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8 Analogies, conventions, and expert systems in medicine: some insights from a XIX century physiologist

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1. INTRODUCTION

Like detective stories, art critique, and history, medical diagnoses rest on highlighting minuscule cues that may shed a beam of light on a grand hidden picture that may be inferred by induction but will never be deduced from general principles. This method is inherently and profoundly different from making *a priori* generalisations, as is often the case in the natural sciences, where empirical regularities inspire the definition of a set of possibilities (Ginzburg, 1986). Probability theory, with its emphasis on games of chance where the set of possibilities is known and given once and for all (e.g. the six faces of a die, or the two faces of a coin), reflects the needs and the methods of natural sciences. As such, it may not be appropriate for investigative practices such as medicine. For this reason, it is interesting to see how a concept of probability was eventually developed by a medical practitioner who was skilled enough in logic to develop one of his own.

In the second half of the nineteenth century, a physiologist named Johannes von Kries was attempting to apply probability theory to the evaluation of the effectiveness of new drugs. Like many developers of expert systems for medical applications today, von Kries realised that the main difficulty lay in the very definition of events (Fox, 2003). What should count as a ‘‘healing’’? That a particular drug prolonged a patient’s life a little longer than if he had not taken it? That a patient recovered from one disease, then succumbed to another? Furthermore, in some cases it is not

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obvious where the border between two or more diseases lies. Consequently, the classification of symptoms is never obvious.

von Kries became deeply involved in this issue and, as a side interest to his academic career as a physiology professor, he became a logician and a probability theorist. von Kries is interesting precisely because, unlike most probabilists, his background was not in mathematics. Contrary to nearly all other probability theorists, he did not consider games of chance as the prototypical situation where uncertainty would arise. As a physician, he knew that in practical situations uncertainty means a lot more than the occurrence of one out of a given list of possibilities.

von Kries viewed probability as a logical relation based on analogy. By drawing analogies between the present and the past, e.g. between the present symptoms and the past ones, a decision-maker may arrive at the conclusion that a certain event, e.g. a particular disease, is more or less "probable".

von Kries was very much ahead of his time. Notably, he did not think of mental categories as sets of elements exhibiting certain commonalities but rather as incremental collections to which a new element is added because of some similarity with existing members. He did so because he had in mind the way a doctor knows a disease, as a series of practical cases that collectively add to a label whose meaning is shared with colleagues. This is not an *ex ante* definition that is the equivalent of the face of a die, but rather a never-ending construction resting on analogies and similarities between a continuous flow of cases.

von Kries stressed that, since similarity is a subjective judgement between phenomena that are objectively different, objective numerical probabilities are not possible for the very same reason why one cannot sum apples to pears. Thus, any assessment of a numerical probability is to some extent arbitrary. More precisely, it is arbitrary to the extent the judgement of similarity on which it rests is itself arbitrary. In medical contexts, this degree of arbitrariness may be very high.

von Kries focused his research activities on the physiology of the sensory organs and their activation by the nervous system. In the nineteenth century, that was the closest thing to a physiology of psychology. He never arrived at a physical foundation of probability judgements. However, he stands as a leading scholarly figure who included logic and psychology in a course of physiology (von Kries, 1923), as well as a man whose interests spanned several disciplinary boundaries (von Kries, 1925). Nowadays, it is still interesting to see how von Kries characterised probability. It is interesting also because expert systems for medical applications face the same kind of difficulties, and their eventual diffusion may establish or stabilise conventions as envisaged by von Kries's probability theory.

2. THE PROBABILITY THEORY OF A PHYSIOLOGIST

According to von Kries, it is by drawing analogies between the present and the past that an individual becomes able to describe that a certain course of events is more or less "probable". However, the reliability of an analogy depends on the similarity between the past cases and the present one: strictly speaking, situations are almost never the same, and analogies are almost never perfect. Consequently, in general, probabilities cannot be expressed numerically:

> "(...) there exist logical relations that link things known with certainty to others for which they provide a large or small probability, without a numerical mass existing for it. Actually only the strict dependency that in deductive reasoning links premise to conclusion can transpose to the second the certainty of the first. With other logical relations, it is not so. Less even with analogy. If we observed one or more cases of a certain kind occurring in a certain way, and we expect the same course for a similar case, this expectation does not share the certainty of the premises on which it is based. Taken for granted these premises, this expectation is more or less probable. But this logical relation cannot be expressed numerically. The probability of an outcome increases with the number of cases that have come to be known, but it depends also on the grade and the kind of similarity of these cases, and especially on the similarity between the case the current expectation refers to, and the cases that occurred in the past. However, there is no reason to assume that the [similarity] assumptions have the same value".[2] (von Kries, 1886: 26)

