Optimal Assessments in VANET: The Oracle

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***Abstract*—We discuss the accuracy of a mechanism, called Oracle, to assess position and transmission ranges in VANET. We prove that, in the best case, the assessments of the oracle are exact. Moreover, we prove that the average case of the Oracle coincides with its best case. Therefore the Oracle is optimal.**

The errors that afflict the assessments of both the positions and the transmission ranges depend on the relative speeds between the vehicles. We prove that the distribution of those relative speeds is an unimodal distribution peaked in zero.

We show and briefly discuss the results of several experiments we made, that fully confirmed our theoretical results.

I. Introduction

In the last years, the idea of taking advantage of vehicular network technologies, VANET [1], to implement vehicular accident warning systems showed an increasing trend. We proposed a system that, without any external supporting infrastructure, could rapidly broadcast alert messages [2], [3].

Our system is optimal under the assumption of a one dimensional and multilane road *e.g.* a freeway or an extra-urban road. Our system delivers alert messages in the minimum number of possible hops, *i.e.* along the fastest path. The main characteristic of our system is that it is optimal even when the transmission ranges vary from vehicle to vehicle, and could change while traveling. In this scenario, the communications between each couple of vehicles in the platoon could be *asymmetric*. That is: a vehicle directly receives messages from a second vehicle, but not *vice versa*.

The basic idea of our system, is that each vehicle in a platoon dynamically asses: the set of its surrounding vehicles, their position, and their transmission ranges. We call *Oracle* the layer of our system that assesses such informations. When a vehicle sends an alert message, it designates, among all the known receivers of that message, the one whose re-transmission will span farther. Recursively applying that strategy leads to the minimum number of hops to propagate the alert message to all the vehicle of the platoon.

In this paper we focus on the properties of the Oracle, showing that it is optimal, *i.e.* that the assessments that it computes are exact in the average case.

This paper is organized as follows. In § II we briefly outline the literature available on this subject. § III describes the logical architecture of the Oracle and shows its main characteristic. The optimality of the Oracle is proved in § IV, while § V show some experimental results. The § VI ends the paper.

II. Related Works

A quite common approach to minimize the number of hops while broadcasting alert messages is that the farther vehicle that receives such a message will relay it. Recursively applying this method leads to a fast broadcast [4]–[6].

These approaches takes advantage of the assumption that the transmission ranges of all the vehicles are both constant and equal for all the vehicles. In [7] the authors suggest an algorithm that assesses the actual transmission range for each vehicle in the platoon.

The accounting for realism in wireless networks has often led to the design of new protocol strategies [8]. The effects on performance of transmission range asymmetries have been investigated in [9]–[11].

Authors in [12] propose a protocol which falls close in principle to our scheme. They introduce *dominant pruning*, an algorithm which uses 2-hop neighbor knowledge to choose 1-hop relays.

Our proposal implements *dominant pruning* keeping in mind that in reality transmission ranges are asymmetric and vary from node to node. In fact, using such information our scheme chooses the 1-hop neighbor that covers most 2-hop neighbors (i.e., the farthest spanning neighbor). Implement *dominant pruning* without carefully considering the nature of wireless communications may result in a severe loss in terms of performance.

III. Oracle's Architecture

Our system is made of two logical modules, both of them running on each vehicle. The *relay module* manages the alert messages relying on the assessments computed by the *oralce module*.

The *oracle module* on each vehicle exchanges messages with the surrounding oracle modules. Based on such exchange of messages, each oracle knows the set of vehicles that receive direct communication from it. Based on the distance with respect to each one of those vehicles, the oracle module can assess its transmission range.

Each oracle module can read the GPS position of the vehicle. All of the oracle assessments are based on a combination of its position and the one of each surrounding vehicle. We consider *exact* the GPS readings.

We are interested in *relative* positions between the vehicles of the platoon, and do not consider ay *absolute* position. We can suppose that all the vehicles undergo the same errors on

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GPS data. Therefore, this assumption makes our system independent of the GPS errors, such as delays and uncertainties on position [13]. Moreover, while the GPS is an underlying module, we could replace it without any modification of the other modules.

For the sake of simplicity, and without loosing in generality, in this paper we consider that all the oracle modules share a common time and sends messages in rounds of duration τ, *i.e.* each oracle sends its k-th message randomly during the time interval $[\tau_k, \tau_{k+1})$.

