We begin with the observation that, in many semantic models of programming languages with effects such as nondeterminism or state, the effects themselves correspond to the presence of certain natural transformations in the categorical semantics. If we are careful and make certain qualifications on the morphisms allowed, then game semantic models of nondeterminism contain a natural transformation $\top_A : I \to A$, while the model of Idealized Algol given by Abramsky and McCusker [1] can be thought of as enriching the game semantics of PCF with new morphisms generated by a natural transformation $\mathsf{cell}_X : I \to !\mathtt{Var}[X]$.

We answer the question: if we have a model of an existing language, and an effect that we would like to add to the model expressed as a particular natural transformation we would like to have, is there a systematic way to construct the new model? It turns out that we can do this in a very abstract category-theoretic way, in such a way that the model we get admits a functor into *any* extension of the original model that has the new effect in it. In fact, our construction is *initial* among such extensions, in a way that we can make precise, and so enjoys useful functorial properties.

We show that it is often possible to prove results such as computational adequacy and full abstraction for this category we have constructed, and then lift them on to other models using the induced functor. Properties of the model in question correspond to properties of the functor out of our abstract category; for example, if a model satisfies a factorization result such as *innocent factorization* [1] or *deterministic factorization* [2], then the induced functor is full.

We can relate this to a number of unsolved problems in Game Semantics. For example, it is difficult to give a direct definition of an innocent nondeterministic strategy in Hyland-Ong games, and the Abramsky-Jagadeesan game semantics model has no obvious analogue for the concept of a visible strategy. One approach, suggested in Russ Harmer's thesis [3], is to give an *indirect* definition instead; for example, by defining an innocent nondeterministic strategy to be the composition of an innocent *deterministic* strategy with $\top_{\mathbb{N}}$. One way of viewing our work is as an abstract and fully general crystallization of this approach, since our category is in some sense formed by freely adding the effects into the original model.

In the nondeterminism case, we examine similarities with the work of Tsukada and Ong [4], and discuss some examples of effects other than nondeterminism and state.

References

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