[2](...) Verhältnisse des logischen Zusammenhanges giebt, welche, indem gewisse Dinge als sicher gelten, für andere eine mehr oder weniger grosse Wahrscheinlichkeit constituiren, ohne dass für diese ein numerisches Maas existirt. In der That kann ja nur der völlig feste Zusammenhang, welcher bei dem deductiven Schlusse die Conclusion an die Prämissen knüpft, die Sicherheit dieser letzteren unverändert und ohne Abzug auf jene übertragen. Bei anderen logischen Verhältnissen ist dies anders. So zunächst beim Analogie-Schlusse. Wenn wir einen oder mehrere Fälle von gewisser Art in einer bestimmten Weise haben verlaufen sehen und für einen ähnlichen Fall den gleichen Verlauf erwarten, so teilt diese Erwartung ja selbstverständlich nicht die Sicherheit derjenigen Voraussetzungen, auf welche sie sich gründet; sie ist – jene als sicher angenommen – immer nur mehr oder weniger wahrscheinlich. Das hier Statt findende logische Verhältniss hat nun gar nichts zahlenmässig Darstellbares; die Wahrscheinlichkeit des früher beobachteten Erfolges steigt mit der Zahl der bekannt gewordenen Fälle und hängt ausserdem von dem Grade und der Art der Aenlichkeit ab, welche die einzelnen Fälle untereinander und insbesondere der gegenwärtig zu beurteilende mit den früheren zeigt. Für eine Aufstellung gleichwertiger Annahmen aber fehlt hier jeder Anhalt.

Similarly, probability cannot be numerical when induction is involved. In fact, induction and analogy are closely linked to one another:

> "The considerations we made regarding analogy hold for induction as well, which is the process whereby we derive general statements from single experiences. This is particularly so when a general statement has numerous consequences and finds a number of applications, which implies that it can be suggested by very many different experiences. In this case, a numerical measure of its soundness does not exist".[3] (von Kries, 1886: 29)

> "The difference between induction and analogy can be expressed by saying that the first is a relation from the particular to the particular, the second a relation from the particular to the general. Thus, the connections involved in the two cases are definitely of a different kind. But however precise and irrefutable this distinction might be, it is quite an unimportant one. On the one hand, the general statement that results from an induction is the summary of an unlimited or unknown number of particular statements, and each of them could be reached by simple analogy. (...) On the other hand, at least in many cases we can recognise in analogy an implicit induction".[4] (von Kries, 1916: 403)

Exceptions exist, and in certain cases probabilities can be expressed numerically. This happens when empirical facts are qualitatively equal to one another, so that it is not necessary to draw any analogy at all. This is the case, for example, of the games of chance we usually refer to when we

[3]Ganz dasselbe, wie für die Analogie, gilt nun auch für das logische Verhältniss, welches bei dem Inductionsverfahren Statt findet; hierunter soll ein solches verstanden sein, bei dem wir aus einem mehr oder weniger ausgedehnten Erfahrungs-Wissen Sätze allgemeinen Inhaltes ableiten. Insbesondere wenn ein solcher Satz sehr mannigfaltige Consequenzen besitzt, sehr vielfach Anwendung findet, also auch durch sehr verschiedenartige Erfahrungs-Resultate begründet werden kann, wird nicht in Abrede zu stellen sein, dass ein numeriscgihes Maass diesere Begründung oder erfahrungsmässigen Bestätigung nicht existirt.

[4]Was das Verhältnis der Induktion zum Aalogie-Schluß anlangt, so wird man den Unterschied beider zunächst dahin zu fixieren geneigt sein, daß in dem einen Falle der Schluß von einem Einzelnen auf ein enderes koordiniertes Einzelnes, im anderen vom Einzelnen auf eine Gesamtheit gehe, daß also die in einen und andern Falle zugrunde liegenden Geltungsbeziehungen streng verschieden seien. Indessen läßt sich doch nicht übersehen, daß wir hiermit eine zwar präzise und formell einwandsfreie, sachlich aber meist wenig belangreiche Unterscheidung machen. Zunächst nämlich versteht sich, daß der allgemeine Satz, der das Ergebnis des Induktions-Schlusses, die Zusammenfassung einer allerdings unbegrenzten oder mindestens unübersehbaren Menge von Einzelsätzen darstellt, deren jeder auch direkt per analogiam erschlossen werden kann. (...) Anderseits aber können wir auch im Analogie-Schluß wenigstens in vielen Fällen die Anwendung eines allerdings nicht ausdrücklich zum Bewußtsein gebrachten und ausgesprochenen, aber doch stillschweigend supponierten Induktions-Schlusses erblicken.