Each vehicle v_i maintains three lists about its surrounding vehicles, that represent its *local* knowledge about the platoon. It is worthwhile to recall that our system does not require any kind of global knowledge, therefore those lists are the sole kind of knowledge necessary to the Oracle. The maintained lists are the following:

- In_i the *input* vehicles, *i.e.* the vehicles whose messages are directly received by v_i;
- Out_i the *output* vehicles, *i.e.* the vehicles that directly receive messages from v_i;
- Be_i the couples of senders and receivers known by v_i, where it lies in between.

Each entry of both the In_i and the Out_i lists represents a vehicle v_j and has the form $\langle j, p_j, b_j, f_j, d_{i,j} \rangle$ where: j is the vehicle unique id, p_j is its position, f_j is the forward transmission range, the b_j is backward transmission range, and $d_{i,j}$ is its distance from v_i, computed by v_i when updating or inserting the entry.

Each entry of Be_i has the form $\langle j, p_j, b_j, f_j, d_{i,j}, s \rangle$. This means that v_i is aware that v_j announced that it has received a message from v_s. Note that, since v_i has received at least one message from v_s, the list In_i contains more data about v_s.

All these entries are function of the time, but for the sake of notation simplicity, we do not explicit that unless strictly necessary.

We denote as $oracle_i$ the oracle module running on vehicle v_i. We also denote $om_i(n)$ the oracle message sent by $oracle_i$ during the round n. At each time n a vehicle v_s sends $om_s(n)$, and vehicle v_r receives that message, then v_r will acknowledges that reception in its next oracle message, $om_r(n+1)$.

Each $om_s(n)$ has the form $\langle s, p_s, In_s, Be_s \rangle$. Upon receiving an $om_s(n)$, vehicle v_r updates its In_r. Moreover, if v_r is contained in In_s, it could update its transmission range in the direction of v_s by comparing their relative positions. It could be the case that v_r updates even Out_r, while it discovers in Be_s that some vehicle v_j received a previous om_r. Finally, v_r could update Be_r when discovering that it lies in between a couple of known sender and receivers.

IV. Optimality of the oracle

We are going to prove the optimality of the Oracle following this path: first we prove that the accuracy of the oracle depends solely on the relative speeds between vehicles, than we prove that these relative speeds, most probably, are zero. Therefore, the average accuracy of the oracle is optimal.

A. Accuracy as function of difference of vehicles' speeds

We focus on a generic couple of vehicles, v_s and v_r.

Lemma IV.1 *The positions, and therefore the distances, of vehicles in each In list are as precise as the GPS reading when recorded in the list.*

Proof: We can safely make the simplification that $om_s(n)$ has an instantaneous propagation with respect to the vehicles' speed. In fact, the distance between any v_s and v_r is at most $1km$, by standard, and any radio message propagates as an electromagnetic wave in the atmosphere. The delay of $om_s(n)$, therefore, depends solely on the sending time of $oracle_s$ and the delivery time of $oracle_r$, *i.e.* the time required to traverse their network stacks. Both of them are in the orders of milliseconds. Vehicle v_s moved at most few centimeters in the period since it read its position, and v_r inserted it in In_r. Therefore, the error of the position of v_s recorded in In_r is largely below the uncertainty of commercial GPS. Recalling that the distances are the difference between two positions, complete that proof. ■

Given a vehicle v_i, we consider the content of its three lists, In_i, Out_i, and Be_i, to measure the accuracy of $oracle_i$. We denote with Greek letters the physical gauges of the values recorded in software variables. The following table shows these correspondences:

TABLE I
PHYSICAL CHARACTERISTICS AND THEIR CORRESPONDING VARIABLES FOR A VEHICLE v_i AT TIME n

characteristic	Physical	Variable
position	$\pi_i(n)$	$p_i(n)$
forward range	$\phi_i(n)$	$f_i(n)$
backward range	$\beta_i(n)$	$b_i(n)$
distance from $v_j(n)$	$\delta_{i,j}(n)$	$d_{i,j}(n)$

The accuracy of the oracle is therefore expressed by the absolute value of the difference between the physical charactersitic and its corresponding variable, *e.g.* the error on the position of vehicle v_i is $||\pi_i - p_i||$.

It is worthwhile to observe that we consider as physical values the ones that could be measured by the Oracle. As an example, consider the physical transmission range of vehicle v_i. It is measured by the farther vehicle that receives messages from v_i, regardless of the actual extent of the transmission, that would be impossible to measure more precisely than that.

