The following text has the corrections for Lemmas 2.4.52 and 2.4.55, and Exercise 2.4.67

Text for Lemma 2.4.52

```
Replace the assertion of the lemma and its proof with the
text below (where before the lemma we now also recall the
definition of F≡FniF≡, and after the lemma we
explain the reason for the non-safety of Fni)
```

Thus

$$(\mathcal{F}_{\equiv}\mathcal{F}_{\mathrm{ni}}\mathcal{F}_{\equiv})(\mathcal{R}) = \{(P,Q) \mid \text{there is a non-input multi-hole context } C \text{, with } P \equiv C\eta,\ Q \equiv C\eta' \text{, and}\} \\ (\eta_i, \eta'_i) \in \mathcal{R} \text{ for all } i\} \;.$$

Lemma 2.4.52 $\mathcal{F}_{\equiv}\mathcal{F}_{\mathrm{ni}}\mathcal{F}_{\equiv}$ is safe.

Proof The proof is similar to that of Lemma 2.3.21 (in the errata notes of the book). The additional use of relation $\equiv$ does not affect much the proof: it is sufficient to apply the Harmony Lemma 1.4.15, so to carry transitions through $\equiv$. We cannot use the function $\mathcal{F}_{\mathrm{ni1}}$ because of the problem with transitive closure in the weak case discussed above. $\square$

In Lemma 2.4.52, the reason why we use $\equiv$ (i.e., $\mathcal{F}_{\equiv}$) is that function $\mathcal{F}_{\mathrm{ni}}$ itself is not safe. It is very nearly safe, but fails to be so due to a problem in replication contexts that appears when one of the processes that fill the context makes a τ transition and the other process answers without making any transitions. As an example, consider the relation $\mathcal{R} \stackrel{\mathrm{def}}{=} \{(\tau.a, a)\} \cup \mathcal{I}$, where $\mathcal{I}$ is the identity relation. We have $\mathcal{R} \subseteq \mathcal{R}$ and $\mathcal{R} \rightsquigarrow \mathcal{R}$. However we do not have $\mathcal{F}_{\mathrm{ni}}(\mathcal{R}) \rightsquigarrow \mathcal{F}_{\mathrm{ni}}(\mathcal{R})$: take the context $!\,[\cdot]$; then $!\,\tau.a \xrightarrow{\tau} a \mid !\,\tau.a$, but the only answer for process $!\,a$ is $!\,a \Longrightarrow !\,a$, and the pair $(a \mid !\,\tau.a, !\,a)$ in not in $\mathcal{F}_{\mathrm{ni}}(\mathcal{R})$.

What is missing, to avoid the counterexample, is the possibility of unfolding a replication; indeed unfolding $!\,a$ once we get $a \mid !\,a$, and then $(a \mid !\,\tau.a, a \mid !\,a) \in \mathcal{F}_{\mathrm{ni}}(\mathcal{R})$. The use of $\equiv$ in Lemma 2.4.52 is precisely to leave us this possibility. Other relations could be used in place of $\equiv$. For instance, we could use the congruence induced by the unfolding axiom for replication (plus the rules for commutativity and associativity of parallel composition); or we could use bisimilarity, or even the expansion relation (Definition 2.4.58). The only requirement is that the relation used should guarantee a tight correspondence on transitions for related processes, akin to what the Harmony Lemma does for $\equiv$.

Text for Lemma 2.4.55

```
Replace the assertion of the lemma and its proof with the
text below (the modifications are similar to those of
Lemma 2.4.52)
```

Lemma 2.4.55 The function $\mathcal{F}_{\equiv}\mathcal{F}_C\mathcal{F}_{\equiv}$ (where $\mathcal{F}_C$ is defined in Lemma 2.3.24) is safe.

Proof The proof is similar to that of Lemma 2.3.24. As in Lemma 2.4.52, the presence of $\equiv$ does not affect much the proof (it suffices to apply the Harmony Lemma). The need of $\equiv$ is as explained after Lemma 2.4.52. $\square$

Text for Exercise 2.4.67

Replace the exercise with the text below
(the modification is minor: in the second item, the
addition of the text about the inclusions of $\equiv$ in $\succeq$ and $\preceq$)

Exercise 2.4.67 Prove Lemma 2.4.66 as follows.

(1) First show that composition (as defined in Lemma 2.3.14) is a secure operator.
(2) Then apply Lemma 2.4.55, the inclusions $\equiv\ \subseteq\ \succeq$ and $\equiv\ \subseteq\ \preceq$, and Exercise 2.4.61(1). $\square$