talk about probability. Nonetheless, in the reality of our daily experiences situations are never perfectly equal to one another.

von Kries acknowledged that, in practice, it is useful to express uncertainty numerically even when the arbitrariness of analogy undermines the theoretical foundations of any numerical probability measure, and that this is what people actually do. The rationale for doing this, according to von Kries, is the application to non-numerical probabilities of a subjective *Taxirung* (evaluation) that expresses the confidence in the reliability of the analogies underlying them. However, von Kries worried about the arbitrariness of this procedure:

> ''We can now examine the question of whether probabilities of any kind can be expressed in numerical form, so that they can be compared with truly numerical probabilities. It seems possible to evaluate a generic probability in such a way as to produce a numerical probability that has the same degree of certainty. In principle there is no objection to such a procedure, but it is necessary to be aware of what it means and of the difficulties that underlie it. If we evaluate numerically the probability of an analogy at $\frac{5}{6}$ and then consider a case of an expectation where the dimensions of the possibility spaces of alternative outcomes are in the ratio 1:5, we are dealing with heterogeneous contexts, that are incomparable by their very nature. Consequently, their point of equivalence is merely a psychological one. What is compared is the psychological certitude in the two contexts: this is all they have in common''.[5] (von Kries, 1886: 181)

If most of our numerical probabilities depend on arbitrary similarity judgements, how do we assess the reliability of a probability value? von Kries built upon Sigwart (1873), Lotze (1874), and Lange (1877), who claimed that probability calculus can be applied after a set of equally possible events has been singled out. Subsequently, higher order probabilities would derive from the combination of these elementary events. von Kries

[5]Es darf nun die Frage aufgeworfen werden, ob nicht eine zahlenmässige Darstellung auch anderer, ganz beliebiger Wahrscheinlichkeiten in der Weise Statt finden kann, dass dieselben mit eigentlich numerischen verglichen werden; es erscheint denkbar, jede Wahrscheinlichkeit zu taxiren, indem man diejenige zahlenmässige Wahrscheinlichkeit angiebt, welche mit ihr gleichen Sicherheits-Grad zu haben scheint. Gegen derartige Abschätzungen ist nun zwar principiell gar nichts einzuwenden; es ist aber notwendig, wol zu beachten, welche Bedeutung sie haben und welchen Schwierigkeiten sie unterliegen. Wenn wir den Wahrscheinlichkeits-Wert eines Analogie-Schlusses zahlenmässig taxiren und auf 5/6 angeben, so sind die logischen Verhältnisse jener Analogie und einer freien Erwartungsbildung, bei welcher die Spielräume in dem Grössen-Verhältniss 1:5 stehen, vollständig heterogen und ihrer Natur nach unvergleichbar. Der Vergleichspunkt beider ist demgemäss ein lediglich psychologischer; verglichen wird die psychologische Gewissheit, welche in dem einen und dem anderen Falle Statt findet; diese ist das einzige beiden Fällen Gemeinsame.

thought of a geometrical representation of possibility spaces as they are conceived by the human mind. He called these possibility spaces *Spielräume*. However, it is of utmost importance to bear in mind that the German word *Spielraum* means "space" as well as "clearance".

Only in the particular case of games of chance do *Spielräume* exist as crisp sets whose sizes can be compared. However, in general, *Spielräume* do not have precise boundaries. Rather, they should be thought as fuzzy sets that entail the necessary clearance for events whose probabilities derive from imperfect analogies. To be absolutely precise, even games of chance cannot be grasped by perfectly crisp *Spielräume*; for instance, because the barycenter of a die is not known with absolute precision, or because frictions on a roulette cannot be distributed with absolute uniformity.