We distinguish two possible scenarios on the basis of symmetric and asymmetric communications. Moreover, for each one of them, we can differentiate with respect to the speeds of the vehicles, that move at the same speed, or that move at different speeds.

Note that the assessment of transmission ranges are based on the distance between the vehicles. Therefore, in the following we mainly discuss the accuracy of the distances, and, unless explicitly stated, we ignore the accuracy of the transmission ranges.

1) Symmetric communications: The simplest case is when the vehicles move at the same speed, *i.e.* their relative positions do not vary.

Since we consider couple of vehicles, any variation of transmission range is not sensed, unless it causes asymmetric communications, that are discussed in § IV-A2.

Lemma IV.2 *The assessments of the oracles are exacts in the case of symmetric communications and vehicles that move at the same speed.*

Proof: Suppose that during the *round n*, vehicles v_f and v_l exchanges oracle messages. Therefore, they insert each other in their In lists. By Lemma IV.1 we know that the distances on those lists are exact when recorded in the list. Since the relative positions of the two vehicles are constant because of their equal speed, this result holds forever.

At time $n+1$, by exchanging their second oracle message, the vehicles can assess their transmission ranges in the direction of each other. Supposing that v_f precedes v_t, then $b_f = ||p_f(n+1)-p_t(n+1)||$ and $f_t = ||p_t(n+1)-p_f(n+1)||$. While depending solely on relative positions, this result holds forever. ■

It is worthwhile, and pretty obvious, to point out that:

Observation IV.3 *The distances between two vehicles changes because their relative speeds are different from zero.*

Without loss in generality, we can suppose that the distances between the vehicles are *constant at intervals*, *i.e.* during each round the vehicles move at the same speed, and they could suddenly accelerate or brake only at the end of each round, therefore modifying their relative distance. The effect of such a model is that the relative distances between vehicles could vary round by round, while they remain constant during each round. Fig. 1 shows an example of such *constant at interval distance* between two vehicles. The horizontal time axis is divided in rounds, while the continuous curve represents the physical distance between the two vehicles. The horizontal segments represent the discontinuous *constant at interval distance*. The circle at one end of each segment represents a discontinuity point.

We can also suppose, without loss in generality, that each $oracle_i$ computes its assessments at the end of each round, by exchanging messages at the very last moment, and without interfere each other. Therefore, during almost the whole duration of each round the assessments are the ones computed at the end of the previous one, *i.e.* $d_{i,j}(n) = \delta_{i,j}(n-1)$. Note that the discontinuity points depicted in Fig. 1 coincide with these computations.

Lemma IV.4 *The accuracy of the oracle depends on the difference of vehicles' speed in the case of symmetric communications.*

Proof: Given two vehicles v_f and v_l, suppose that the latter follows the former.

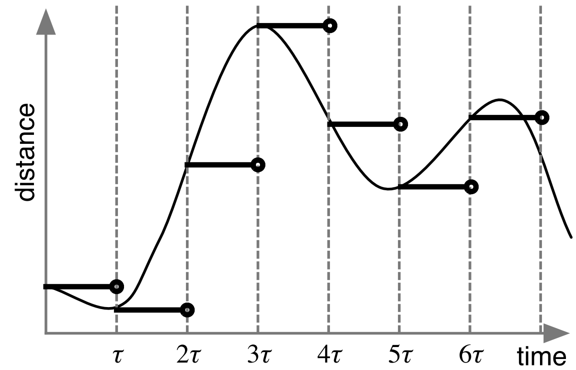

Fig. 1. Constant at intervals distance

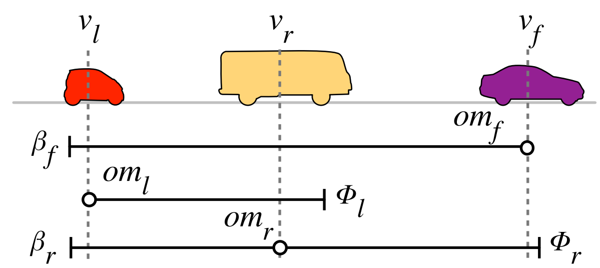

Fig. 2. Asymmetric communications

Let, now, focus on vehicle v_f, during a generic *round n*. The accuracy of the distance assessed by $oracle_f$ is $||\delta_{f,l}(t) - d_{f,l}(n-1)|| \geq 0$ during the whole duration of the round. That value becomes zero for an instant, just at the beginning of the round. The accuracy of transmission range is $||\beta_f(t) - b_f(n-1)|| \geq 0$. Since $b_f = ||p_f - p_l|| = d_{f,l}$, even the accuracy of transmission range depends on the distance.

