On Context Bisimulation for Parameterized Higher-order Processes

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June, 2013

All processes in this talk are higher-order.

Parameterized higher-order processes are the subject of this talk.

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- Π: $0 \mid X \mid u(X).P \mid \overline{u}P'.P \mid P| P' \mid (c)P$
- Π_n^D: parameterization on processes
  Higher-order abstraction: $\langle X_1, X_2, ..., X_n \rangle P$ $\quad$ ($\lambda X_1, X_2, ..., X_n.P$)
  Application: $P\langle Q_1, Q_2, ..., Q_n \rangle$
  ($X_1, X_2, ..., X_n$ are process variables)

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- Π: $0 \mid X \mid u(X).P \mid \overline{u}P'.P \mid P|P' \mid (c)P$
- Π_n^D: parameterization on processes
  Higher-order abstraction: $\langle X_1, X_2, ..., X_n\rangle P$ $(\lambda X_1, X_2, ..., X_n.P)$
  Application: $P\langle Q_1, Q_2, ..., Q_n\rangle$
  ($X_1, X_2, ..., X_n$ are process variables)
- Π_n^d: parameterization on names
  First-order abstraction: $\langle x_1, x_2, ..., x_n\rangle P$
  Application: $P\langle u_1, u_2, ..., u_n\rangle$
  ($x_1, x_2, ..., x_n$ are name variables)

Main result

- Π_n^D: normal bisimulation characterizes context bisimulation
- Π_n^d: similar technique cannot be applied
  (no obvious characterization is known)

Organization

- BACKGROUND
- RESULTS ON Π_n^D
- RESULTS ON Π_n^d
- DISCUSSION

1 Background

Context bisimulation

$P \approx Q$:

Input, τ: $P \xrightarrow{\alpha} P'$ implies $Q \overset{\widehat{\alpha}}{\Longrightarrow} Q'$ s.t. $P' \approx Q'$

Output: $P \xrightarrow{(\widetilde{c})\overline{a}A} P'$ implies

$Q \overset{(\widetilde{d})\overline{a}B}{\Longrightarrow} Q'$ s.t. $\forall E[X].$ $\boxed{(\widetilde{c})(E[A] \mid P') \approx (\widetilde{d})(E[B] \mid Q')}$

In Π: normal bisimulation simplifies context bisimulation [San92,94]

E.g. $P \cong Q$

Output: $P \xrightarrow{(\widetilde{c})\overline{a}A} P'$ implies

$Q \overset{(\widetilde{d})\overline{a}B}{\Longrightarrow} Q'$ s.t. $\boxed{(\widetilde{c})(P' \mid !m.A) \cong (\widetilde{d})(Q' \mid !m.B)}$ (m fresh)

Comparing the output clauses

$$(\widetilde{c})(P' \mid !m.A) \quad \overset{\cong}{\cdots\cdots} \quad (\widetilde{d})(Q' \mid !m.B)$$

$$R_1 \stackrel{def}{=} (\widetilde{c})(P' \mid (m)(E[Tr_m] \mid !m.A)) \quad \overset{\approx}{\cdots\cdots} \quad (\widetilde{d})(Q' \mid (m)(E[Tr_m] \mid !m.B)) \stackrel{def}{=} R_2$$

$$\approx \qquad\qquad \approx$$

$$(\widetilde{c})(P' \mid E[A]) \quad \overset{\approx}{\cdots\cdots} \quad (\widetilde{d})(Q' \mid E[B])$$

Comparing the output clauses

$$(\widetilde{c})(P' \,|\, !m.A) \;\cdots\cdots \cong \cdots\cdots\; (\widetilde{d})(Q' \,|\, !m.B)$$

$$R_1 \stackrel{def}{=} (\widetilde{c})(P' \,|\, (m)(E[Tr_m] \,|\, !m.A)) \;\cdots \approx \cdots\; (\widetilde{d})(Q' \,|\, (m)(E[Tr_m] \,|\, !m.B)) \stackrel{def}{=} R_2$$

$$\approx \qquad\qquad \approx$$

$$(\widetilde{c})(P' \,|\, E[A]) \;\cdots\cdots \approx \cdots\cdots\; (\widetilde{d})(Q' \,|\, E[B])$$

$\approx$: Factorization property $\quad \boxed{E[A] \approx (m)(E[Tr_m] \,|\, !m.A)}$

$$Tr_m \stackrel{def}{=} \overline{m}.0 \;\; \text{(trigger)}$$

Now, we move on to Π_n^D and Π_n^d.

