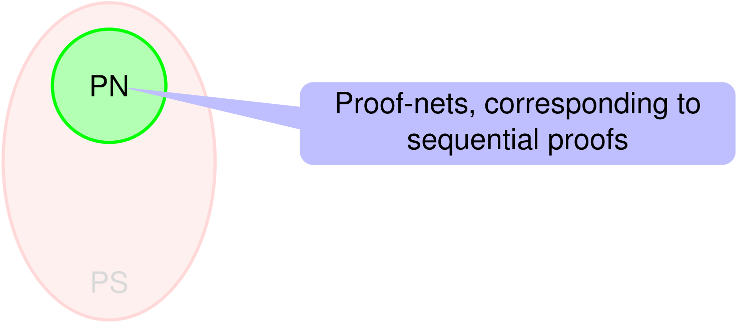
The framework

- Linear Logic (Girard, 1987) has always shown a persistent tendency to link with computer science. Its very roots are in the Curry-Howard isomorphism.
- Denotational semantics: giving mathematical invariants for the execution of programs (and cut-elimination of proofs).
- Proof-nets: the desequentialized representation of proofs of LL.
- We here work with the truly linear fragment of LL (no structural rules, i.e. no erasing or duplicating).

MLL is robust

- The multiplicative fragment (without units) works like a charm.
- There is a robust pairing between syntax – proof-nets – and its main denotational semantics – coherent spaces.
- Coherent spaces: sets with a symmetric reflexive relation, the coherence (i.e. graphs). The states of the spaces are its cliques.
- Coherent spaces validate the MIX rule, which correspond to unconnected proof-nets.
- From now on, we will regard only cut-free proofs and structures (typical of semantical investigations).

The picture



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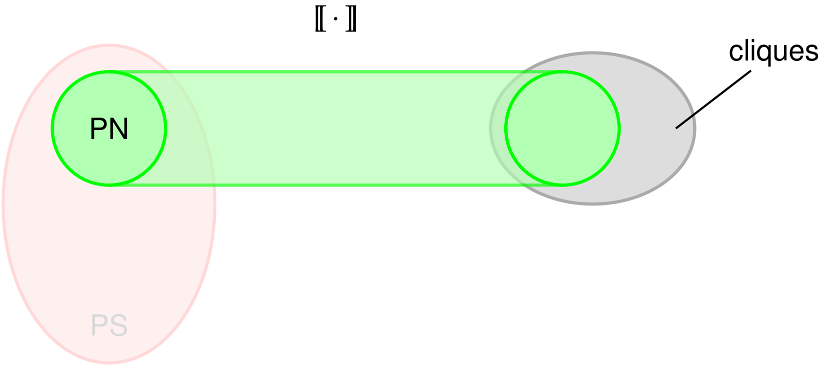

Sequents as syntactical forests

$$\Gamma = \quad A \parr B, \quad (B^\perp \otimes C) \parr (C^\perp \parr A^\perp)$$

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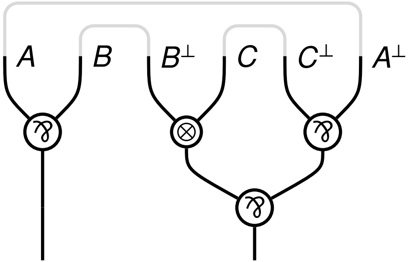

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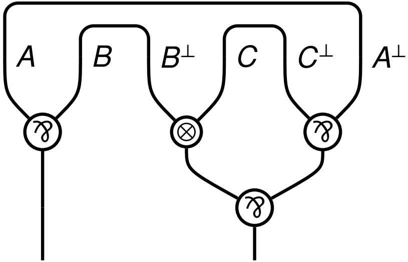
Proof-nets as linkings



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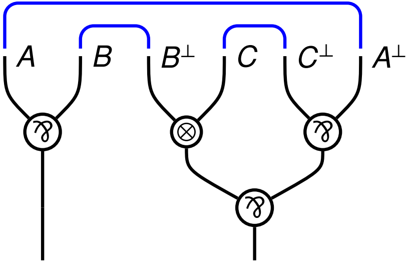

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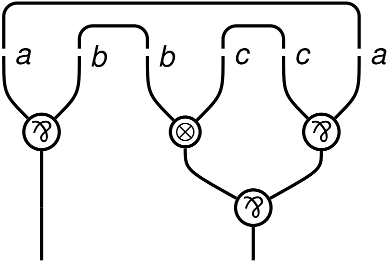
Experiments



$e(\pi) = \big((a, b), \quad ((b, c), (c, a))\big)$

$a \in |[\![A]\!]|, \quad b \in |[\![B]\!]|, \quad c \in |[\![C]\!]| \quad \Rightarrow \quad e(\pi) \in |[\![\Gamma]\!]|$

$[\![\pi]\!] := \{\, e(\pi) \mid e \text{ experiment on } \pi \,\}$

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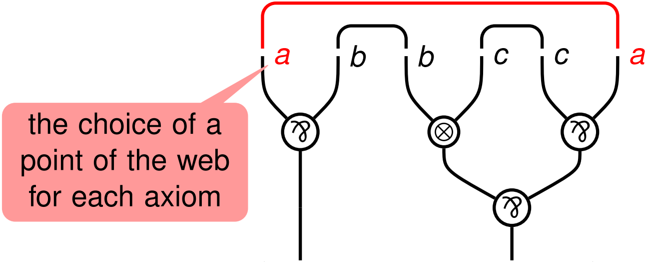

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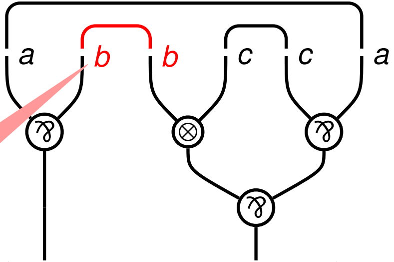

the choice of a point of the web for each axiom

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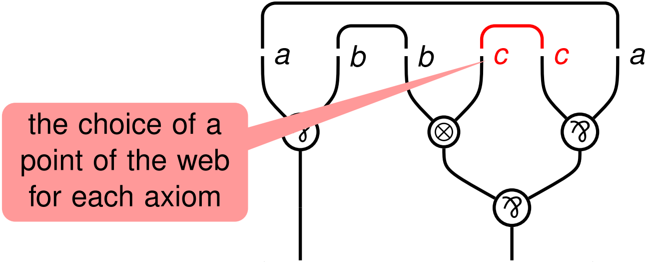