Similar considerations holds for vehicle v_l, therefore all the computation of the oracles are based on relative distances between the two vehicles.

By the Observation IV.3, the accuracy of the oracle depends on the relative speed between v_f and v_l. ■

2) Asymmetric communications: We consider the simple case three vehicles, v_f, v_r, and v_l that travel in that order on a road. We suppose, as shown by Fig. 2, that: the backward transmission range of v_f spans till v_l, the forward transmission range of v_l spans till v_r, and the transmission ranges of v_r spans till the other two vehicles. We could ignore the forward transmission range of v_f, and the backward transmission range of v_l because these do not reach any vehicle. The transmission ranges are represented as segments, originating from the position of each vehicle and ending where labelled. When a transmission range crosses a dotted line representing the position of a vehicle, we consider that that vehicle receives the corresponding messages.

For the sake of simplicity, we can ignore the accuracy of the transmission ranges, that derives from the accuracy on distances.

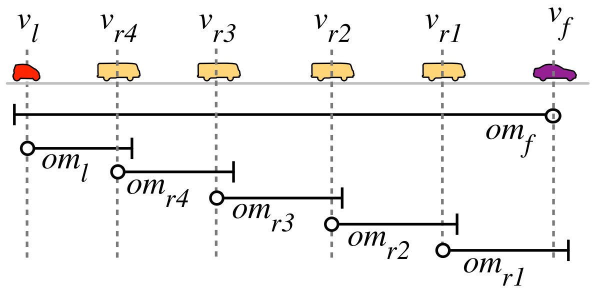

Fig. 3. Asymmetric communications with long relay chain

Lemma IV.5 *The assessments of the oracles are exacts in the case of asymmetric communications and vehicles that move at the same speed.*

Proof: Let focus on the case of a single relay vehicle. Suppose that during the *round n*, the vehicles v_f, v_r and v_l travel in that order at constant speeds. At the end of the round, they will send their oracle messages. While v_r receives both om_f and om_l, in the next round it will announce that v_l received om_f. Therefore, at the end of the *round n+1*, the vehicle v_f can set $d_{f,l}(n+1) = d_{f,r}(n) + d_{r,l}(n-1)$. Due to the Observation IV.3, these distances are constant with respect to the time. Moreover, due to Lemma IV.2, these distances are exact.

Let consider now the case of multiple relay vehicles, say v_{r1}, v_{r2}, $\ldots$, v_{rR}. In the worst case, it could be that the transmission range of each relay spans solely till the next one. Fig. 3 shows that case for $R = 4$. Therefore, $d_{f,l}(n+1) = d_{f,r1}(n) + d_{r1,r2}(n-1) + \cdots + d_{rR,l}(n-r)$. As in the previous case, these relative distances do not changes with respect to the time and are exact. ■

Lemma IV.6 *The accuracy of the oracle depends on the difference of vehicles' speed in the case of asymmetric communications.*

Proof: As shown above, in the general case of a multiple relay, we have: $d_{f,l}(n+1) = d_{f,r1}(n) + d_{r1,r2}(n-1) + \cdots + d_{rR,l}(n-r)$. By Observation IV.3, these distances solely depend on difference of speeds. ■

B. Average relative speed

Suppose a platoon of vehicles, and group their speeds at intervals of constant amplitude A m/sec. The number of vehicles whose speeds belong to each interval is directly proportional to the probability that a given vehicle of the platoon moves at that speed. We could represent that distribution of speeds as an array S of integer, where the $i-th$ element, s_i, contains the number of vehicles that move at a speed in the interval $[iA, (i+1)A)$. We suppose also that S has N elements.

We are interested in the distribution of all the possible differences of speeds between each couple of vehicles. To build this distribution, we need to analyze each vehicle and compute its relative speed with respect to all the vehicles of the platoon. As an example, consider all the possible couples of vehicle moving at the same speed. Since the vehicles are different from each other, all the possible couples of vehicles in s_i is s_i^2. Therefore, all possible couples of vehicles in the platoon that move at the same speed is $\sum_{i=1,\ldots,N} s_i^2$.

