Motivation

Contracts

Check-points

Retractable contracts

Further directions

Contracts with roll-back

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based on:

- F. Barbanera, M. Dezani, U. de’Liguoro,
  Compliance for reversible client/server interactions, BEAT’14.
- F. Barbanera, M. Dezani, I. Lanese, U. de’Liguoro,
  Retractable Contracts, PLACES’15.
A *contract*\(^1\) is the abstract description of the behaviour of either a *client* or a *server*.

A client *complies* with a server if all her requirements are fulfilled, either by reaching a distinguished satisfaction state or by running an infinite communication without ever getting stuck.

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A contract\(^1\) is the abstract description of the behaviour of either a client or a server.

A client complies with a server if all her requirements are fulfilled, either by reaching a distinguished satisfaction state or by running an infinite communication without ever getting stuck.

What about allowing client and server to change their mind, rolling back to some previous choice and progress differently?

There are at least two alternatives (but possibly more):

- the **conservative approach**, extending the contract language without making compliant more roll-back free contracts
- the **adaptive approach**, where roll-back makes more contracts compliant

\(^1\)In the theory proposed by Castagna, Laneve, Padovani and others.
Syntax

\[ \sigma, \rho \ ::= \ 1 \mid \sum_{i \in I} a_i \cdot \sigma_i \mid \bigoplus_{i \in I} \overline{a}_i \cdot \sigma_i \mid x \mid \text{rec } x \cdot \sigma \]

LTS

\[ \sum_{i \in I} a_i \cdot \sigma_i \xrightarrow{a_i} \sigma_i \quad \bigoplus_{i \in I} \overline{a}_i \cdot \sigma_i \rightarrow \overline{a}_j \cdot \sigma_j \quad \overline{a} \cdot \sigma \xrightarrow{\overline{a}} \sigma \]

Communication semantics:

\[ \frac{\rho \xrightarrow{\alpha} \rho' \quad \sigma \xrightarrow{\overline{\alpha}} \sigma'}{\rho \parallel \sigma \rightarrow \rho' \parallel \sigma'} \quad \frac{\rho \rightarrow \rho'}{\rho \parallel \sigma \rightarrow \rho' \parallel \sigma} \quad \frac{\sigma \rightarrow \sigma'}{\rho \parallel \sigma \rightarrow \rho \parallel \sigma'} \]
Compliance

The client $\rho$ is compliant with the server $\sigma$, written $\rho \vdash \sigma$, if

$$\forall \rho', \sigma'. \rho\parallel\sigma \xrightarrow{*} \rho'\parallel\sigma' \not\rightarrow \Rightarrow \rho' = 1$$
The client $\rho$ is compliant with the server $\sigma$, written $\rho \vdash \sigma$, if
\[
\forall \rho', \sigma'. \; \rho \parallel \sigma \xrightarrow{*} \rho' \parallel \sigma' \not\rightarrow \Rightarrow \rho' = 1
\]
Duality

Definition

\[ \bar{1} = 1, \quad \sum_{i \in I} a_i \cdot \sigma_i = \bigoplus_{i \in I} \bar{a}_i \cdot \bar{\sigma}, \quad \bigoplus_{i \in I} \bar{a}_i \cdot \sigma_i = \sum_{i \in I} a_i \cdot \bar{\sigma}. \]
Duality

**Definition**

\[
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\]

**Fact**

Duality is involutive; moreover

1. \( \forall \sigma. \bar{\sigma} \vdash \sigma \quad \& \quad \sigma \vdash \bar{\sigma} \),
2. \( \rho \vdash \sigma \quad \& \quad \bar{\sigma} \vdash \tau \Rightarrow \rho \vdash \tau \).

Decidability theorem

The compliance relation is axiomatisable by an algorithmic system, hence it is decidable.
Duality

**Definition**

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\overline{1} = 1, \quad \sum_{i \in I} a_i \cdot \sigma_i = \bigoplus_{i \in I} \overline{a_i} \cdot \overline{\sigma}, \quad \bigoplus_{i \in I} \overline{a_i} \cdot \sigma_i = \sum_{i \in I} a_i \cdot \overline{\sigma}.
\]

**Fact**

Duality is involutive; moreover

1. \( \forall \sigma. \overline{\sigma} \vdash \sigma \quad \& \quad \sigma \vdash \overline{\sigma} \),
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**Decidability theorem**

The compliance relation is axiomatisable by an algorithmic system, hence it is decidable.
A wider scenario: in a communication contracts can roll-back:

- at any moment (for unpredictable reasons)
- to a checkpoint ▲ (the last crossed one)
Contracts with roll-back