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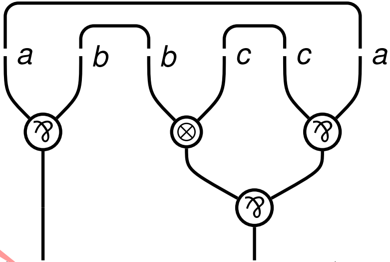

which gives a re-sult, collected at the conclusions

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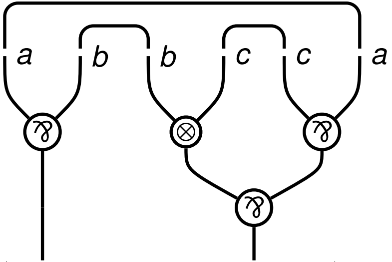

the set of results gives the interpretation

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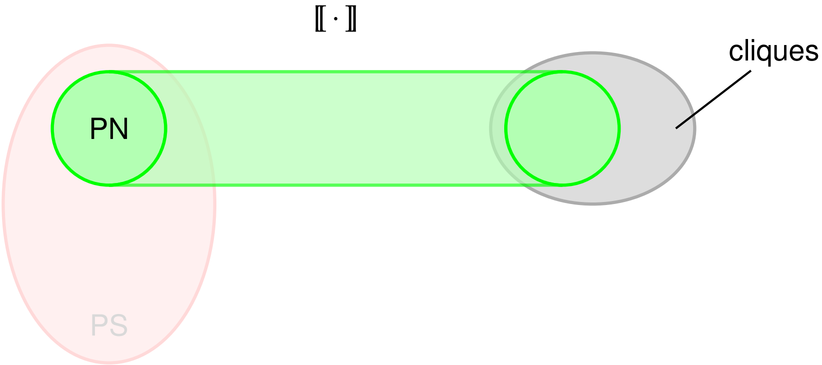

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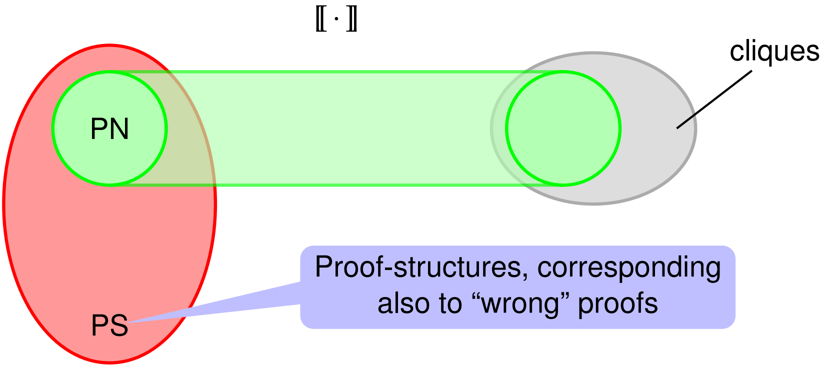
$[\![\cdot]\!]$
cliques
PN
PS
Proof-structures, corresponding also to “wrong” proofs

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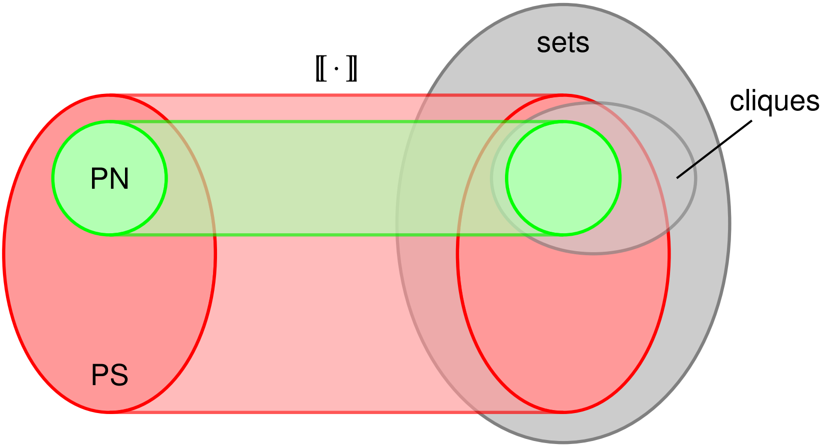

The importance of allowing mistakes

- Proof correctness is established via a "geometric" sequentializability criterion (ex: long trip, Girard 1987, or switching acyclicity and connectedness, Danos & Regnier 1989).
- "Making mistakes" $\Longrightarrow$ richer syntax, better understanding of what "doing right" really means.
- It also allows to consider different ways of "doing right".

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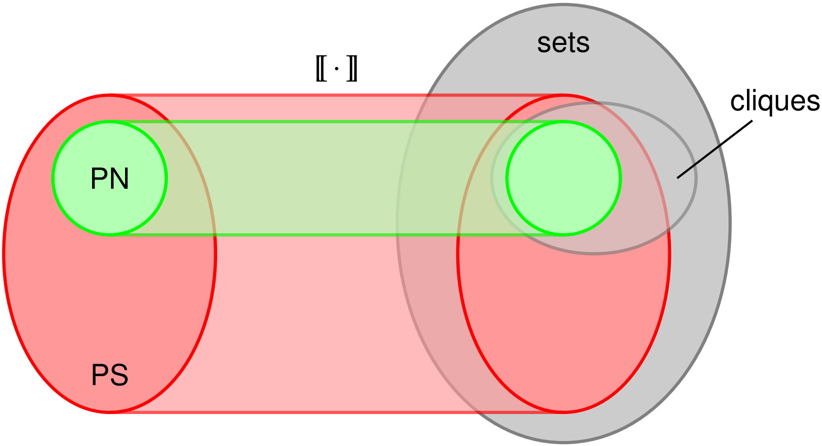

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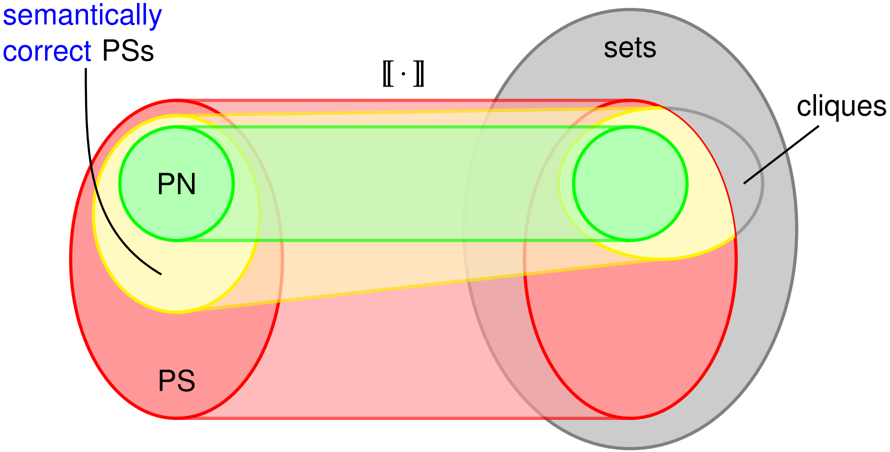

Semantic correctness

As proof-structures $\xmapsto{[\![\cdot]\!]}$ sets, it makes sense to define:

π semantically correct $\iff \forall [\![\cdot]\!] : [\![\pi]\!]$ is a clique.

The fact that proof-nets $\xmapsto{[\![\cdot]\!]}$ cliques is reworded as

Theorem (Girard 1987)

π correct $\Rightarrow [\![\pi]\!]$ semantically correct.