Games of chance offer the simplest instances of *Spielräume*. In this case, the *Spielräume* conceived by a human mind can be deduced from the objective description of a game:

> "Games of chance can be seen as the best approximation to an ideal case where the relationships between *Spielräume* can be deduced, with a great improvement of our understanding. In fact, the meaning of the *Spielraum* principle is much easier to grasp if we can proceed from a description of *Spielräume*".[6] (von Kries, 1916: 621)

In the limiting case of *Spielräume* that correspond to elementary events that exhaust all possibilities, and if the dimensions of these *Spielräume* can be compared to one another, probabilities can be expressed numerically:

> "The final result of our logical research is that hypotheses can be cast into numerical probability relations if they correspond to elementary *Spielräume* whose dimensions can be compared to each another, and that particular probability values arise if these hypotheses exhaust all possibilities".[7] (von Kries, 1886: 36)

[6] Ja, man kann sagen, daß das, was in den realen Zufalls-Spielen gegeben und beachtenswert ist, gerade als eine weitgehende Annäherung an jenen idealen Fall am einfachsten und zutreffendsten beschrieben wird. Und namentlich darf auch bemerkt werden, daß die deduktive Betrachtung der Spielraums-Verhältnisse unser Verständnis in wertvoller Weise vervollständigt. Denn die Bedeutung des Spielraums-Prinzips gewinnt ohne Zweifel in hohem Maße dadurch an Greifbarkeit und Anschaulichkeit, daß wir die Spielräume, um die es sich handelt, in ganz direkter Weise angeben und bezeichnen können.

[7] Als Gesamt-Ergebniss der logischen Untersuchung erhalten wir somit den Satz, dass Annahmen in einen zahlenmässig angebbaren Wahrscheinlichkeits-Verhältniss stehen, wenn sie indifferente und ihrer Grösse nach vergleichbare ursprüngliche Spielräume umfassen, und dass bestimmte Wahrscheinlichkeits-Werte sich daher da ergeben, wo die Gesammtheit aller Möglichkeiten durch eine Anzahl solcher Annahmen erschöpft werden kann.

von Kries was eager to stress that *Spielräume* are by no means objective properties of dice or anything else we can express a probability about. On the contrary, they are constructed by the person who evaluates a probability subjectively, though not without reference to empirical knowledge:

> "Any probability statement requires first of all a list of cases that appear equally possible to our present and individual state of knowledge. Thus, they are of a subjective nature. However, since probability statements are only possible in connection with a particular knowledge having an objective meaning, and since this knowledge contributes to probability expressions, we can say that probability statements have an objective meaning, too".[8] (von Kries, 1886: 76)

Contrary to modern subjectivists, however, von Kries did not prescribe any numerical probability judgement in cases where no empirical knowledge is available. The following discussion of a well-known paradox highlights that the "principle of sufficient reason" is actually not sufficient to establish numerical probabilities everybody would agree upon:

> "If we know that an urn contains an equal number of black and white balls, we estimate the probability of drawing a black (white) ball as ½. According to the principle [of sufficient reason], we should reach the same result if we only knew that the urn contains black and white balls, without knowing anything about their proportions".[9] (von Kries, 1886: 8)

> "Let us come back to the example of an urn containing black and white balls in unknown proportions. The idea that each single ball could be indifferently black or white appears to be just as sensible as the idea that black and white balls are present in equal proportions. But this consideration leads to a uniform probability over all possible proportions of black and white balls! Moreover, further reflections lead us to still different claims. We could say equally well

[8]Wahrscheinlicjkeits-Satz enthält zunächst und unmittelbar eine Aufstellung von Fällen, welche bei unserem gegenwärtigen individuellen Wissens-Stande gleich möglich erscheinen, hat also eine subjective Bedeutung. Da aber eine solche Aufstellung num im Anschluss an gewisse Kenntnisse von objectiver Bedeutung möglich ist, so gelangen auch diese in dem Wahrscheinlichkeitssatze zum Ausdruck, und man darf daher sagen, dass derselbe implicite auch einen objectiven Sinn besitzt.

[9]Wenn wir wissen, dass in einem Gefässe gleich viele schwarze und weisse Kugeln enthalten sind, so setzen wir die Wahrscheinlichkeit, bei einer Ziehung eine weisse oder eine schwarze Kugel zu erhalten, gleich, und beziffern beide mit ½. Dieselbe Ansetzung der Wahrscheinlichkeit würde unserem Princip nach auch dann gerechtfertigt sein, wenn wir überhaupt nur wüssten, dass schwarze und weisse Kugeln in dem Gefässe sind, während ihr Zahlen-Verhältniss uns ganz unbekannt wäre.

> that the simplest assumption is that the urn is either filled with balls of only one color, or with a mixture of balls of both colors. Hence, if the urn contains 1000 balls we should attach higher probabilities to the urn containing a thousand, five hundred or no black balls than, to say, the urn containing 873 black balls. Clearly, the attempt of a comprehensive decomposition of all possibilities gets lost in a maze''.[10] (von Kries, 1886: 33)

Neither a frequentist nor a subjectivist, von Kries was essentially an advocate of the logical view of probability. But his was a very original version of the logical view, one that called for a novel logic.

