# Algorithms and Data Structures 2010-2011

Lesson 1: *Introduction to algorithms and basic data structures*

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*(slide credits: these slides are a revised version of slides created by Dr. Gabriele D'Angelo)*

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## **Outline** of the lesson

- Introduction to algorithms
- Introduction to data structures and abstract data types
- Abstract data type List
- Basic data structures
  - arrays
  - linked lists

## Algorithm: **informal definition**

A good "informal definition" of **algorithm** is the following:

- an algorithm is any well-defined computational procedure that takes some value (or set of values) as input and produces some value (or set of values) as output
- an algorithm is thus a sequence of computational steps that transforms the **input** into the **output**

**Another definition**: an algorithm is a tool for solving well specific computational problems



## Algorithm: **etymology**

- **Muḥammad ibn Mūsā al-Khwārizmī** was a Persian Islamic mathematician, astronomer, astrologer and geographer. He was born around 780 in Khwārizm (now Khiva, Uzbekistan) and died around 850
- The words **algorism** and **algorithm** stem from Algoritmi, the Latinization of his name


(source: wikipedia)

## Example: **sorting problem**

- Example: **sorting problem**
- **INPUT:** a sequence of *n* numbers $< a_1, a_2, \ldots, a_n >$
- **OUTPUT:** a permutation $< a_1', a_2', \ldots, a_n' >$ of the input sequence such that $a_1' \leq a_2' \leq \ldots \leq a_n'$
- Many algorithms can be used to solve this problem, some of them are really simple (and slow) others are very complex (and fast)



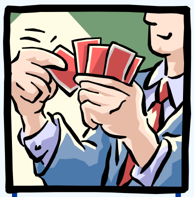
## The "problem" and the algorithm: definitions

- An **instance of a problem** consists of **all inputs needed to compute a solution** (to the problem)
- An algorithm is said to be **correct** if for **every** input instance, it **halts** with the **correct output**
- A correct algorithm **solves** the given computational problem. An **incorrect algorithm** might **not halt at all** on some input instance, or it might **halt with other than the desired answer** (wrong output)

## Example: **sorting** of numbers

### INPUT

- sequence of numbers
  - $a_1, a_2, a_3, \ldots, a_n$
  - 2 5 4 10 7



### OUTPUT

- a permutation of the sequence of numbers
  - $b_1, b_2, b_3, \ldots, b_n$
  - 2 4 5 7 10

- **Correctness**
- For any given input the algorithm halts with the output:
  - $b_1 < b_2 < b_3 < \ldots < b_n$
  - $b_1, b_2, b_3, \ldots, b_n$ is a permutation of $a_1, a_2, a_3, \ldots, a_n$

- **Running time**
- Depends on
  - number of elements ($n$)
  - how (partially) sorted they are
  - algorithm



## Example: **insertion sort**

The **insertion sort** is a very simple sorting algorithm. This algorithm is not very **efficient**

**A**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 6 | 8 | 9 | 7 | 2 | 5 | 1 |

1 ... j→ ... n

←i

### Strategy

1. **Start "empty handed"**
2. **Insert a card in the right position of the already sorted hand**
3. **Continue until all cards are inserted/sorted**

## Some **problems** solved by algorithms

- **The Human Genome Project**: identification of all the 100,000 genes in the human DNA
- **Internet Search Engines**: the Google PageRank is a link analysis algorithm that assigns a numerical weighting to each element of a hyperlinked set of documents, such as the World Wide Web, with the purpose of "measuring" its relative importance within the set (from wikipedia)
- **Electronic commerce**: public-key cryptography and digital signatures (implemented in all Internet browsers)
- **Communication and transmission protocols**: routing algorithms, encoding, data compression etc.