A *wider* scenario: **in a communication contracts can roll-back:**

- at any moment (for unpredictable reasons)
- to a checkpoint ▲ (the last crossed one)

\[
\sigma, \rho ::= 1 \mid \sum_{i \in l} a_i.\sigma_i \mid \triangle \sum_{i \in l} a_i.\sigma_i \mid \bigoplus_{i \in l} \overline{a_i}.\sigma_i \mid \triangle \bigoplus_{i \in l} \overline{a_i}.\sigma_i \mid x \mid \text{rec } x.\sigma
\]
LTS for contracts with roll-back

\[ \sigma \prec \sigma' \xrightarrow{\text{rbk}} \circ \prec \sigma \ (\text{rbk}) \]

where \( \circ = \text{no checkpoint crossed yet, i.e. no roll-back is possible} \)

**Implies:**

*No two consecutive roll-backs*

*So, memory can be cleared after “crossing” a ‘▲’.*

In fact

\[
\begin{align*}
\gamma \prec \sigma & \xrightarrow{\alpha} \gamma \prec \sigma' \quad \alpha \in \mathcal{N} \cup \overline{\mathcal{N}} \\
\therefore \gamma \prec \triangleleft \sigma & \xrightarrow{\alpha} \triangleleft \sigma \prec \sigma' 
\end{align*}
\]

Possible extension: multiple roll-backs handling \( \gamma = \gamma_1 : \cdots : \gamma_k \) as a stack.
Roll-back is synchronous

Roll-back from a partner should not be hidden to the other one: it is a \textit{synchronous} transition:

\[
\rho \prec \rho' \xrightarrow{\text{rbk}} \circ \prec \rho \quad \sigma \prec \sigma' \xrightarrow{\text{rbk}} \circ \prec \sigma
\]

\[
\rho \prec \rho' \parallel \sigma \prec \sigma' \xrightarrow{\text{rbk}} \circ \prec \rho \parallel \circ \prec \sigma
\]

Many difficulties of reversible computations are overcome in our context, where, for instance, both client and server reduce in a sequential way.
Checkpoint compliance \(\downarrow\)

\[\downarrow(a.b.c + b) \parallel \overline{a}.\overline{b}.\overline{c}\]

\[\rightarrow b.c \parallel \overline{b}.\overline{c}\]

\[\rightarrow c \parallel \overline{c}\]

\[\mathsf{rbk}\]

\[\rightarrow (a.b.c + b) \parallel \overline{b}.\overline{c}\]

\[\rightarrow 1 \parallel \overline{c}\]

\[\checkmark\]
Relating $\rightarrow\mathfrak{a}$ to $\rightarrow$

We expect the following to hold:

**Duality** \[ \forall \sigma, \rho. \bar{\sigma} \rightarrow\mathfrak{a} \sigma \ & \ \rho \rightarrow\mathfrak{a} \bar{\rho} \]

**Conservativity** \[ \forall \sigma, \rho. \rho \rightarrow\mathfrak{a} \sigma \Rightarrow erase(\rho) \rightarrow erase(\sigma) \]
We expect the following to hold:

**Duality** \( \forall \sigma, \rho. \overline{\sigma} \vdash \sigma \& \rho \vdash \overline{\rho} \)

**Conservativity** \( \forall \sigma, \rho. \rho \vdash \sigma \Rightarrow erase(\rho) \vdash erase(\sigma) \)

But

\[
\circ \prec \mathcal{A} a.(b + c) \parallel \circ \prec \overline{\mathcal{A}} \overline{\mathbf{a}}.(\overline{b} \oplus \overline{c})
\]

\[
\rightarrow \mathcal{A} a.(b + c) \prec \mathcal{A} (b + c) \parallel \overline{\mathcal{A}} \overline{\mathbf{a}}.\overline{b} \oplus \overline{c} \prec \overline{b} \oplus \overline{c}
\]

\[
\rightarrow \mathcal{A} a.(b + c) \prec \mathcal{A} (b + c) \parallel \mathcal{A} (b \oplus \overline{c}) \prec \overline{b}
\]

\[
\xrightarrow{rbk} \circ \prec \mathcal{A} a.(b + c) \parallel \circ \prec \overline{\mathcal{A}} \overline{b} \oplus \overline{c}
\]

\[
\not\rightarrow
\]

hence

\[
\mathcal{A} a.(b + c) \not\vdash \overline{\mathcal{A}} \overline{\mathbf{a}}.\overline{b} \oplus \overline{c} = \overline{\mathcal{A}} a.(b + c)
\]
Relating $\rhd$ to $\vdash$

To solve the problem of saving **Duality**, we may redefine the LTS by putting:

\[
\gamma \prec \sum_{i \in I} a_i \cdot \sigma_i \xrightarrow{a_k} \gamma \prec \sigma_k \quad \gamma \prec \bigoplus_{i \in I} \bar{a}_i \cdot \sigma_i \xrightarrow{\bar{a}_k} \gamma \prec \sigma_k
\]
Relating $\lnot \uparrow$ to $\lnot$