In MLL also the reverse hold!

Theorem (Rétoré 1997)

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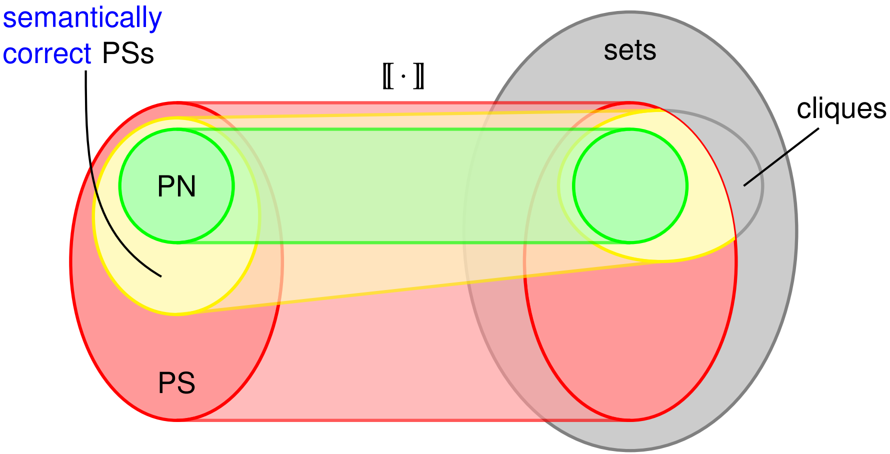

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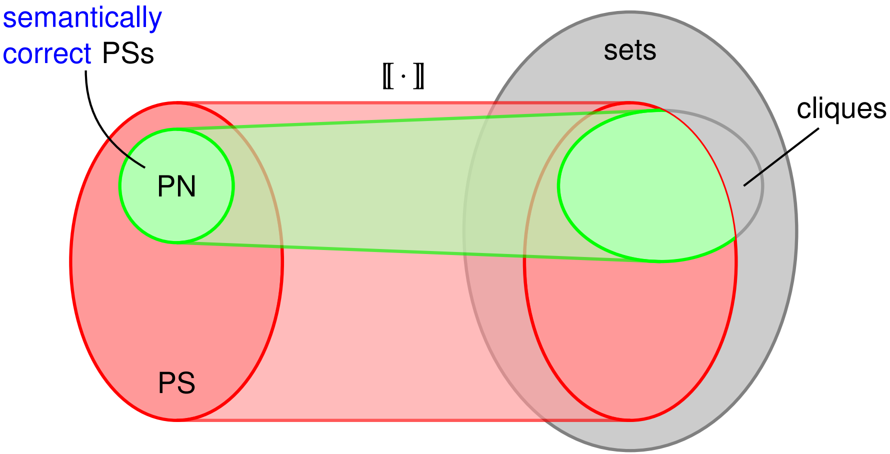
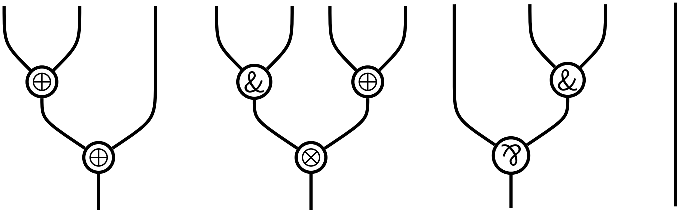


Outline

Hughes – van Glabbeek's proof-structures



$(A \oplus B) \oplus A, \quad (A^{\perp} \,\&\, B^{\perp}) \otimes (C \oplus C), \quad C^{\perp} ⅋ (A \,\&\, A), \quad A^{\perp}$

Slices are MLL proof-structures with unary additives

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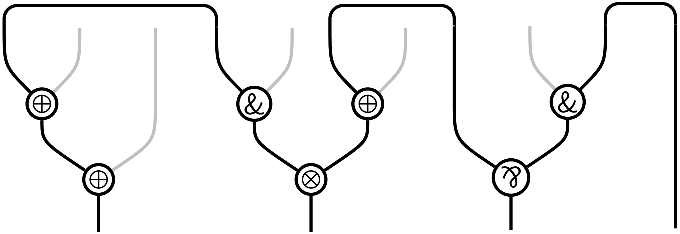

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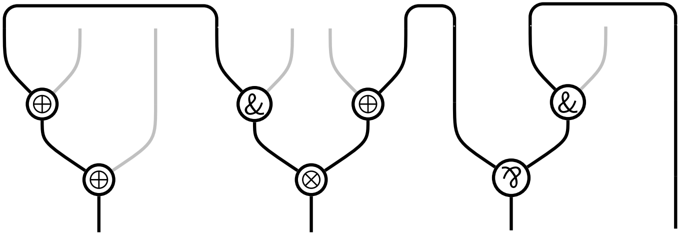

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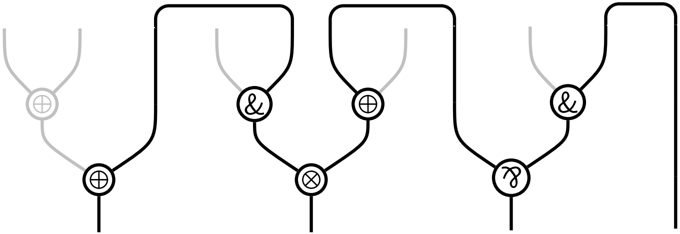

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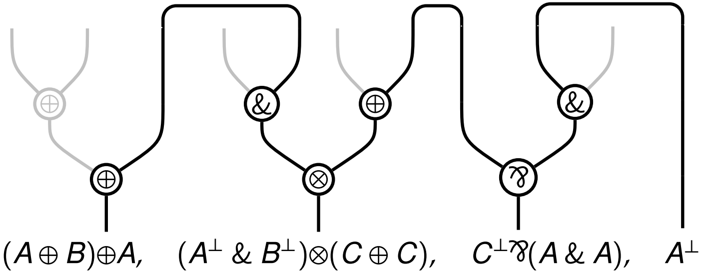

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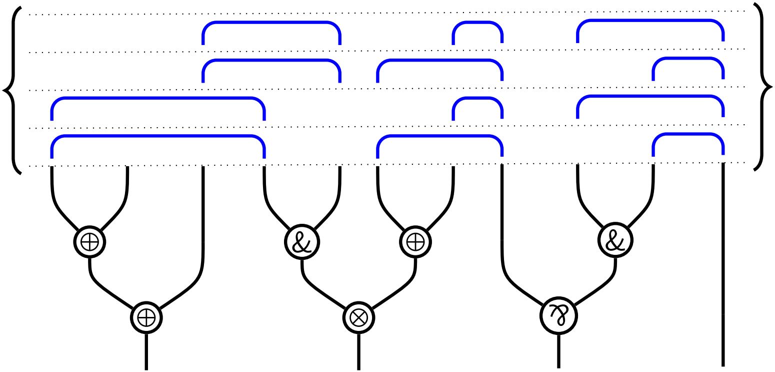

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Proof structures are sets of slices (or equivalently, linkings.)