## Data structures

The title of this course is "Algorithms and **Data Structures**"

- Until now we have tried to define what is an algorithm, but what is a "data structure"?
- **DEFINITION:**

  A data structure is a way to store and organize data in order to facilitate operations on them (e.g. data access and modification)

- **VERY IMPORTANT**: no single data structure works well for all purposes, and so it is important to know the **strengths** and **limitations** of several of them

## What is a **data structure**?

- **Definition**: a *representation* and *organization* of data
  - **representation**:
    - data can be stored variously according to their type (for example signed, unsigned, etc.)
    - *example: the representation of integers in memory*
  - **organization:**
    - the way of storing data changes according to the organization (ordered, not ordered, list, tree, etc.)
    - *example: if you have more than one integer?*



## **Properties** of a data structure?

- **Efficient utilization of memory and disk space**
- **Efficient algorithms for:**
  - creation
  - manipulation *(e.g. insertion / deletion)*
  - data retrieval *(e.g. find)*
- **A well-designed data structure uses less resources**
  - computational: *execution time*
  - spatial: *memory space*

## Data structures and algorithms: a little of **terminology**

- **Algorithm:**

  outline, the essence of a computational procedure, step-by-step instructions
- **Program:**

  an implementation of an algorithm in some programming language
- **Data structure:**

  organization of data needed to solve the problem
- **Abstract Data Type (ADT):**

  is the **specification of a set of data** and the set of **operations** that can be performed on the data



## Overall picture

**Data structure and algorithms design goals**

**Implementation goals**

- Correctness

- Efficiency

- Robustness

- Adaptability

- Reusability

## Data structures

- Example of **basic data objects**:

| | |
|---|---|
| Boolean | {false, true} |
| Digit | {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} |
| Letter | {A, B, …, Z, a, b, …, z} |
| NaturalNumber | {0, 1, 2, …} |
| Integer | {0, +1, +2, …, -1, -2, …} |
| String | {a, b, …, aa, ab, ac, …} |

- **Data structures** are composed by **basic data objects**
- Representation of data objects should facilitate an **efficient** implementation of the algorithms



## Abstract data type: **linear List**

**DEFINITION** of linear list (**Abstract Data Type List**):

- Instances are of the form $\{e_1, e_2, \ldots e_n\}$ where $n$ is a finite natural number and represents the **length of the list**
- **In this case** the elements are viewed as atomic, it means that their individual structure is not really relevant
- List is empty → $n=0$
- Relations:

  $e_1$ is first element and $e_n$ is the last (precedence relation)

## Abstract data type: **linear List,** example of **operations**

- Create a list — Create(L)
- Delete a list — Destroy(L)
- Determine if a list is empty — IsEmpty(L)
- Determine the length of the list — Length(L)
- Find the *k-th* element — Find(L, k)
- Search for a given element — Search(L, x)
- Delete the k-th element — Delete(L, k)
- Insert a new element just after the k-th element — Insert(L, x, k)

- Other useful operations could be: **append**, **join**, **copy** …

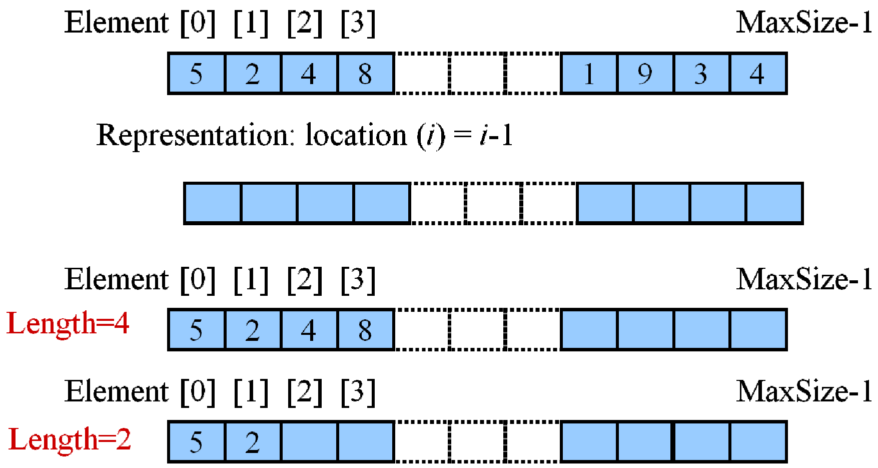


## Data structures: **arrays**

Given such a definition of the **linear List abstract data type**, what is a good data structure to use for its implementation?

