# Safe Dependency Atoms for Logics with Team Semantics

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Team Semantics [12] generalizes Tarski's Semantics for First Order Logic by allowing formulas to be satisfied or not satisfied with respect to sets of assignments (called *teams*), rather than with respect to single assignments. First Order Logic with Team Semantics is easily shown to be equivalent to First Order Logic with Tarski's semantics, in the sense that a first order formula is satisfied by a set of assignments in Team Semantics if and only if it is satisfied by all assignments in the set with respect to Tarski's semantics.

The richer nature of the satisfaction relation of Team Semantics, however, makes it possible to extend First Order Logic in novel ways, such as by introducing new operators or quantifiers [15, 5, 4, 1] or new types of atomic formulas which specify dependencies between different assignments contained in a team. Important logics obtained in the latter way are *Dependence Logic* [14], *Inclusion Logic* [6], and *Independence Logic* [10]. Despite the semantics of the atoms which these logics add to the language of First Order Logic being first order (when understood as conditions over the relations corresponding to teams), these logics are strictly more expressive than First Order Logic. This, in brief, is due to the second order existential quantifications implicit in the Team Semantics rules for disjunction and existential quantification. Thus, exploring the properties of fragments of such logics (as done for instance in [2, 9, 11, 13, 3]) provides an interesting avenue to the study of the properties and relations between fragments of Second Order Logic.

I will present a contribution towards the more systematic study of the properties of first order definable dependency atoms and of the logics they generate. Building on the work of [7, 8], in which I described dependencies which are *strongly first order* in that they do not increase the expressive power of First Order Logic if added to it, I will provide partial answers to the following

**Question:** Let $\mathcal{D} = \{\mathbf{D}_1, \mathbf{D}_2, \ldots\}$ be a set of first order definable dependencies. Can we characterize the sets of dependencies $\mathcal{E} = \{\mathbf{E}_1, \mathbf{E}_2, \ldots\}$ which are **safe** for $\mathcal{D}$, in the sense that every sentence of $FO(\mathcal{D}, \mathcal{E})$ is equivalent to some sentence of $FO(\mathcal{D})$?

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