To solve the problem of saving **Duality**, we may redefine the LTS by putting:

\[
\gamma \prec \sum_{i \in I} a_i \cdot \sigma_i \xrightarrow{a_k} \gamma \prec \sigma_k \quad \gamma \prec \bigoplus_{i \in I} \bar{a}_i \cdot \sigma_i \xrightarrow{\bar{a}_k} \gamma \prec \sigma_k
\]

but this immediately breaks **Conservativity**:

\[
a \lnot \uparrow \bar{a} \oplus \bar{b} \text{ where } a \not\! \lnot \bar{a} \oplus \bar{b}
\]
Constraining communication

With the new LTS we constrain communication rules:

\[
\begin{align*}
\begin{array}{ccc}
\rho \xrightarrow{a} \rho' & \quad & \sigma \xrightarrow{\bar{a}} \sigma' & \quad & \mathcal{A}^\oplus(\sigma) \subseteq \mathcal{A}^+(\rho) \\
\rho \parallel \sigma \rightarrow \rho' \parallel \sigma' & \end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{ccc}
\rho \xrightarrow{\bar{a}} \rho' & \quad & \sigma \xrightarrow{a} \sigma' & \quad & \mathcal{A}^\oplus(\rho) \subseteq \mathcal{A}^+(\sigma) \\
\rho \parallel \sigma \rightarrow \rho' \parallel \sigma' & \end{array}
\end{align*}
\]

where

\[
\begin{align*}
\mathcal{A}^+(\mathbf{1}) = \mathcal{A}^+(\bigoplus_{i \in I} \bar{a}_i \cdot \sigma_i) &= \emptyset & \mathcal{A}^+(\sum_{i \in I} a_i \cdot \sigma_i) &= \{a_i \mid i \in I\} \\
\mathcal{A}^\oplus(\mathbf{1}) = \mathcal{A}^\oplus(\sum_{i \in I} a_i \cdot \sigma_i) &= \emptyset & \mathcal{A}^\oplus(\bigoplus_{i \in I} \bar{a}_i \cdot \sigma_i) &= \{a_i \mid i \in I\}
\end{align*}
\]
Definition

Define $\vdash^\bullet$ exactly as $\vdash$ but w.r.t. the semantics of contracts with checkpoint.
Results

**Definition**

Define $\vdash$ exactly as $\vdash$ but w.r.t. the semantics of contracts with checkpoint

**Theorem**

- $\vdash$ satisfies both Duality and Conservativity principles
Results

**Definition**

Define $\dashv$ exactly as $\vdash$ but w.r.t. the semantics of contracts with checkpoint

**Theorem**

- $\dashv$ satisfies both Duality and Conservativity principles
- $\dashv$ can be characterized coinductively
Results

Definition
Define $\vdash$ exactly as $\vdash$ but w.r.t. the semantics of contracts with checkpoint.

Theorem
- $\vdash$ satisfies both Duality and Conservativity principles.
- $\vdash$ can be characterized coinductively.
- There is a formal system for deducing whether $\rho \vdash \sigma$, which is sound and complete.
Results

**Definition**

Define $\vdash^\blacktriangle$ exactly as $\vdash$ but w.r.t. the semantics of contracts with checkpoint

**Theorem**

- $\vdash^\blacktriangle$ satisfies both Duality and Conservativity principles
- $\vdash^\blacktriangle$ can be characterized coinductively
- there is a formal system for deducing whether $\rho \vdash^\blacktriangle \sigma$, which is sound and complete
- derivability in the system is decidable, hence $\vdash^\blacktriangle$ is decidable
A different motivation for rolling back is to recover from a failure:

\[
\begin{align*}
\text{Buyer} &= \text{bag.price.}(\text{card} \oplus \text{cash}) \oplus \text{belt.price.}(\text{card} \oplus \text{cash}) \\
\text{Seller} &= \text{bag.price.}(\text{card} + \text{cash}) + \text{belt.price.cash}
\end{align*}
\]
Retractable contracts

A different motivation for rolling back is to recover from a failure:

\[
\begin{align*}
\text{Buyer} & = \text{bag} \cdot \text{price} \cdot \text{card} \oplus \text{cash} \oplus \text{belt} \cdot \text{price} \cdot \text{card} \oplus \text{cash} \\
\text{Seller} & = \text{bag} \cdot \text{price} \cdot \text{card} + \text{cash} + \text{belt} \cdot \text{price} \cdot \text{cash}
\end{align*}
\]

Then Buyer $\not \dashv$ Seller because, by choosing $\text{belt} \cdot \text{price}$ on Buyer’s side

\[
\text{Buyer} \parallel \text{Seller} \xrightarrow{*} \text{card} \oplus \text{cash} \parallel \text{cash} \rightarrow \text{card} \parallel \text{cash}
\]