- **Array**: an array is a data structure consisting of a *group of elements that are accessed by indexing*
- Usually arrays are **fixed-size**, that is: their size cannot change once their storage has been allocated (i.e. it is not possible to insert new elements in the middle of the array)

## Data structures: **arrays**






## Linear list, **array-based implementation**, operations

| | |
|---|---|
| ■ Create a list | Create(L) |
| ■ Delete a list | Destroy(L) |
| ■ Determine if a list is empty | IsEmpty(L) |
| ■ Determine the length of the list | Length(L) |
| ■ Find the *k-th* element | Find(L, k) |
| ■ Search for a given element | Search(L, x) |
| ■ Delete the k-th element | Delete(L, k) |
| ■ Insert a new element just after the k-th element | Insert(L, x, k) |

**What is the "cost" of such operations given an array-based implementation of the list?**

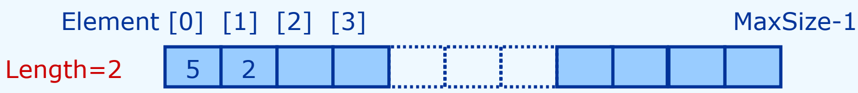
## **Operations**: Search, Delete and Insert

1. **Search(L, x)**
2. **Delete(L, k)**
3. **Insert(L, x, k)**

- These operations could require to scan / modify as much elements as the length of the list!
  - **Therefore, their cost is linear in size of the list**
- What is the cost of the **Find(L, k)** operation?



## Arrays: **inefficient use of space**

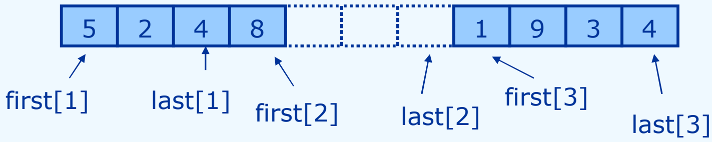



- **EXAMPLE**:
  - assume that we need **3 lists**
  - together will **never** have more than **5000 elements**
  - each list may have **up to 5000** elements at some time or the other

Simple implementation = *15,000 elements* → ***INEFFICIENT***

## **Arrays**: a more efficient solution

- One of the **many possible solutions**:
  - represents all the lists using a **single array**
  - use **two additional arrays** *first* and *last* to index into this one




  - What happens if the list 2 is empty?
  - How to add elements to list 2 when there is no additional space between list 2 and 3?
  - One solution would to "shift" all the elements of 3, what if it is not possible (i.e. the array boundary has been reached)?
  - ➔ **Insertions take more time (at least in the worst case)**



## **Limitations** of the Array data structure

**Advantages** and **disadvantages** of the **array data structure**

- **PRO**:
  - **simple** to use
  - **fast** (in the case of direct access to a defined location)
- **CONS**:
  - must specify **size** at construction time
  - **reorganizations** are quite complex and **costly**

**We need a more flexible data structure!**

## **Dynamic arrays**: general idea

- A possible (and **often wrong**) solution is to implement a sort of **dynamic array**
- In this case, the size of the array depends on its load factor (that is how many elements are in the array)
- It is necessary to modify the operations used to insert and delete elements
- **Problem**: due to implementation constraints the amount of memory allocated for the array is predefined and can not be modified (e.g. increased or decreased) at runtime
- Given an array of length ***MaxSize***



## **Dynamic arrays**: implementation details

- Given an array of length ***MaxSize***

**Insert() operation**

- If we already have *MaxSize* elements in the list:
  1. allocate a new array of size *MaxSize * 2*
  2. copy the elements from old array to new one
  3. delete the old array

**Delete() operation**

- If the list size drops to one-half of the current *MaxSize*
  1. create a smaller array of size *MaxSize / 2*
  2. elements are copied and the old array is deleted

## **Dynamic arrays**: problems

- What are the PRO and CONS of the **dynamic array data structure**?
- How much does it cost each **Insert()** or **Delete()** operation?
- What happens if the number of element in the array is always “near to MaxSize”?