Without pretending to lay down a comprehensive list of the forms of human thinking, von Kries distinguished between *Werturteile* (value judgements), *Real-Urteile* (reality judgements), and *Beziehungs-Urteile* or *Reflexions-Urteile* (relation judgements). Value judgements, which include aesthetic and moral judgements, fell outside the scope of his investigation. Reality judgements represent ''the first, basic way a subject grasps reality'' (von Kries, 1916: 36), purely subjective understanding such as ''sweet'' or ''red'' (von Kries, 1916: 38), symbols that acquire a meaning when they are linked to the corresponding properties of an empirical object (von Kries, 1916: 40). von Kries devoted much attention to reality judgements, but not the whole of his *Logik* (von Kries, 1916) can be reported here. Rather, we shall focus on relation judgements, which include probability judgements.

Relation judgements connect mental representations to one another (von Kries, 1916: 33). von Kries did not claim that it is possible to provide an exhaustive list of relation judgements, but deemed it sensible to focus on the following ones: *analytische Urteile* (analytical judgements), *Subsumtions-Urteile* or *Inzidenz-Urteile* (subsumption judgements),

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[10]Kommen wir nunmehr auf das Beispiel eines Gefässes, welches mit teils schwarzen teils weissen Kugeln gefüllt ist, nochmals zurück. Auch hier erscheint zunächst die Vorstellung, dass jede einzelne Kugel ebensowol schwarz wie weiss sein könne, im Allgemeinen ebenso berechtigt, wie die oben erwähnte; nach dieser letzteren sollte die Annahme, dass irgend eine beliebige Zahl der Kugeln schwarz und die anderen weiss seien, einen bestimmten, für jede hier gewählte Zahl gleichen Wahrscheinlichkeits-Wert haben; nach der ersteren würde mit überwiegender Wahrscheinlichkeit zu erwarten sein, dass annährend gleich viele schwarze und weisse Kugeln vorhanden sind. Auch hier aber führen weitere Ueberlegungen zu noch ganz anderen Ansätzen. Wir könnten recht wol auch sagen, dass die Füllung des Gefässes mit Kugeln bloss einer Sorte, und anderseits eine zufällige Durcheinandermischung beider Sorten am ehesten anzunehmen sei; es würde danach, wenn tausend Kugeln vorhanden sind, den Annahmen, dass tausend, dass fünfhundert oder dass gar keine schwarz sei, grössere Wahrscheinlichkeit zugeschrieben werden müssen, als etwa der, dass 873 schwarz seien. Der Versuch einer vollständigen Zergliederung aller Möglichkeiten würde sich in ein endloses Labyrinth verlieren und notwendig resultatlos bleiben.

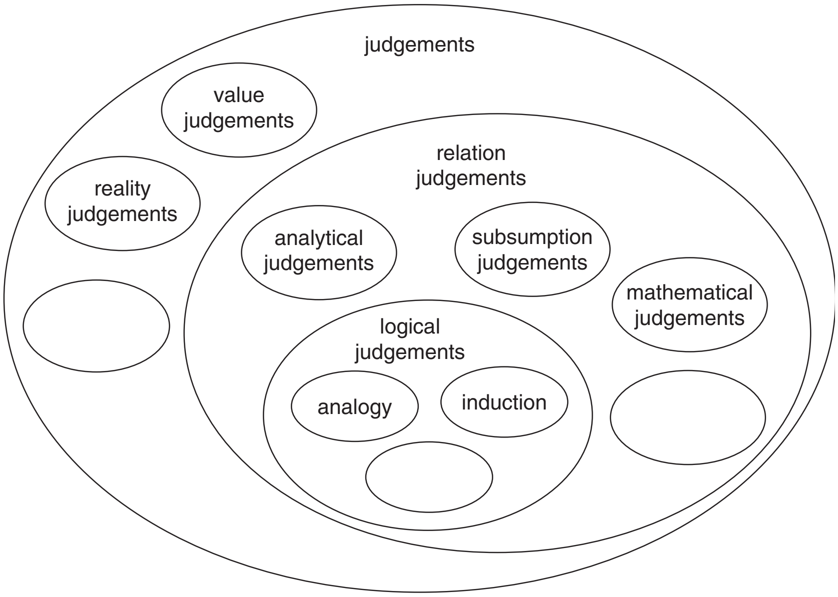

Fig. 1. von Kries's classification of relation judgements.

mathematische Urteile (mathematical judgements) and judgements that establish a dependence, of which the *logische Urteile* (logical judgements) are the most important ones. Among logical judgements we find analogy and induction, which originate probability statements. Figure 1 summarises von Kries's classification, where empty ovals are there to remind that von Kries did not pretend it to be exhaustive.