Transplant the characterization technique to Π_n^D and Π_n^d?

Π_n^D: YES Π_n^d: NO

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Transplant the characterization technique to Π_n^D and Π_n^d?

Π_n^D: YES Π_n^d: NO

Two points:

- Factorization: $E[A] \approx (m)(E[Tr_m] \| !m.A)$? A can be an abstraction
- Processes are strictly higher-order (e.g. abstraction-passing)

2 Results on Π_n^D

Factorization in Π_n^D

Trigger: $Tr_m \stackrel{def}{=} \langle Z \rangle \overline{m} Z$

A, B: abstraction, $E[X]$: non-abstraction, m: fresh.

$$E[A] \;\approx\; (m)(E[Tr_m] \,|\, !m(Z).A\langle Z \rangle)$$

Factorization in Π_n^D

Trigger: $Tr_m \stackrel{def}{=} \langle Z \rangle \overline{m} Z$

A, B: abstraction, $E[X]$: non-abstraction, m: fresh.

$$E[A] \;\approx\; (m)(E[Tr_m] \,|\, !m(Z).A\langle Z \rangle)$$

Similar if $E[X]$ is an abstraction.

Normal bisimulation in Π_n^D

$P \cong Q$:

Output: $P \xrightarrow{(\widetilde{c})\overline{a}A} P'$ implies $Q \stackrel{(\widetilde{d})\overline{a}B}{\Longrightarrow} Q'$ s.t.

$$(\widetilde{c})(P' \mid !m(Z).A\langle Z\rangle) \cong (\widetilde{d})(Q' \mid !m(Z).B\langle Z\rangle) \qquad (m \text{ fresh})$$

Normal bisimulation in Π_n^D

$P \cong Q$:

τ: $P \xrightarrow{\tau} P'$ implies $Q \Longrightarrow Q'$ s.t. $P' \cong Q'$;

Input: $P \xrightarrow{a(Tr_m)} P'$ (m fresh) implies $Q \overset{a(Tr_m)}{\Longrightarrow} Q'$ s.t. $P' \cong Q'$;

Output: $P \xrightarrow{(\widetilde{c})\overline{a}A} P'$ implies $Q \overset{(\widetilde{d})\overline{a}B}{\Longrightarrow} Q'$ s.t.

$$(\widetilde{c})(P' \mid !m(Z).A\langle Z\rangle) \cong (\widetilde{d})(Q' \mid !m(Z).B\langle Z\rangle) \qquad (m \text{ fresh})$$

Theorem 1. *In* Π_n^D*, normal bisimilarity* ($\cong$) *coincides with context bisimilarity* ($\approx$).

3 Results on Π_n^d

Failure in Π_n^d

Failure of Trigger: $Tr_m \stackrel{def}{=} \langle z \rangle \overline{m} z$ is impossible.

Failure of Factorization: $E[A] \approx (m)(E[Tr_m] | !m(z).A\langle z \rangle)$ is unavailable.

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Failure of Trigger: $Tr_m \stackrel{def}{=} \langle z \rangle \overline{m} z$ is impossible.

Failure of Factorization: $E[A] \approx (m)(E[Tr_m] \,|\, !m(z).A\langle z \rangle)$ is unavailable.

There is no way to define normal bisimulation (at least in its original sense).

Crux: purely higher-order; no name-passing (at least in a direct manner)

So the method of normal bisimulation cannot be extended to Π_n^d.

4 Conclusion

- A separation between Π_n^D and Π_n^d concerning the normal technique (sheds light on their expressiveness)
- calls for an alternative (simple) characterization for Π_n^d

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Further direction:

- In Π_n^D, the strong versions of normal bisimulation and context bisimulation also coincide.
- In Π_n^d, examine the expressiveness.
  E.g. comparing with (first-order) π-calculus.
  To collect clue: to what extent it can 'simulate' name-passing.

Thank you