**We need an even more flexible data structure!**

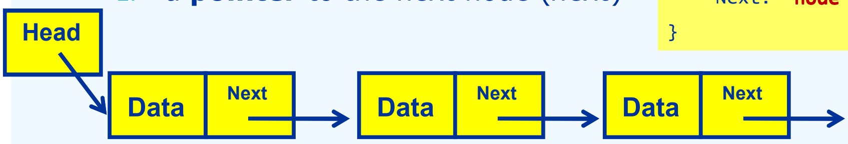


## **Linked lists**: general idea

- **Flexible space use**: dynamically allocate space for each element as needed
- **Linked list**
  - A **list** is a pointer to the head node
  - Each **node** of the list contains:
    1. the **data item** (*data*)
    2. a **pointer** to the next node (*next*)

```
define type list
{
    head: *node := NULL
}
```

```
define type node
{
    Data: integer
    Next: *node
}
```



## **Singly linked list**: definition

- The collection structure has a pointer to the list **head**, that is initially set to **NULL**
- To add the **(first?) new item to the end of the list**:
  1. allocate space for node
  2. set Data as required (initialization of data)
  3. set Next to NULL
  4. set Head to point to the node
- **Be careful in case of first node**
- **...but why to add to the end?**


```
List L;
Integer x;
Function Add_item_to_end(L, x)
{
      new node n
      n.Data = x
      n.Next = NULL
      If (L.Head == NULL) {
             L.Head = *n
      }
      Else {
            pointer = L.Head
            while (pointer <> NULL) {
                  Prev_pointer = pointer
                  pointer = pointer.Next
            }
            Prev_pointer.Next = *n
            }
}
```



## **Singly linked list**: definition

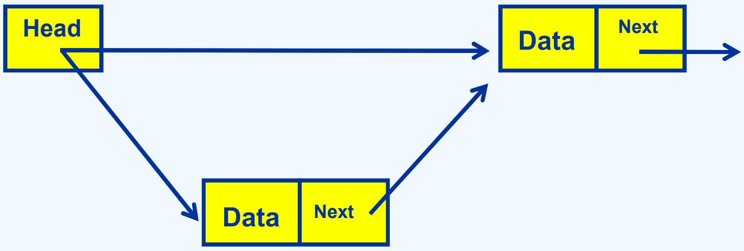
- Check this equivalent solution adding new nodes to the front:
- To add **any** new **item**:
  1. allocate space for node
  2. set Data as required (initialization of data)
  3. set Next to (previous) Head
  4. set Head to new node

```
List L;
Integer x;
Function Add_item_to_front (L, x)
{
      new node n
      n.Data = x
      n.Next = L.Head
      L.Head = *n
}
```

List L : insertion of first item

## **Singly linked list**: implementation

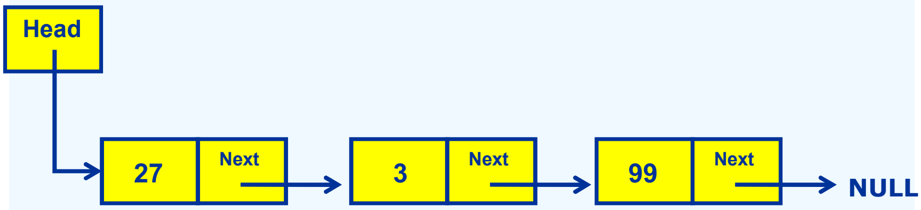
- To add a second item (in front of the list):
  1. allocate space for node
  2. set Data as required (initialization of data)
  3. set Next to current Head
  4. set Head to point to new node






## **Unsorted singly linked list**

- If we suppose that the elements in the list are **unsorted**, the time required to **add a new element** to the list is ***constant***, that is independent of *n* (the size of the existing list)
- In this case, the time required to **search an element** in the unsorted list ***depends on the size of the list***. In the worst case all elements in the list have to be checked (that is, *n*)