According to this scheme, probability arises as a logical judgement, from analogy and induction. However, in order to understand analogy and induction, it is necessary to explain the difference between analytical judgements and subsumption judgements. Analytical judgements establish a relation between a synthetic concept and its components. A synthetic concept can be explained by means of a definition, and its parts can be derived by means of deductive logic. Subsumption judgements, on the other hand, establish what von Kries called a "synchytic" concept. In contrast to synthetic concepts, synchytic concepts cannot be explained by means of a definition, but only by means of examples.

> "The [subsumption] judgement is to the construction of a synchytic concept like the analytical judgement is to the construction of a synthetic concept. Both processes provide the material of our thinking out of given elements, but [in the case of the construction of a synchytic concept] we are not able to overview these elements.

> Consequently, we can describe the ensuing concepts and relationships only by means of examples".[11] (von Kries, 1916: 11)

Furthermore, synchytic concepts cannot be crisp and clear-cut:

> "In most cases we are not able to overview all the elements that such concepts entail. And where we can provide a systematic description of the multiplicity of our sensations, as e.g. for the concept "red", we are not able to fix a precise sensation as the border of the concept. Even less with more complex concepts like accident, State, etc. We must conclude that, because of their very construction process, all synchytic concepts are to some extent imprecise".[12] (von Kries, 1916: 12)

Although von Kries did not use the modern terminology, he was actually dealing with mental categories and the way they aggregate information. In particular, the two passages quoted above anticipate the following basic tenets of modern psychology:

1. In general, mental categories have fuzzy boundaries;
2. Each member of a mental category has a similarity with some other members of the category, but not necessarily with all of them. Consequently, mental categories may not entail a group of objects that share common features. Both circumstances prevent categories like. "game" from being described by means of a definition, although a few examples suffice to convey their meaning.

Using his personal terminology, von Kries defined probability judgements as subsumption judgements that, in general, would arrive at synchytic concepts. As such, they are not susceptible of a numerical evaluation:

> "When we judge two expectations or two hypotheses to be equally probable, we make a comparison of the same kind as when we

[11]Wie schon bemerkt, steht ja das Inzidenz-Urteil in der nämlichen Beziehung zu der synchitischen, wie das analytische zu der synthetische Begriffsbildung. Mit diesen beiden Arten der Begriffsbildung sind die Funktionen bezeichnet, vermöge deren wir das begriffliche Material unseres Denkens aus irgendwelchen in anderer Weise gegebenen Elementen bilden. Wir sind aber nicht in der Lage, diese Elemente selbst ohne weiteres zu übersehen oder aufzuzählen. Und wir können dem gemäß jene logischen Funktionen und die aus ihnen resultierenden Beziehungen zunächst nur an einzelnen Beispielen aufweisen.

[12]In der Tat sind wir ja meist nicht in der Lage, die Gesamtheit des Einzelnen, was ein solcher Begriff umfassen soll, übersichtlich darzustellen. Und wo wir, wie z.B. beim Begriffe Rot, eine systematische Darstellung der hierhergehörigen Empfindungs-Mannigfaltigkeit geben können, sind wir doch nicht in der Lage, irgendeine bestimmte Empfindung zu fixieren, die die Grenze des rot zu nennende ausmachte. Noch weniger können wir bei verwickelteren Begriffen, wie Unfall, Staat u. dgl., daran denken, sie als Zusammenfassung einer aufzählbaren Menge scharf angebbarer Beispiele zu betrachten. Alle synchytischen Begriffe sind also, wie das mit der ganzen psychologischen Natur ihrer Bildung zusammenhängt, mehr oder weniger unbestimmt.