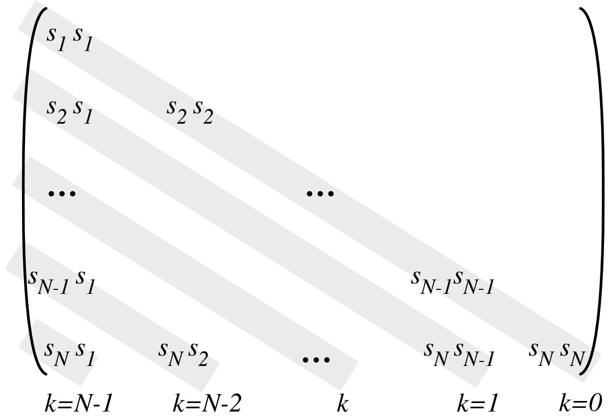

Fig. 4. The Δ array, and its diagonals

The simplest way to compute all the relative speeds between the vehicle of the platoon, is by constructing a bi-dimensional array Δ of size $N \times N$, and then considering its diagonals. Let Δ be the vectorial product of S times itself. Therefore, each element of Δ is $\delta_{i,j} = s_i s_j$. The array is symmetric with respect to the main diagonal, and we can consider solely its lower triangular part. Note that the sum of the elements of the main diagonal of Δ is the number of couple of vehicles that move at the same speed, such ad we computed above.

For the sake of simplicity, we define *k-th diagonal* of Δ the one whose elements are $\{\delta_{i,i-k}, \forall i = k+1, k+2, \ldots, N-1\}$. Following that definition, the main diagonal is the $0-th$ one. Fig 4 shows the array Δ and highlights its diagonals.

To compute the number of couples of vehicles that move at relative speed of A m/sec, we need to sum all the possible couples between vehicles into two consecutive elements of the array S. That is, $\sum_{i=2,\ldots,N} s_i s_{i-1}$, *i.e.* the sum of the elements of the *1-th diagonal* of Δ. Recursively applying this observation, we have that the sum of the elements of the *k-th diagonal* is the number of couples of vehicles whose relative speed is (kA) m/sec.

The following Lemma shows that the highest probable value of the relative vehicles' speed is zero.

Lemma IV.7 *The sum of the elements of the main diagonal is greater of the sum of the elements of each k-th diagonal.*

Proof: Consider the *1-th diagonal*, following the thesis, we could write

$$\sum_{i=2,\ldots,N} s_i s_{i-1} \leq \sum_{i=1,\ldots,N} s_i^2$$

Considering that for all the x and y positive integers it holds that: $2xy \leq x^2 + y^2$, we can rewrite the above inequality as

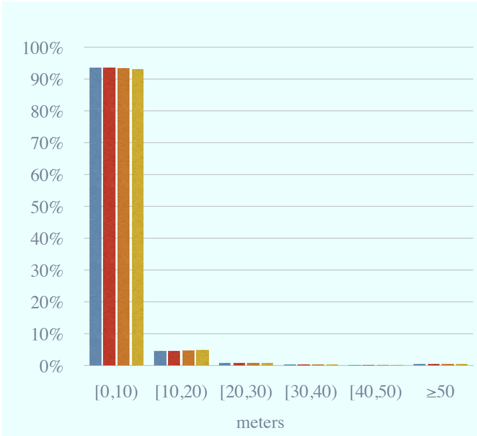

Fig. 5. Experiments on accuracy of the positions

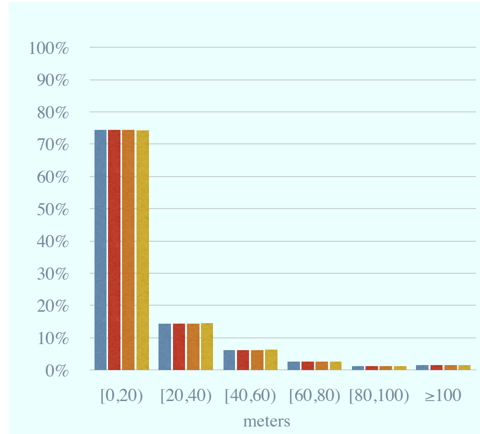

Fig. 6. Experiments on accuracy of the ranges

follows:

$$\sum_{i=2,\ldots,N} s_i s_{i-1} \leq \frac{1}{2} \sum_{i=2,\ldots,N} (s_i^2 + s_{i-1}^2) < \sum_{i=1,\ldots,N} s_i^2$$

Applying analogous considerations to the subsequent *k-th diagonal*, with $k = 1, \ldots, (N-1)$, proves the lemma. ■

The consequence of the above Lemma IV.7 is that the distribution of the relative vehicles' speeds is modal with respect to zero. In other words in the average case, the Oracle deals with vehicles that move at the same speed, *i.e.* the Oracle is exact.

