Comparing Strategies and Böhm Trees in a Probabilistic Setting

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A well-known result [1] in the model theory of the pure λ-calculus is the formal correspondence between the interpretation of a term as a strategy and its infinitely extensional Böhm tree.

The PhD thesis of Leventis [2] suggests a definition of Böhm trees for *probabilistic* λ-terms, *i.e.* terms enriched with a probabilistic choice operator $+_p$. In a probabilistic version of Böhm's Theorem, Leventis shows that infinitely extensional probabilistic Böhm trees are a *fully abstract* representation, meaning that they precisely characterise the observational equivalence of terms.

Recent work on probabilistic concurrent game semantics allows us to give a model for the probabilistic λ-calculus, with an interpretation of terms as probabilistic strategies. One naturally expects the correspondence of [1] to extend to the probabilistic setting. However:

(1) There is a mismatch: the level of intensionality of probabilistic Böhm trees is strictly between that of play-based strategies and that of concurrent (or presheaf-based) strategies.

(2) Proving such a correspondence result in the deterministic case involves a continuity argument, using that strategies and trees are lubs of chains of finite-depth approximants. With probability this method does not suffice: strategies of finite depth may still have *infinite width*, for instance in the case of a term reducing to an infinite probabilistic sum $M_1 +_{p_1} M_2 +_{p_2} \dots$.

We will describe our attempts at solving those two problems. This is work in progress.

References

[1] Andrew D Ker, Hanno Nickau, and C-H Luke Ong. Innocent game models of untyped λ-calculus. *Theoretical Computer Science*, 272(1-2):247–292, 2002.

[2] Thomas Leventis. *Probabilistic lambda-theories.* PhD thesis, Aix-Marseille Université, 2016.