A semantic criterion for MLL and MALL full-completeness

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***Abstract*—This paper focusses on defining a concrete fullycomplete model of linear logic. Until now, those obtained rely crucially on a notion of quotient or 2-categorical tools such as dinatural transformations. This research presents a new categorical construction mixing double-glueing and Chu-construction. We expose a relational semantic of permutations, and make use of the former construction to discriminate them. The resulting model is fully-complete for MLL without units. Our model does not depend on any 2-categorical argument or quotient. Furthermore, it can be enriched with a hypercoherent structure to get a static fully complete model of MALL without units. We make use of this result to prove the Pagani conjecture about hypercorrectedness.**

I. Introduction

Linear logic is one of the most studied logics in computer science, especially from the denotational semantics point of view. Several important models, such as coherence spaces [1] or games [2], have been first introduced as denotational models for linear logic, before being reused successfully for programming languages. Undoubtedly an important criterion to judge the relevance of a model is the property of full completeness, presented in [2], and described as “the tightest possible connection between syntax and semantics”. The main goal of the present research is to find a “concrete” fully complete model. Precisely, we want to avoid the use of 2-categorical tools to model type variables, together with not relying on a quotient. We hope that such a model could provide an important insight to the semantics of programming languages directly, or indirectly linked to linear logic through the Curry-Howard correspondence, such as PCF or the session types. Finally, such a model might be reused across the neighbourhood field of semantics of programming languages to provide models of programming languages that are likely to be fully-abstract.

As of today, most models of logics with type variables are built using di-natural transformations [3], [4], [5], with the intuition that a proof should behave uniformly at variable types [6]. Consequently, the interpretations of the proofs are not concrete, regardless of how simple the morphisms of the initial category were. The first goal of this study was to shift from a 2-categorical setting to a first-order category. As MLL^- proof structures are permutations, we first needed a semantic account of permutations. We developed a way to model them that took inspiration from the concurrent game model of Abramsky and Méllìès[3]. The morphisms of this model were closure operators σ originating from a stable output function f: $\sigma(x) = x \sqcup f(x)$. Our morphisms can alternatively be seen as permutations, lattices, linear functions, output functions associated with a closure operators, a model of data-flow. Notably, we can compose them relationally, through tracing of permutations, or through the Kahn-semantics of data-flow. Similarly we can compose them by composing their associated closure operators.

We wish to discriminate those morphisms that come from a denotation of a proof. We refine them using ideas relating games and static models, especially those developed by Hyland and Schalk [7] [8] when studying the abstract games. We enrich the domains, on which the strategies act, with a new notion of Opponent (O) and Player (P) positions, defined through a correspondence with Chu-dualisation. We furthermore would like to keep the double-glueing construction on which the original concurrent games model rested. This one consists of equipping each object of the category with a set of strategies and counter-strategies. In order to mix the two together successfully, we introduce introduce a new categorical construction that showcase how the double-glueing construction and the Chu-dualization can be interleaved together by requiring that the strategies only act on P-positions, and counter-strategies on O-positions.

This model admits a canonical extension that models the additive connectives of MALL^-. We conjecture that the resulting model is fully-complete. We display some early results in this direction.

Once we obtain our model of MLL^-, we demonstrate how it can be extended to provide full completeness of MALL^-. This follows the method previously established in [4] for double-glued hypercoherences. However, this method was designed with di-natural transformations in mind, while here we show that it adapts well to our previously defined model. In contrast to the games model of MALL^- [3] [9] where morphisms should be considered “up to equivalence”, the model that we have developed is static; therefore each element in the function space model defines a morphism.

This new category provides a proof of the Pagani conjecture about proof structure and hypercoherences [10]: hypercorrectedness together with connectedness entails sequentializability. In particular, as Tranquilli successfully characterized hypercorrectedness of proof-structures [11], this leads to a new definition of proof net for MALL^-.

In the first section of our talk, we remind briefly the Chu-dualization and the double-glueing construction, before exposing how we interleave them together. During the second one we present our semantic of permutations, and how the previous construction can be applied to it. We then proceed to show how the resulting model is fully-complete. In the third section, we show some preliminary results about the canonical extension of the previous model with products. Finally, in the last section, we remind the Pagani conjecture. We then present a new model with hypercoherence, such that the hypercoherence and Chu-condition are orthogonal: they do not interfere with one another. We show how the resulting model is fully-complete. We conclude that the Pagani conjecture is true.

References

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