We can superimpose slices...

...and register additive dependancies via jumps

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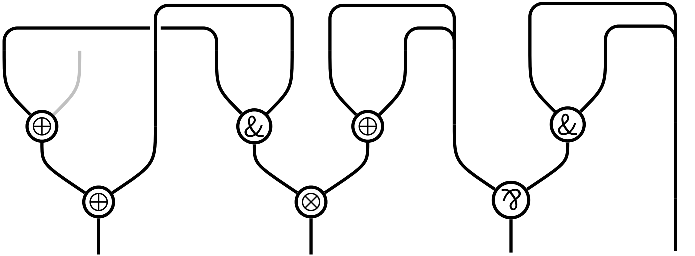

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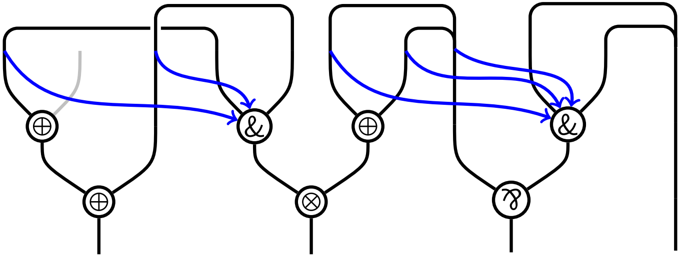

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Hughes – van Glabbeek's proof-nets

From HvG 2003: a set θ of linkings is a PS if

&-compatibility and fullness (or *resolution*)

Every choice on the &s has a unique $\lambda \in \theta$ agreeing with it

A PS θ is correct (i.e. a PN) iff

MLL correctness

Every $\lambda \in \theta$ is switching acyclic and conncted

Toggling

$\forall \Lambda \subseteq \theta : \exists w \in \&2(\mathcal{G}_\Lambda)$ out of all switching cycles in $\mathcal{G}_\Lambda$

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To

Without connectedness PNs sequentialize in MALL+MIX

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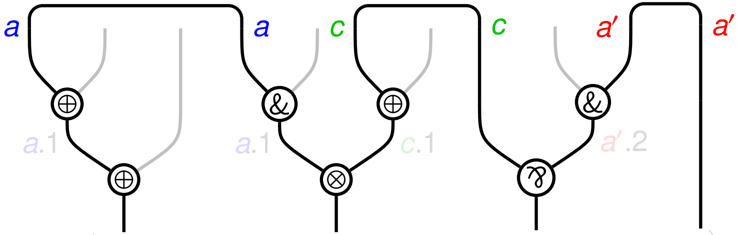
Experiments

Multiplicative experiments extend to slices and proof-structures



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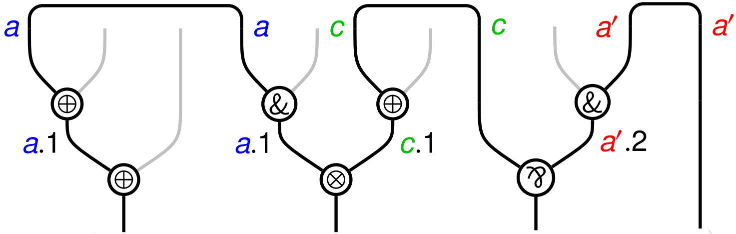

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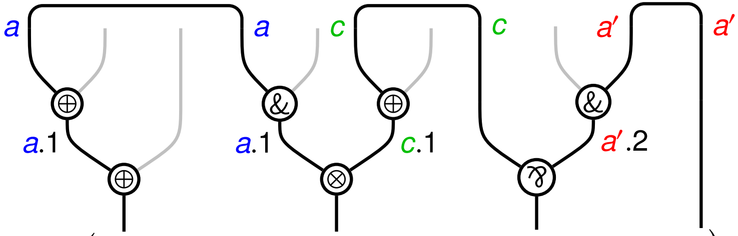

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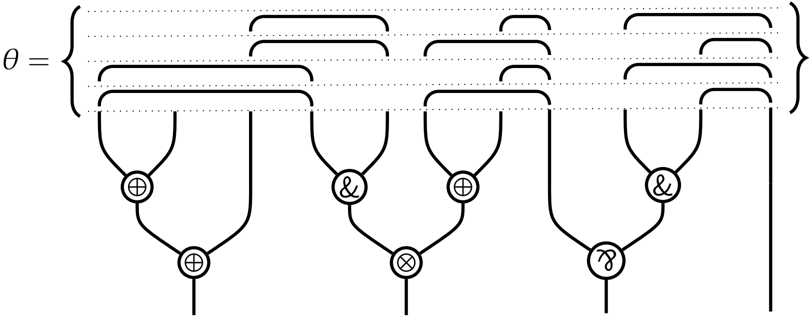


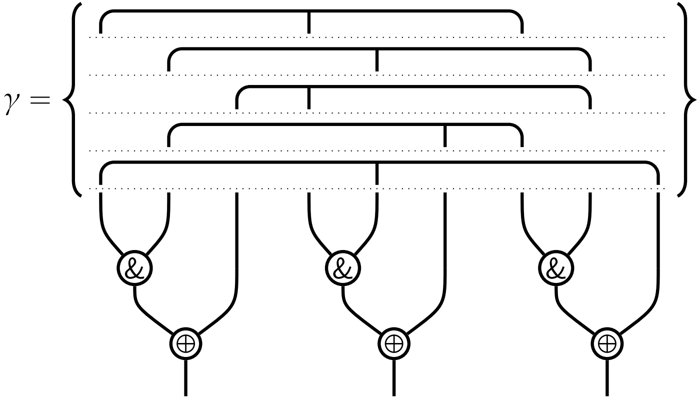
$$[\![\theta]\!] := \bigcup_{\lambda \in \theta} [\![\lambda]\!]$$

Additive proof-structure and coherent spaces

- Though θ correct $\Rightarrow [\![\theta]\!]$ is a clique, the inverse is far from true.
- The most famous counterexample is the Gustave proof-structure.
- It is the counterpart of the unsequentializable function in the stable model of PCF.

$$G(\mathtt{t}, \mathtt{f}, \bot) := \mathtt{t}$$
$$G(\mathtt{f}, \bot, \mathtt{t}) := \mathtt{t}$$
$$G(\bot, \mathtt{t}, \mathtt{f}) := \mathtt{t}$$

The Gustave proof-structure



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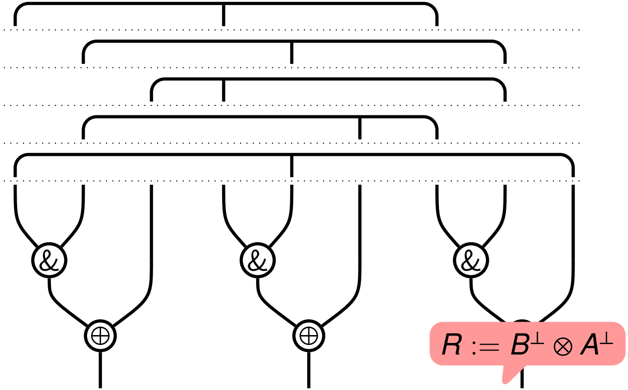