If Buyer will insist in paying by card, we could change her contract

\[
\text{Buyer}' = \text{bag} \cdot \text{price} \cdot \text{card} \oplus \text{cash} + \text{belt} \cdot \text{price} \cdot \text{card} \oplus \text{cash}
\]

and allow roll-back to (all) external choices whenever a communication failure occurs.
Retractable contracts: syntax

\[ \sigma, \rho ::= 1 \mid \sum_{i \in I} a_i \cdot \sigma_i \mid \sum_{i \in I} \overline{a}_i \cdot \sigma_i \mid \bigoplus_{i \in I} \overline{a}_i \cdot \sigma_i \mid x \mid \text{rec } x \cdot \sigma \]
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\]

LTS (where \( \gamma = \gamma_1 : \cdots : \gamma_k \)):

\[
(+) \quad \gamma \prec \alpha . \sigma + \sigma' \xrightarrow{\alpha} \gamma : \sigma' \prec \sigma \quad (\oplus) \quad \gamma \prec \overline{a} . \sigma \oplus \sigma' \xrightarrow{} \gamma \prec \overline{a} . \sigma
\]

\[
(\alpha) \quad \gamma \prec \alpha . \sigma \xrightarrow{\alpha} \gamma : \circ \prec \sigma \quad (\text{rbk}) \quad \gamma : \sigma' \prec \sigma \xrightarrow{\text{rbk}} \gamma \prec \sigma'
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\]

\[
(\alpha) \quad \gamma \prec \alpha \cdot \sigma \xrightarrow{\alpha} \gamma : \circ \prec \sigma \quad \text{ (rbk) } \quad \gamma : \sigma' \prec \sigma \xrightarrow{\text{rbk}} \gamma \prec \sigma'
\]

Communication:

\[
\gamma \prec \rho \xrightarrow{\text{rbk}} \gamma' \prec \rho' \quad \delta \prec \sigma \xrightarrow{\text{rbk}} \delta' \prec \sigma'
\]

then applies only if \( \rho \) and \( \sigma \) are in the failure condition:

\[ \rho \neq 1 \quad \& \quad \text{ neither communication nor internal actions may occur.} \]
Derivation system for $\neg \text{rbk}$

$\Gamma \vdash 1 \vdash \sigma$

$\Gamma, \rho \vdash \sigma \vdash \rho \vdash \sigma$

$\Gamma \vdash \alpha.\rho + \rho' \vdash \overline{\alpha}.\sigma + \sigma' \vdash \rho \vdash \sigma$

$\forall i \in I. \Gamma, \bigoplus_{i \in I} \overline{a_i}.\rho_i \vdash \sum_{j \in I \cup J} a_j.\sigma_j \vdash \rho_i \vdash \sigma_i$

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$\Gamma \vdash \sum_{j \in I \cup J} a_j.\sigma_j \vdash \bigoplus_{i \in I} \overline{a_i}.\rho_i$
Decidability of $\neg r^\text{rbk}$

Definition (Compliance of retractable contracts)

$\gamma \prec \rho \not\vdash^\text{rbk} \delta \prec \sigma$ if and only if

$$\forall \gamma', \delta' \prec \sigma'. \gamma \prec \rho \parallel \delta \prec \sigma \rightarrow \gamma' \prec \rho' \parallel \delta' \prec \sigma' \nrightarrow$$

implies $\rho' = 1$. 
Decidability of $\not\vdash_{rbk}$

**Definition (Compliance of retractable contracts)**

$\gamma \prec \rho \not\vdash_{rbk} \delta \prec \sigma$ if and only if

$$\forall \gamma', \delta' \prec \sigma'. \; \gamma \prec \rho \parallel \delta \prec \sigma \implies \gamma' \prec \rho' \parallel \delta' \prec \sigma' \not\rightarrow$$

implies $\rho' = 1$.

**Theorem**

The derivation system is sound and complete w.r.t. $\not\vdash_{rbk}$, and derivability is decidable, hence $\not\vdash_{rbk}$ is decidable.
Further directions

The sub-contract relation is defined:
\[ \sigma_1 \leq \sigma_2 \iff \forall \rho. \rho \vdash \sigma_1 \Rightarrow \rho \vdash \sigma_2 \]

Bernardi, Hennessy [MSCS 20??] have established that it coincides with must-testing preorder.

How can be characterized \( \leq \wedge \) and \( \leq_{\mathsf{rbk}} \)?

Can the compliance relation be refined w.r.t. infinite contracts, while remaining decidable?

Are contracts with roll-back and reversible processes related?

To what extent roll-back compliance can model adaptability?
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Thank you!