Finite play nondeterministic strategies form a final coalgebra

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It is well-known that the set of deterministic strategies for a game G form a final coalgebra for a polynomial functor $P(G)$ on $\mathbf{Set}^Q$, where Q is the set of positions. Let us take a *finite play nondeterministic strategy* for a game to be a prefix-closed set of finite plays, cf. [2]. Then it turns out that the set of total finitely nondeterministic strategies is the final coalgebra for the corresponding polynomial functor on $\mathbf{Semilatt}^Q$. For example, suppose the set of deterministic strategies is the final coalgebra for $X \mapsto \prod_{i \in I} \sum_{j \in J_i} X$ on $\mathbf{Set}$. Then the set of nondeterministic strategies is the final coalgebra for $A \mapsto \prod_{i \in I} \bigoplus_{j \in J_i} A$ on the semilattice category. Here $\bigoplus$ is the semilattice coproduct. Explicitly, $\bigoplus_{j \in J} B_j$ is the set $\sum_{K \subseteq_{f+} J} \prod_{k \in K} B_k$ with the evident join operation.

A similar story works for countably nondeterministic strategies (or higher cardinalities), and for finite and countable probabilistic strategies, cf. [1], using polynomial functors on the category of convex spaces.

If the nondeterministic strategies are not total, then we use the adjunction $F \dashv U : \mathbf{Semilatt}^{\perp} \to \mathbf{Semilatt}$. In our example, set of nondeterministic strategies will be the final coal gebra for $\prod_{i \in I} F \oplus_{j \in J_i} U$ on $\mathbf{Semilatt}^{\perp}$. A similar story holds for subdistributions.

References

[1] V. Danos and R. Harmer. Probabilistic game semantics. *ACMTCL: ACM Transactions on Computational Logic*, 3, 2002.

[2] R Harmer and G McCusker. A fully abstract game semantics for finite nondeterminism. In *14th Symposium on Logic in Comp. Sci.* IEEE, 1999.