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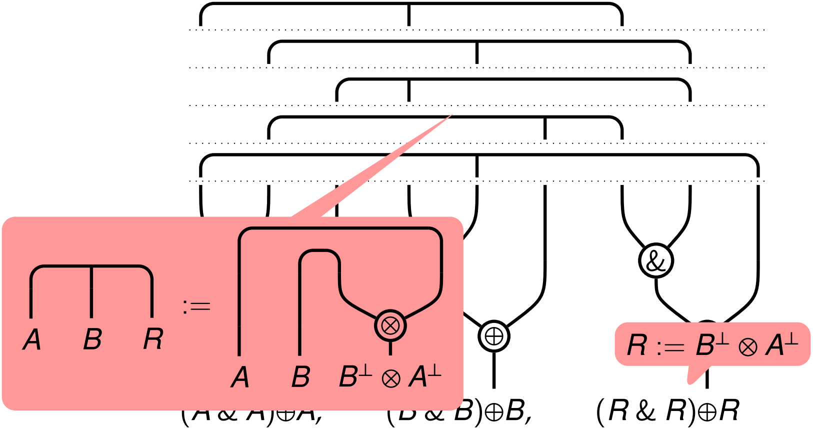

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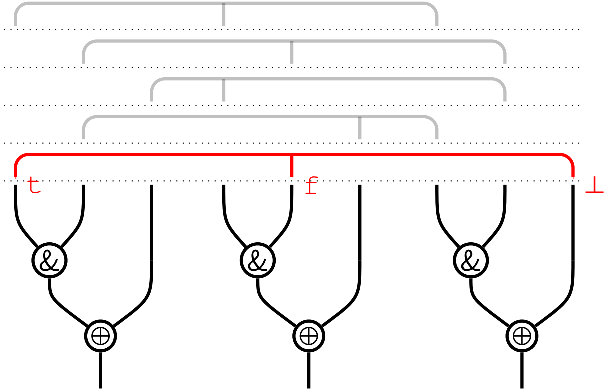

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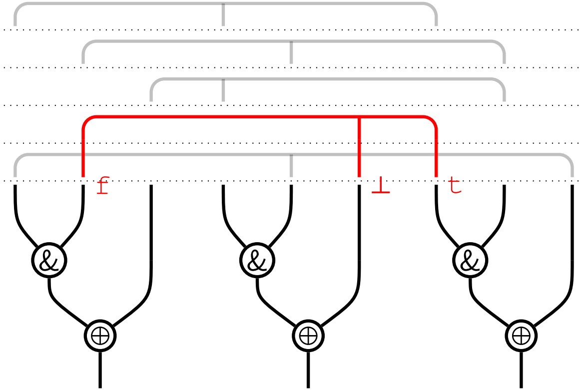

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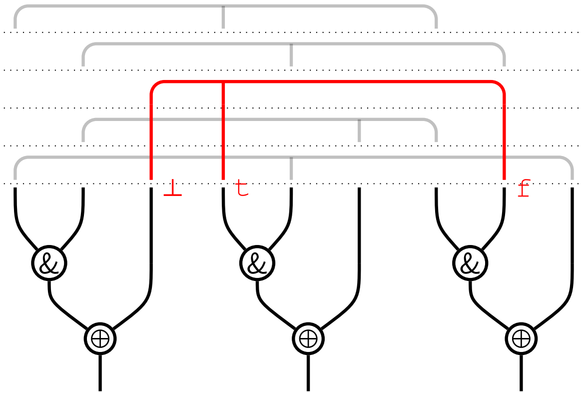

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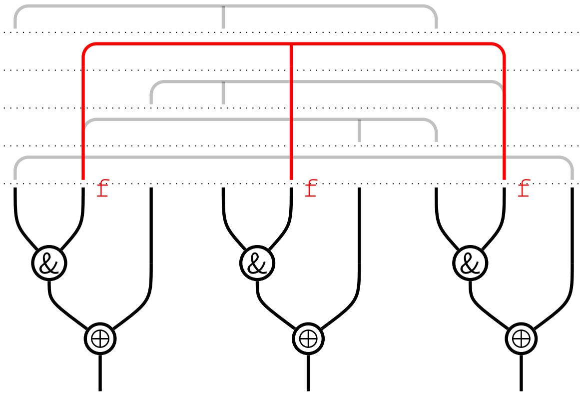

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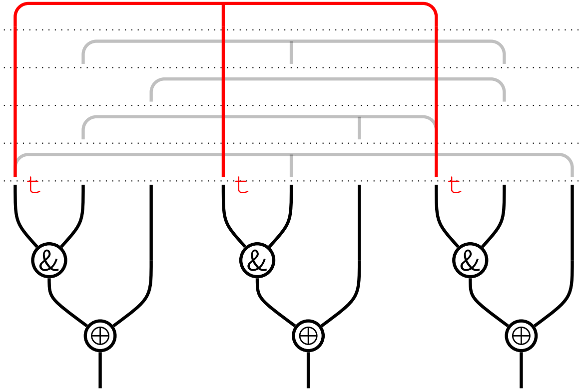

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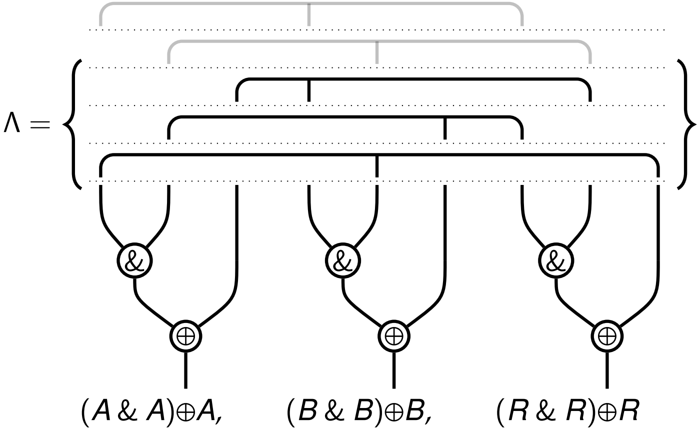

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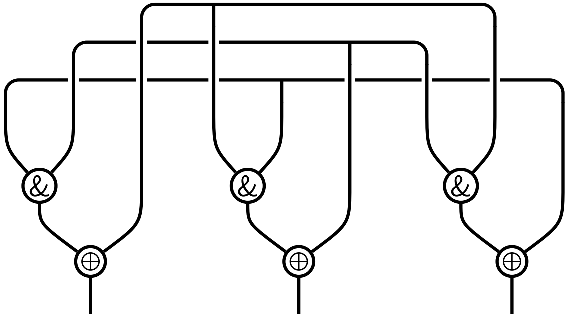

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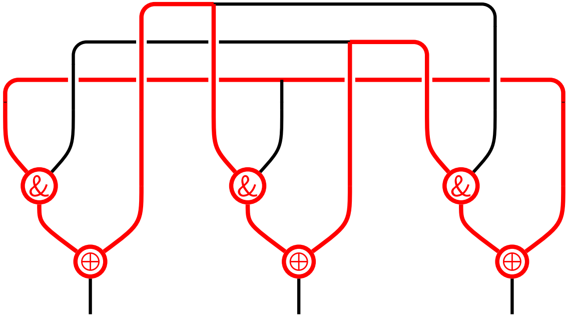

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Outline

Hypercoherent spaces

- Coherent spaces:
  $(|X|, \coh)$, with $\coh \subseteq |X| \times |X|$ a binary relation
- Hypercoherent spaces (Ehrhard 1995):
  $(|X|, \coh)$ with $\coh \subseteq \mathcal{P}_{\text{fin}}(|X|)$ a predicate on finite sets
- Additives: and
- The strongly stable model of hypercoherent spaces (Bucciarelli & Ehrhard 1991) rejects Guastave's function, and correspond to sequentializable functions, maybe it can help with MALL?