V. Experimental results

The above theoretical results has been confirmed by several experiments we carried, based on *Ns2* and *GTNetS*. The number of vehicles involved in the simulations is 400 on a portion of road of $8km$. Fig. 5 and Fig. 6 show the precision of the Oracle for the positions and for the ranges respectively. The bars of the figures represent different conditions on the frequency and size of the overlying safety application; we do not report the intervals of confidence while those are very thin.

The relevant points in those figure are that: the accuracy of the positions is less than $10m$ in about the 94% of the cases, and that the accuracy of the ranges is less than $20m$ in about the 75% of the cases, and less than $40m$ in about the 90% of the cases. As expected, the accuracy of the ranges is lower with respect to the one of positions, while it could depend on chains of relays, that last for several rounds, making the original data less up-to-date.

VI. Conclusion

We proved that the Oracle is optimal. Firstly we proved that the errors on the assessment computed by the Oracle are zero in the best case. Then we proved that in its average case, the Oracle behaves as in its best case.

Mreover, we showed that the above theoretical results are fully confirmed by our experiments.

Our results are general, while they are not based on any assumption on the distribution of the speeds. Therefore, the Oracle is optimal in any traffic condition.

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References

[1] http://grouper.ieee.org/groups/802/11/Reports/tgp_update.htm
[2] A. Amoroso, M. Ciaschini, M. Roccetti. "The farther relay and oracle for VANET. preliminary results". In *Proceedings of the 4th Annual international Conference on Wireless internet (Maui, November 17 - 19, 2008)*.
[3] M. Roccetti, G. Marfia, A. Amoroso, "An Optimal 1D Vehicular Accident Warning Algorithm for Realistic Scenarios", *Proc. IEEE Symposium on Computers and Communications (ISCC'10)*, IEEE, Riccione, June 2009.
[4] G. Korkmaz, E. Ekici, F. Ozguner, U. Ozguner. "Urban multi-hop broadcast protocol for inter-vehicle communication systems", *Proceedings of the 1st ACM international Workshop on Vehicular Ad Hoc Networks*. October 2004, 76-85.
[5] E. Fasolo, R. Furiato, A. Zanella, "Smart broadcast algorithm for intervehicular communication", *Proceedings of 2005 Wireless Personal Multimedia Communication*, September 2005.
[6] J. J. Blum, A. Eskandarian, "A reliable link-layer protocol for robust and scalable intervehicle communications", *IEEE Transactions on Intelligent Transportation Systems*, Vol. 8, N. 1, March 2007, 4-13.
[7] C.E. Palazzi, S. Ferretti, M. Roccetti, "An Inter-Vehicular Communication Architecture for Safety and Entertainment", *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, no. 1, pp. 90-99, Sep. 2009.
[8] S. Ramanathan, "Multicast tree generation in networks with asymmetric links". In *IEEE/ACM Transactions on Networking* 4, 4 (Aug. 1996), 558-568.
[9] A. Hamidian, C.E. Palazzi, T. Y. Chong, J. M. Navarro, U. Krner, M. Gerla, "Deployment and Evaluation of a Wireless Mesh Network, *Proceedings of the 2nd IARIA/IEEE International Conference on Advances in Mesh Networks (MESH 2009)*, Athens, Greece, June 2009.
[10] D.S. De Couto, D. Aguayo, J. Bicket, R. Morris, 2005. "A high-throughput path metric for multi-hop wireless routing". In *Wireless Networks* 11, 4 (Jul. 2005), 419-434.
[11] B. M. Maggs. http://www-2.cs.cmu.edu/bmm/wireless.html
[12] H. Lim, C. Kim, 2000. "Multicast tree construction and flooding in wireless ad hoc networks". In *Proceedings of the 3rd ACM international Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems, MSWIM '00*, Boston, Massachusetts, August 2000.
[13] S. Savasta, M. Pini, G. Marfia, "Performance Assessment of a Commercial GPS Receiver for Networking Applications", *Proceedings of the 5th IEEE Consumer Communications and Networking Conference* (CCNC'08), Las Vegas, Nevada, January 10-12, 2008.