

Algorithms and Data Structures  
2015-2016

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Graph Algorithms

# DAGs as Dependency Graphs

Tasks to be completed can be arranged into DAGs  
(Direct Acyclic Graphs)

- 1) vertices are tasks
- 2) an edge  $(v,w)$  means that  $v$  must be completed before  $w$   
(or is required by  $w$  or ...)

Question : can we use a general directed graph instead?

Exercise : draw the graph to cook your favourite meal

# Topological Sorting

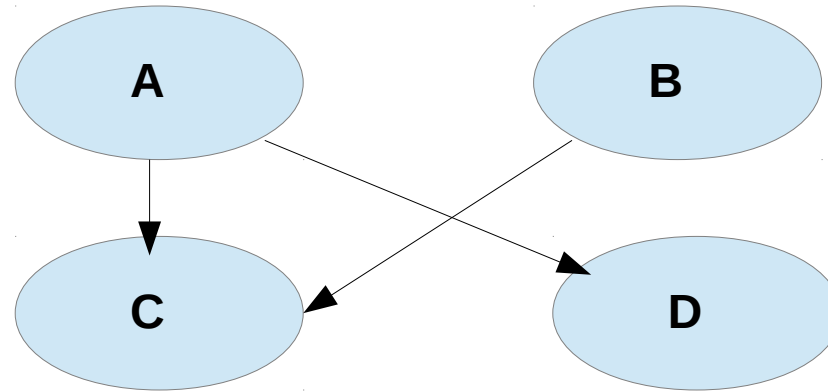
Given a DAG  $(V,E)$ ,  $(v_1, \dots, v_n)$  is a **topological sorting** of the nodes in  $V$  iff

- 1)  $\{v_1, \dots, v_n\} = V$
- 2) for each  $i < j$ , there is no path from  $v_j$  to  $v_i$

Tasks can be executed in topological order without violating the dependency graph.

Question : can a DAG be topologically sorted in two different ways?

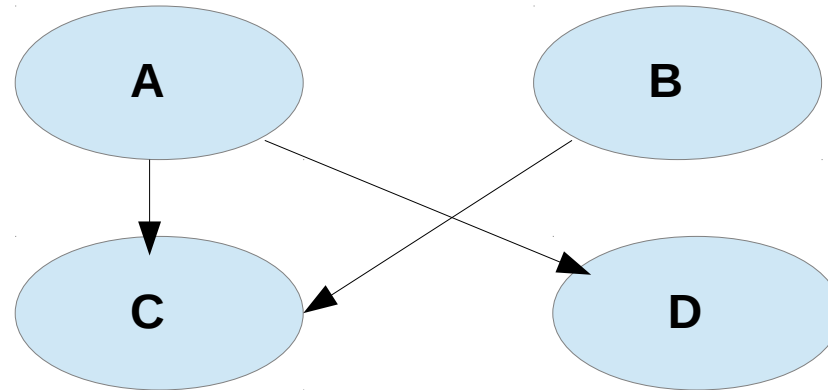
# Topological Sorting



$\{A, B, C, D\}$ ,  $\{B, A, D, C\}$ ,  $\{A, D, B, C\}$  are all valid topological orders

Question: are there more?

# Topological Sorting



Topological sorting algorithm – ideas :

1. nodes are added one by one in front of the result list
2. a node is to be added only **after** adding all nodes that are **reachable** from it

Question: how to know what nodes are reachable from a given one?

# Topological Sorting

```
TopoSort(G) =
```

```
  L = empty_list()
```

```
  for u in vertices(G)
```

```
    modified_dfs(G,u,L)
```

```
  return L
```

```
modified_dfs(G,u,L) =
```

```
  mark(u)
```

```
  for v in AdjSet(G,u)
```

```
    If not marked(v)
```

```
      modified_dfs(G,v,L)
```

```
  add(L,v) // v is added to L after all nodes
```

```
    // reachable from v
```

# Topological Sorting

**Exercise** : run the previous algorithm on the graph of Slide 5 multiple times, changing only the order in which `vertices(G)` returns the node.