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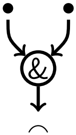
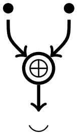
- Additives: 

 and 

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Hypercoherent semantic correctness

Again we can define

θ semantically correct $\iff \forall [\![\cdot]\!] : [\![\theta]\!]$ is a hyperclique.

and again

Theorem

θ correct $\Rightarrow [\![\theta]\!]$ semantically correct.

In MALL the reverse does not hold! (for HvG PS: Pagani 2006)

~~**Theorem ("Retoré")**~~

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In MA... (for HvG PS: Pagani 2006)

Theo...

π inc... rrect.

We give a direct proof of this, rather than passing via sequentialization, more on this later

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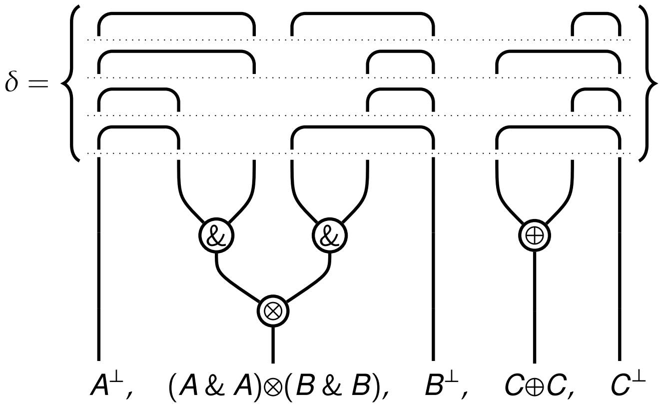

Taking δ, superimposing, adding jumps, we get a (bad) cycle.
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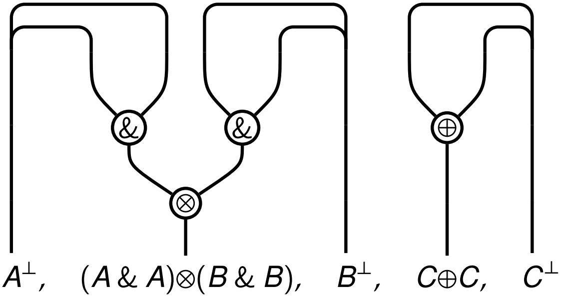

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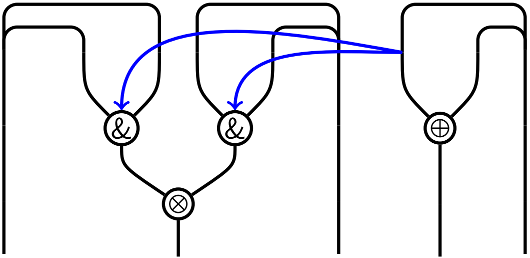

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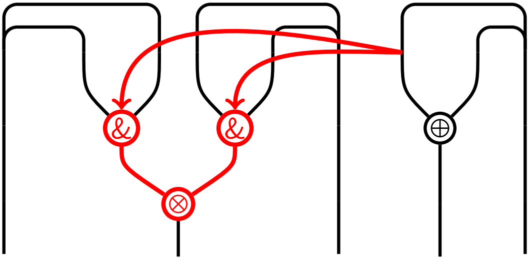

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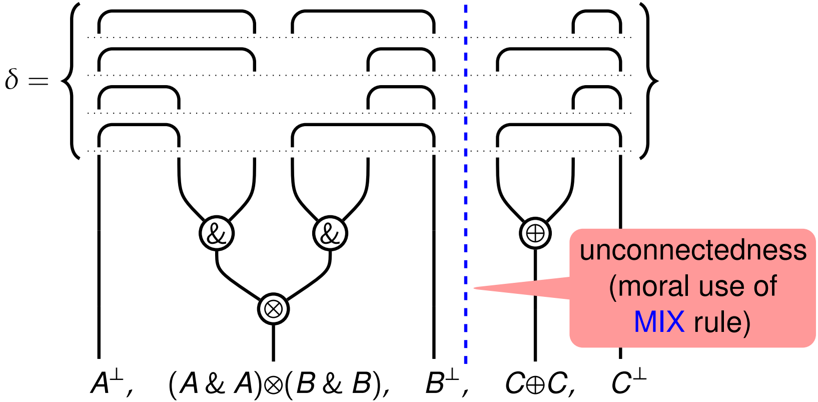

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The conjecture and its factorization

Conjecture (Pagani 2006)

For θ proof-structure with every slice switching connected

$[\![\theta]\!]$ *semantically correct* $\Rightarrow$ θ *correct*

We have “factorized” the conjecture by finding the geometric criterion for semantic correctness, that we call hypercorrectness (definition in the next slides).

Theorem

θ *hypercorrect* $\Leftrightarrow$ θ *semantically correct.*

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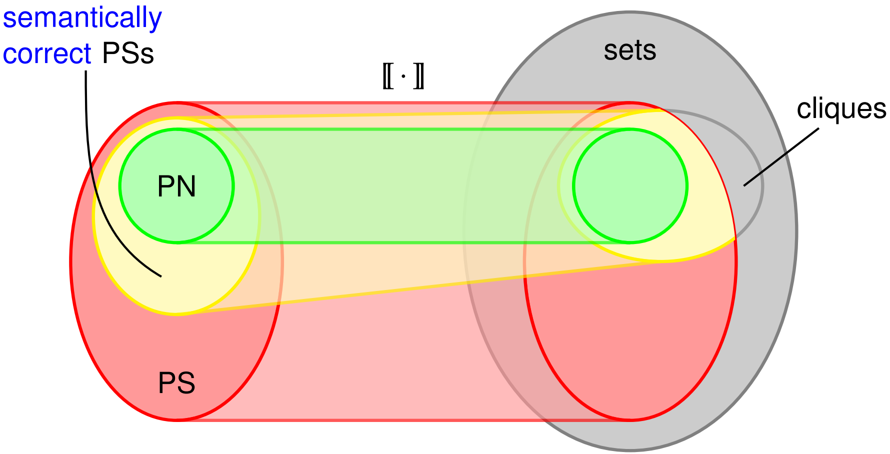
So now we can try to prove it all inside graphs

Theorem

θ hypercorrect $\Leftrightarrow$ θ semantically correct.

