# Algorithms and Data Structures, Academic Year 2012/2013 

## International Bologna Master in Bioinformatics

## June 19, 2015

Please complete the following exercises by applying the concepts that have been illustrated to you during the classes. The score associated with each exercise and the expected time for completion is reported in the first line. Do NOT copy/exchange results (the parameters of each exercise are different).

Exercise 0 (2 points): write your name and surname in the first row of all the sheets you use.
Name: $\qquad$ Surname: $\qquad$

Exercise 1 (48 points, 60 minutes): please design the data structures that you would implement to efficiently realize a "balanced pairs" service as follows. A string containing numbers [0..9] and letters [A..Z] like the following "A27HG53H8A3725G" is provided as input to your program. Your task is to design the pseudo code of the algorithm and the additional data structures required to efficiently answer to the following questions.

1. Is the string containing "pairs" of each letter or number? In other words, is the number of occurrences of each letter or number even?
2. Which letters or numbers occur a odd number of times in the string? (list all of them in output)
3. Extra bonus: try to implement a version of the solution which is computationally $\mathrm{O}(\mathrm{N})$, given N the size of the input string, and another version of the solution which is spatially $\mathrm{O}(\mathrm{K})$, given K the number of different symbols in the input string (no matter the size of N ).

For each implementation of the solutions to the questions above, please provide a motivation for your design, and a sketchy discussion of average/worst-case complexity in space and computation.
(use additional sheets for this exercise)

Name: $\qquad$ Surname:

Exercise 2 (10 points, 15 minutes): please provide the ordered sequence of visited nodes in a visit of the three, in pre- in- and post-order respectively:


Exercise 3 (20 points, $\mathbf{3 0}$ minutes): please provide the ordered sequence of visited nodes in a Depth-First-Search (DFS) visit of the directed graph G, starting from node A, by using the Adjacency List implementation (also show the Adjacency list data structure, and plot the graph).
$\mathrm{G}=(\mathrm{V}, \mathrm{E}), \mathrm{V}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}\}$, $\mathrm{E}=\{(\mathrm{A}, \mathrm{B})(\mathrm{A}, \mathrm{C})(\mathrm{A}, \mathrm{D})(\mathrm{B}, \mathrm{D})(\mathrm{C}, \mathrm{E})(\mathrm{D}, \mathrm{A})(\mathrm{C}, \mathrm{B})(\mathrm{D}, \mathrm{F})(\mathrm{E}, \mathrm{G})(\mathrm{F}, \mathrm{D})(\mathrm{G}, \mathrm{F})(\mathrm{G}, \mathrm{B})\}$,

Name: Surname:

Exercise 4 ( $\mathbf{1 0}$ points, 10 minutes): provide a demonstration that the following assertion is true or false: $3 \mathrm{~N}^{\wedge} 2$ is $\mathrm{O}\left(\mathrm{N}^{\wedge} 3 \log \mathrm{~N}^{\wedge} 2\right)$.

Name: $\qquad$ Surname:

Exercise 5 ( 10 points, 10 minutes): write the pseudo code of a recursive function which returns the value TRUE if the first argument X is a Fibonacci Number and FALSE if the first argument X is not a Fibonacci number. Comment on the time complexity of the function and discuss the advantage of using a divide et impera or a greedy strategy for the proposed algorithm.
example: $f(5)$ returns TRUE, $f(7)$ returns FALSE.
int $f($ int $x)$
begin