> say that two sensations are equally strong, or that two differences of sensations are equally large or equally clear, and so on. Now, (...) we must distinguish very carefully this concept of equality from the strict and definitive equality concept of mathematical expressions. The equalities that are expressed in these statements basically mean that the relationships between the objects to be compared can neither be expressed in terms of more nor in terms of less, but the broad range spanned by these comparisons rests on an even broader and correspondingly imprecise meaning of this more or less, stronger or weaker, and so on. Thus, we can consider these comparisons as [subsumption] judgements (...) that construct an imprecise [equality] concept".[13] (von Kries, 1916: 596)

At this point, we can summarise von Kries's probability theory along the following lines. Probability judgements are analogy and induction relations, linking statements that can be represented by *Spielräume* in a possibility space. Depending on the goodness of the underlying analogies, *Spielräume* can be grasped with varying degree of precision. Strictly speaking, only in the abstract case of games of chance that are known with absolute precision (i.e. a perfect die) *Spielräume* can be compared in a mathematical sense and probabilities can be calculated. In all other cases, probabilities are subsumption judgements, which are not numerical because their comparison is inherently imprecise. However, even in these cases a subjective *Taxirung* can provide a numerical, although arbitrary, probability value.

In practice, real-world games of chance are so close to being perfectly known that the amount of *Taxirung* is negligible. Consequently, probabilities can be safely considered to be numerical. In other fields, such as medical or social statistics, a remarkable amount of subjective *Taxirung* is necessary in order to yield numerical values but arbitrariness (e.g. in the definition of macroeconomic magnitudes) may be covered up by conventions. Finally, there are situations where analogies are so poor that the

[13]Das Urteil, das wir aussprechen, indem wir zwei Erwartungen oder zwei Annahmen in diesem Sinne gleich wahrscheinlich nennen, ist offenbar ein Vergleichungs-Urteil in dem früher des Genaueren besprochenen Sinne, ganz ähnlich wie wenn wir zwei Empfindungen gleich stark, zwei Empfindungs-Unterschiede gleich groß oder gleich deutlich nennen usw. Nun wurde schon oben betont, daß wir den in solche Sätze eingehenden Gleichheits-Begriff wohl unterscheiden müssen von dem strengen und endgültigen der mathematischen Sätze. Die Gleichsetzungen, die in diesen Sätzen ausgesprochen werden, bedeuten im Grunde, daß die Beziehungen der beiden verglichenen Objekte sich weder als ein Mehr noch als ein Minder bezeichnen lassen. Der weite Umfang, in dem solche Vergleichungen ausgeführt werden können, beruht auf dem überaus weiten, aber auch entsprechend unbestimmten Sinne dieses Mehr oder Weniger, Stärker oder Schwächer usw. Wir können solche Vergleichungen also auch auffassen als Aussagen einer Inzidenz (einer durch den Sinn des allgemeinen Begriffes gegebenen Zugehörigkeit) zu jenen unbestimmten Begriffen.

amount of *Taxirung* necessary to yield a numerical probability is so high, and conventions so shaky, that individuals are not able to express any numerical probability at all. This does not mean that they are unable to point out that a certain event is "probable", but merely that this probability cannot take a numerical value.

3. SOME CONSEQUENCES FOR EXPERT SYSTEMS

Through a series of passages and in spite of serious misunderstandings, von Kries had a strong influence on John Maynard Keynes (Fioretti, 1998, 2001, 2003), the economist whose name is associated with the possibility of underemployment equilibria (Keynes, 1936). Keynes realised that, if numerical probabilities arise out of subjective evaluations that may have a tenuous relationship with the objective reality, then the only possibility for reaching an agreement is that of sticking to a convention. If we are in a traditional domain where age-old conventions exist, they will probably persist. But how do people behave if a convention has to be established?

Keynes illustrated the point by means of a simple example: a beauty contest launched in his times by a popular magazine. Readers were confronted with a series of pictures of women whose beauty they were asked to rate. The winning reader was the one who ranked the pictures in the same order as the average reader would. Readers would not be motivated to express their vote according to a personal idea of beauty, but rather according to what they *thought* the mean idea of beauty probably was.

Clearly, this game has no fixed point. The idea of beauty can never settle at a stable equilibrium. Even if a conventional equilibrium is eventually reached, it will not be stable with respect to exogenous perturbations. In general, the oscillations of the idea of beauty will depend on the structure of the communications between the subjects involved, on how many magazines there are and how many readers each has or, in our age, on more powerful visual media, such as television or the internet.

Many domains of applied research rely heavily on conventions in order to yield numerical values. Computational models of climate change are a case in point. Variables are numerous, there are multiple feedback loops among variables and, as if that is not enough, climate is characterised by sharp and unpredictable phase transitions. Under such conditions, computer models may potentially yield any number of conceivable results. Instead, researchers have observed some convergence towards a series of conventional estimates. This is not to say that models never differ in their outcomes: differences do occur, and frequently enough to spur strident policy debates. However, researchers apparently care that some key outputs of their models such as the prospective annual variations of temperature in particular areas of the planet are not too distant from the outcomes of previous models. Too different a result may suggest that the new model is wrong, so it is safer for a researcher to maintain received wisdom rather than reject it altogether (van der Sluijs, 1997).

