Finite play nondeterministic strategies form a final coalgebra

Paul Blain Levy, University of Birmingham

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It is well-known that the set of deterministic strategies for a game $G$ form a final coalgebra for a polynomial functor $P(G)$ on $\text{Set}^Q$, where $Q$ is the set of positions. Let us take a finite play nondeterministic strategy for a game to be a prefix-closed set of finite plays, cf. [2]. Then it turns out that the set of total finitely nondeterministic strategies is the final coalgebra for the corresponding polynomial functor on $\text{Semilatt}^Q$. For example, suppose the set of deterministic strategies is the final coalgebra for $X \mapsto \prod_{i \in I} \sum_{j \in J_i} X$ on $\text{Set}$. Then the set of nondeterministic strategies is the final coalgebra for $A \mapsto \prod_{i \in I} \bigoplus_{j \in J_i} A$ on the semilattice category. Here $\bigoplus$ is the semilattice coproduct. Explicitly, $\bigoplus_{j \in J} B_j$ is the set $\sum_{K \subseteq \text{fin}(J)} \prod_{k \in K} B_k$ with the evident join operation.

A similar story works for countably nondeterministic strategies (or higher cardinalities), and for finite and countable probabilistic strategies, cf. [1], using polynomial functors on the category of convex spaces.

If the nondeterministic strategies are not total, then we use the adjunction $F -\dashv U : \text{Semilatt}^\perp \to \text{Semilatt}$. In our example, set of nondeterministic strategies will be the final coalgebra for $\prod_{i \in I} F \bigoplus_{j \in J_i} U$ on $\text{Semilatt}^\perp$. A similar story holds for subdistributions.

References