The picture



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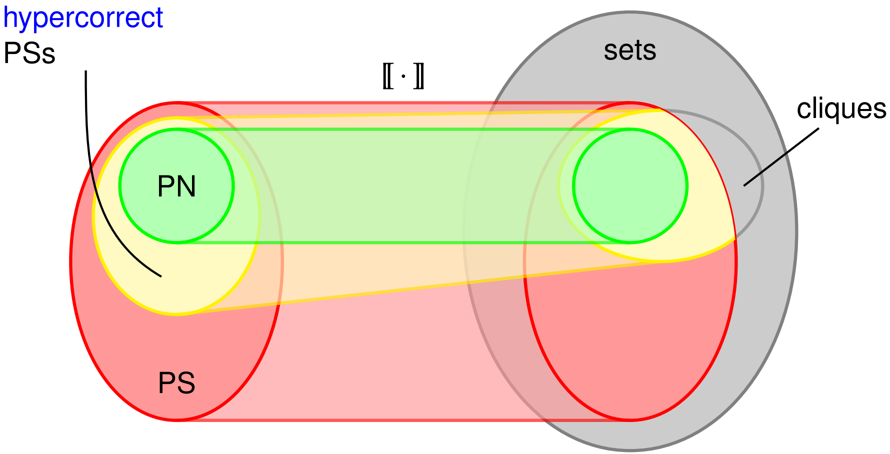
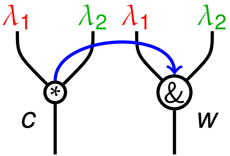


Outline

Orientating the cycles

- The idea is consider switching oriented paths.
- Other works (Abramski & Mèllies 1999, Blute, Hamano & Scott 2005) suggest semantics "sees" cycles with jumps oriented in the same sense.
- For a technical reason we change the definition of jumps.

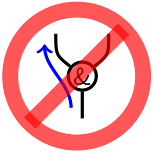


in $\mathcal{G}_\Lambda$

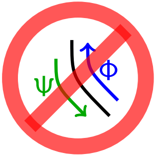
where $\lambda_1, \lambda_2 \in \Lambda$, c is a $\oplus$ or an atomic leaf (an additive contraction), and w is the only with binary for λ_1 and λ_2.

&-oriented paths

- An oriented switching path Φ is &-oriented if binary &s in it are traversed from premise to conclusion (in particular all jumps are traversed in the same direction)



- Φ and Ψ oriented switching paths on $\mathcal{G}_\Lambda$ are compatible if every time they traverse the same edge, they do so in the same direction. A union of paths is compatible if they are pairwise so.



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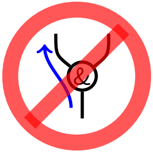

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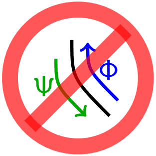

In the paper there is a dual condition on contractions, which however can be dropped

The criterion

A proof-structure θ is a proof-net if

Toggling

$\forall \Lambda \subseteq \theta : \forall S \neq \emptyset$ union of switching cycles in $\mathcal{G}_\Lambda$

$\exists w \in \&2(\mathcal{G}_\Lambda) : w \notin S$

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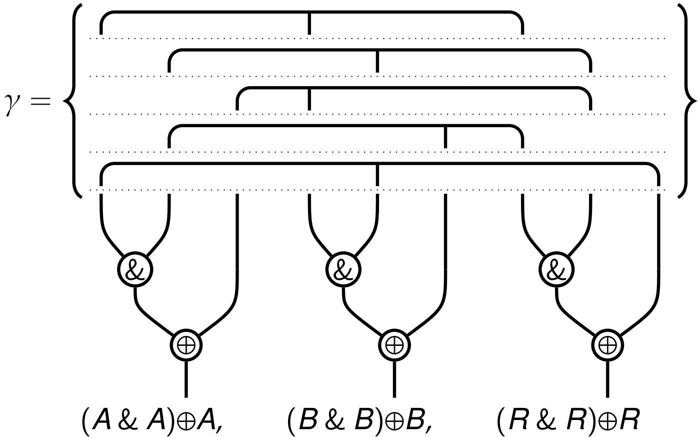
The Gustave PS revisited



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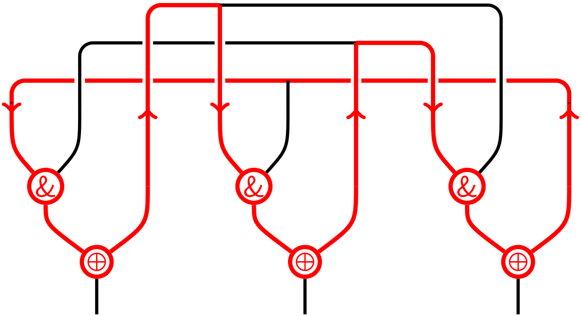

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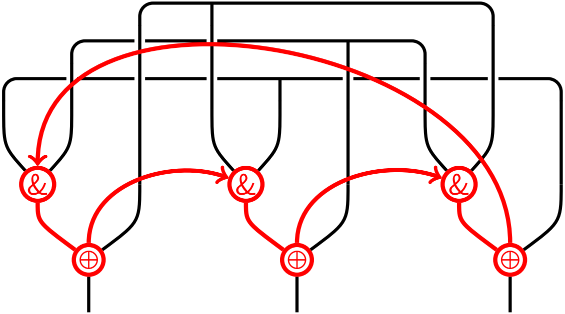

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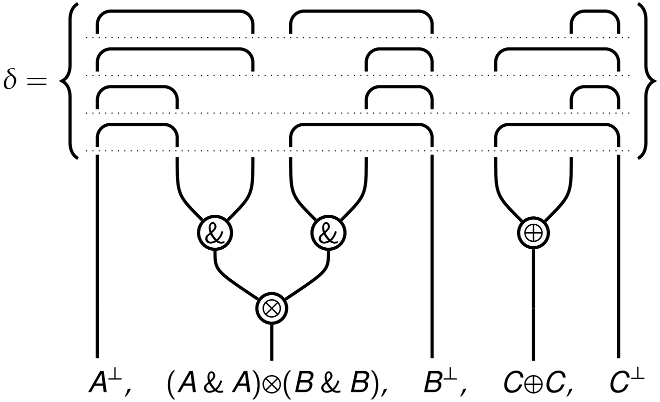

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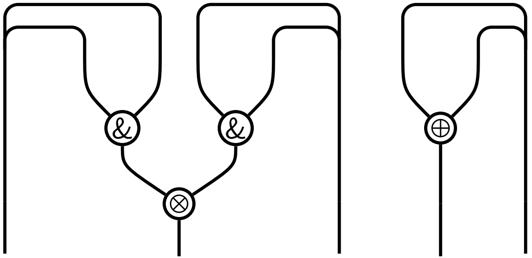

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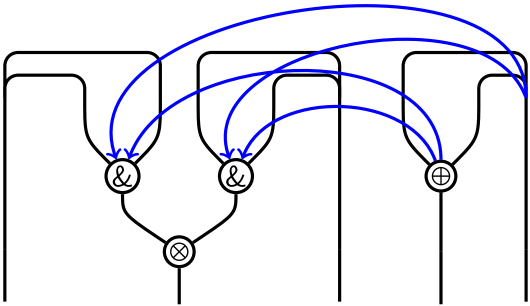

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Future work

1. Is the second part of the conjecture true?
   For θ sw. connected proof structure, θ hypercorrect iff correct?
   There is evidence (AM 1999, BHS 2005)
2. Employ the new jumps for a more general syntax (no η-expansion, exponentials)
3. Has the criterion significance for cut reduction?
   Probably, semantics usually lift to good properties. A very good recent example is Pagani 2006 and his current work on differential interaction nets (visible acyclicity corresponding to fintary relations)

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Thank you.