Medicine may be no less complex than global warming. As von Kries pointed out long ago, not only do a large number of variables interact with one another to produce surprising outcomes, but even the very definition of outcomes is subject to interpretation. How may expert systems impact on such a field? Surely, they will help eliminating wrong diagnoses. May be they will eliminate innovative, better diagnoses as well?

A main rationale for making use of expert systems in medical diagnosis is that they may reduce the variance of treatments by diffusing best practices (Fox and Das, 2000). At the time expert systems are first introduced, it is clear what the best practices are. Thus, the initial adoption of expert systems has only advantages for patients. The question is, what happens after that? How will the knowledge embedded in expert systems be updated? In particular, the following two difficulties may be envisioned:

1. As von Kries pointed out, the identification of a set of possibilities may not be straightforward in medicine. Indeed, the very process of defining a new category for a disease, of recognizing a new symptom or aggregating well-known symptoms in a novel way, may conduct to fundamental discoveries and substantial improvements. It is a "synchytic" judgement, one that requires openness of mind and readiness to accept and formulate novel hypotheses. To the extent that a physician delegates diagnosis to an expert system, the process of hypothesis formulation is constrained by the accepted wisdom. Innovating may become difficult.
2. Suppose that an innovation is made, but not necessarily in a major research centre of big science. A better procedure for making diagnoses, a more effective drug for a particular disease, an improvement of a surgical procedure. How can this innovation be included in the next generation of expert systems? Innovations may be many and not very visible to software houses, they may compete with one another, a judgement on their validity may be influenced by personal idiosyncrasies or preferential commercial relations, and the very relevance of an innovation may be subject to subjective guess and unwarranted beliefs. Similar to the case of the beauty contest, the adoption of innovations by expert systems depends on the shape of the network of relations that channels and distributes information.

Hopefully, we may find ways to avoid such undesirable developments. Most importantly, expert systems should be so designed that physicians understand what the machine is doing and are allowed to re-program it, perhaps by means of a visual programming language. If flexible enough, expert systems may even help physicians to formulate novel ''synchytic'' judgements.

Possibly, expert systems for medical diagnoses should not be monopolised by a few software houses. Indeed, the successful experience of open source and free software stands as a strong indication of what may be done in order to keep network structures flexible and productive (Raymond, 1999). Creative expert systems that are able to stimulate tentative ''synchytic'' judgements by their users would have a positive impact on information networks, from professional associations to chat rooms to mailing lists. To the extent that people understand what a machine is doing and are empowered to change what it is doing, they can ensure that conventions, as encoded in expert systems, evolve rapidly towards the emerging best practice.

4. CONCLUSIONS

von Kries stands as a prominent figure in the development of probability theory, a very original thinker who has been unduly neglected. His greatest originality lies in considering the cognitive processes that originate a set of possibilities in the mind of a decision-maker. Essentially, he singled out conditions for the validity of the premises from which every probability theory starts.

His experience as a physician enabled him to think creatively because he was not biased to consider throwing dice or playing roulette as prototypical situations in which uncertainty arises. Incidentally, it is interesting to note that another fundamental development in the mathematics of uncertain reasoning, Evidence Theory, originated from a scenario in which the thinking process of a judge evaluating testimonies is used as the prototypical setting for uncertainty (Shafer, 1976). In effect, a physician evaluating symptoms is in quite a similar situation as a judge evaluating testimonies: moreover, both of them are conceptually far from playing dice. Once again, we find a similarity between medicine and other disciplines in which the development of problem solving strategies begins with the recognition of clues.

von Kries was incredibly ahead of his time and it is curious that his work has received so little scholarly attention. To my knowledge, no English translation of his works yet exists. Only recently have English-speaking cognitive scientists arrived at the same understanding of mental categories that von Kries developed over a century ago. Perhaps von Kries would have received greater credit for his work, had not the destruction of German culture under the Nazis and the ensuing disaster of World War II occurred.

A more important reason may be the predominance of subjective probability theory, which emerged in the 1930s. Within a paradigm that pretends to mathematise any form of uncertainty by forcing people to make bets, the questions posed by von Kries make no sense. Nowadays, expert systems for medical applications are making us understand that these questions do make sense, and that they are very important indeed.

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