Interpretpreting Substructural Logics by Games of Information Extraction

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Resource consciousness is routinely cited as a motivation for substructural logics (see, e.g., [7]). However, frequently the reference to resources is kept informal, like in Girard's well-known example of being able to buy a pack of Camels and/or a pack of Marlboro [4] with a single dollar, illustrating linear implication as well as the ambiguity of conjunction between a "multiplicative" and an "additive" reading. Game semantics for fragments and variants of linear logics (see, e.g., [1, 2]) redress this situation by identifying formulas with games and connectives with operators on games. While this supports the study of computational features, the propositional aspect of formulas as assertions seems to get out of sight. On the other hand, dialogical logic (see [6]), which may be viewed as the oldest type of game semantics, sticks with the interpretation of formulas as statements by strategic agents, but hardly cares for resource consciousness.

Motivated by these findings, we have introduced in [3] a new type of two-person games – Client/Server games – based on the idea that a proof is an action-plan i.e. a strategy for one of the players (the "Client") to establish that a particularly structured information package is contained in the information provided by the other player (the "Server"). The interpretation of game states as (single conclusion) sequents leads to variations of the basic game, that match various fragments of intuitionistic linear logic, but also other substructural logics based on variants of Lambek's calculus [5].

The purpose of this contribution is not only to review and extend the results of [3], but rather to explore more generally to which extent the indicated type of games leads to a better understanding of different forms of 'substructurality' as different ways of providing, extracting, and checking structured information in a resource conscious manner. Among the results and ideas that we want to present are the following:

- game based characterizations of a wide range of substructural logics, from intuitionistic linear logic to full intuitionistic logic, in a uniform manner,
- discussion of various alternatives for bundling and structuring information that may or may not correspond to known connectives and types of sequents,
- an analysis of the !-exponential of linear logic with respect to the intended interpretation as 'arbitrarily (but finitely) many times',
- a new interpretation of so-called subexponentials,
- games of information extraction that do not just serve as interpretations of known sequent systems, but rather give rise to new connectives and new proof systems.

References

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