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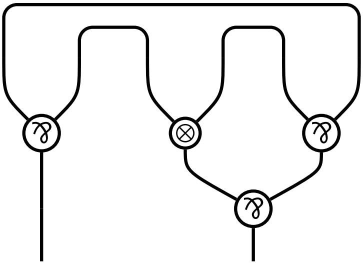

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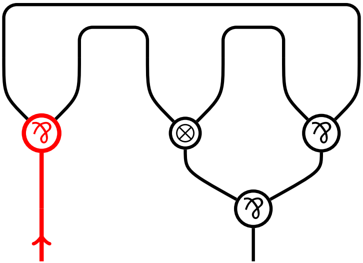

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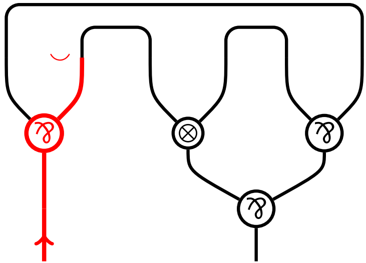

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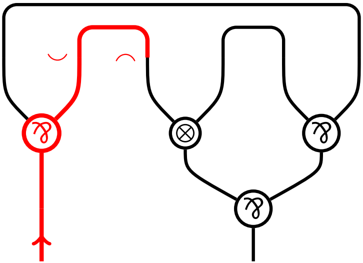

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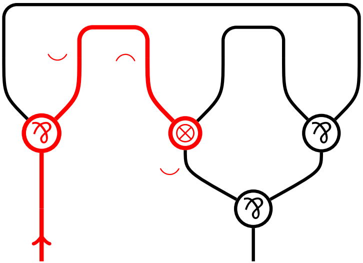

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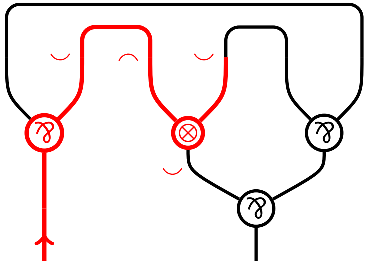

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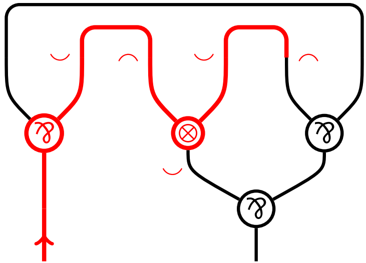

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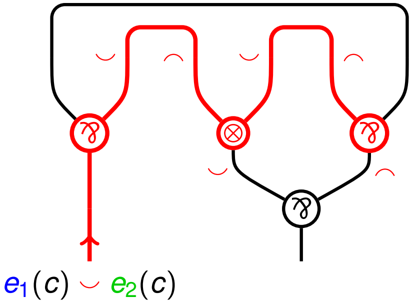

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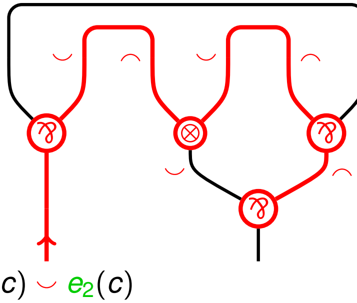

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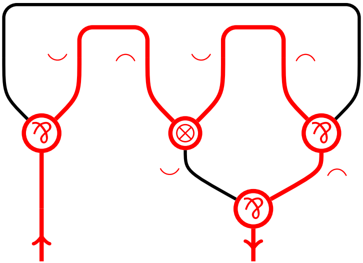

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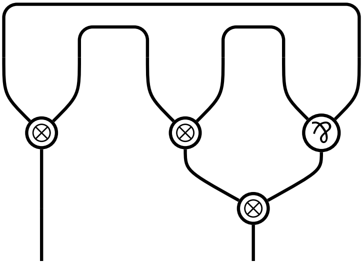

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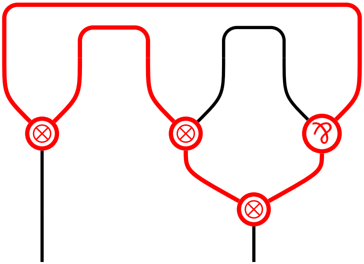

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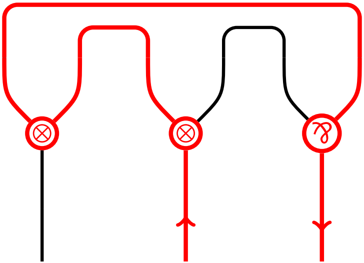

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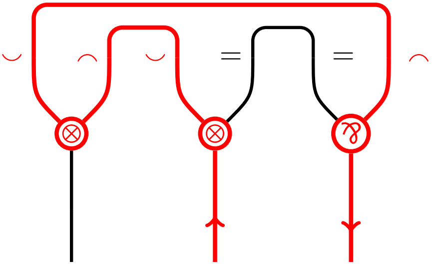

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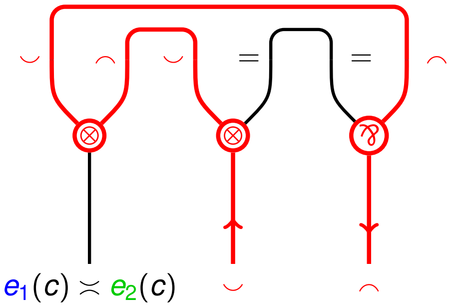
- …so that "closing" again, $e_1(\pi) \smile e_2(\pi)$

Incorrect implies semantically incorrect



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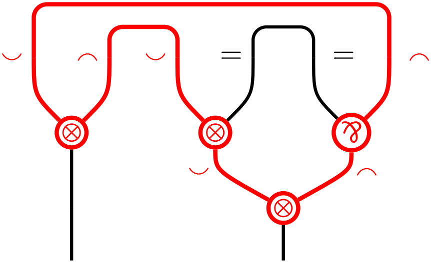

$e_1(c) \asymp e_2(c) \quad e_1(d) \smile e_2(d)$

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