## Unsorted singly linked list: Search() operation

```
Function Search(L, x)
{
    pointer = L.Head
    while (pointer <> NULL) {
        if (pointer.Data = x) then
                return(pointer)
        pointer = pointer.Next
    }
    return(NULL)
}
```

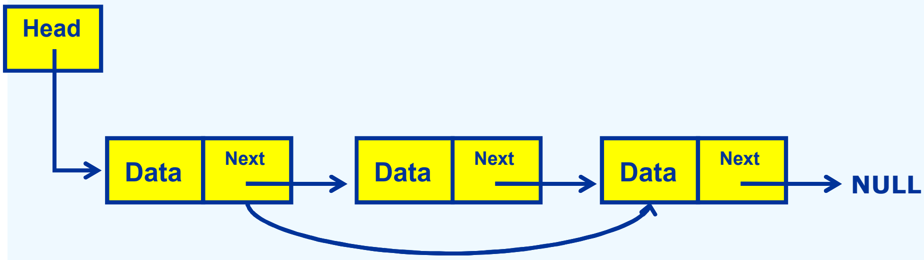
- **L** = list
- **x** = value to find in the list
- **return value** = the pointer to the element or NULL if missing

- This version of the Search() function is **iterative**, also a **recursive** version can be designed



## Unsorted singly linked list: Delete() operation

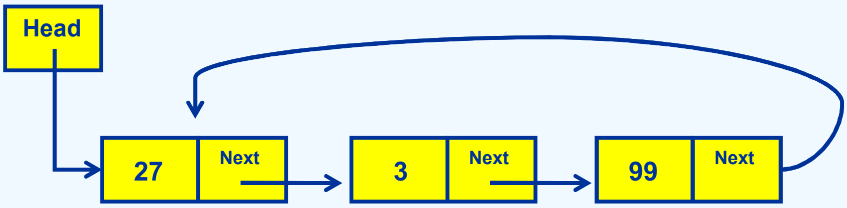
- What happens when an element of the list has to be deleted?




- **IMPORTANT**: the deletion of the first and the last element are special cases that have to be managed very carefully

## **Circular list**: definition

- In the case of a **circular list** the Next pointer of the last element is not NULL: it points to the first element of the list




- Without the NULL pointer as a trailer, how is it possible to check the end of list?

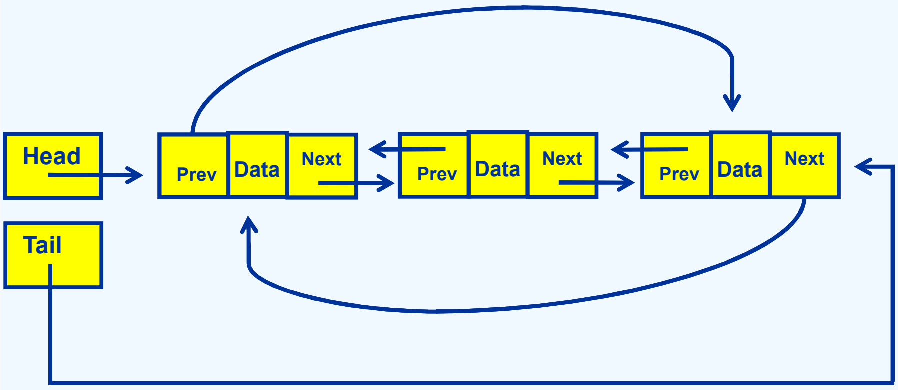


## **Doubly linked lists**: definition

**Doubly linked lists**, each element is composed by:

1. a **pointer** (Prev) to the previous element in the list
2. a **field** to contain the Data
3. a **pointer** (Next) to the next element in the list

## Doubly linked lists






## References

- Part of this material is inspired / taken by the following freely available resources:
  - http://www.cs.rutgers.edu/~vchinni/dsa/
  - http://www.cs.aau.dk/~luhua/courses/ad07/
  - http://www.cs.aau.dk/~simas/ad01/index.html
  - http://140.113.241.130/course/2006_introduction_to_algorithms/courseindex.htm

# Algorithms and Data Structures 2008 - 2009

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