Cooperation and Competition

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Introduction

- Agents need to choose among several options
- Agents do not choose in isolation but the outcome of their decisions (actions) depends on the choices made by other agents they are interacting with
  - pricing a new product in a competitive market
  - biding in an auction
  - choosing a route in a data network
  - choosing a stance in international relations
  - deciding to resort to doping or not
- Want to study notions like "cooperation" in a world where agents are in perpetual competition

Exam or Project

- Student needs to decide whether to study for exam or prepare project (cannot do both)
- Project prepared jointly with a partner
- Exam:
  - if you study, expected grade is 92
  - if you do not study, expected grade is 80
- Project:
  - if both of you work on it, expected joint grade is 100
  - if only one of you works on it, expected joint grade is 92
  - if no one works on it, expected joint grade is 84
- Each of you needs to decide *independently*, knowing that the other will also be making a decision

Exam or Project

Your partner

You		*Project*	*Exam*
	Project	90, 90	86, 92
	Exam	92, 86	88, 88

Ingredients of a Game

- A set of participants called *players*
- Each player has a set of options for behavior called *strategies*
- For each choice of strategies, each player receives a *payoff* that may depend on the strategies selected by other players. Summarized in the form of a *payoff matrix*

Considerations for Games

- How many players? For now, consider only *two-player* games
- How many encounters? For now, consider only *one-shot* games (as opposed to *dynamic* or *iterated* games)

Considerations for Games

- What do the players know?
- For now, assume each players knows *everything* about the structure of the game: who the other players are, the set of strategies, the payoff matrix
  - But not the strategies of the other players
- Each players tries to maximize her own payoff, given her beliefs about the strategies used by other players — *rational* players

Back to "Exam or Project"

- Consider what you should do for each possible choice of strategy by your partner:
  - if you knew that she was going to study, you should study for the exam as well
  - if you knew that she was going to work on the project, you should still study for the exam
- *Strictly Dominant Strategy*: strategy that is the best choice regardless what the other player does

Back to "Exam or Project"

- "Study for exam" is a strictly dominant strategy for both players, meaning each will get an average grade of 88
- Yet, there is an outcome that is better for both (both worked on project and obtain an average grade of 90) that *cannot* be achieved by rational players

Prisoner's Dilemma

- Two robbery suspects apprehended by police, being interrogated in separate rooms
- There is not enough evidence to convict either one
- But each can be charged with a lesser crime (resisting arrest)
- You need to decide whether to confess or not

Suspect 1 \ Suspect 2	*NC*	*C*
NC	-1, -1	-10, 0
C	0, -10	-4, -4

Prisoner's Dilemma

- "Confessing" is a strictly dominant strategy for PD
- Like the "Exam or Project" game, there is an outcome that is better for both (not confess) but that cannot be achieved under rational play
- PD captures the essence of seeking cooperation among selfish individuals:
  - Doping among athletes
  - Nuclear disarmament

Best Responses

- Let S be the strategy chosen by *Player* 1 and T be the strategy chosen by *Player* 2
- Let $P_1(S,T)$ denote the payoff to *Player* 1
- Strategy S for *Player* 1 is a *best response* to a strategy T for *Player* 2 if S produces at least as good a payoff as any other strategy paired with T:

  $P_1(S,T) \geq P_1(S',T)$ for all other strategies S' of *Player* 1

Best Responses

- *Strict best response*: $P_1(S,T) > P_1(S',T)$
- *Dominant strategy* for Player 1 is a strategy that is a best response to *every* strategy of Player 2
- *Strictly dominant strategy* for Player 1 is a strategy that is a strict best response to *every* strategy of Player 2

Nash Equilibrium

- Even in games where there are no dominant strategies, we should expect players to use strategies that are best responses to each other
- If players chose strategies that are best responses to each other, then no player will have an incentive to deviate to an alternative strategy and the system will remain in an "equilibrium"

Nash Equilibrium

- Pair of strategies (S,T), S for *Player* 1 and T for *Player* 2, is a *Nash equilibrium* if S is a best response to T and T is a best response to S
- John Nash won the 1994 Nobel Prize in Economics for this idea that he developed in 1950

Coordination Games

- Two individuals are trying to meet at a shopping mall with two entrances, a North entrance and a South entrance

A \ B	*North*	*South*
North	1, 1	0, 0
South	0, 0	1, 1

- Reasonable to expect that players will play strategies in the Nash equilibrium. But this games has two Nash equilibrium: (North, North) and (South, South)

More Coordination Games

- Unbalanced Coordination Game

		B: North	B: South
A	North	1, 1	0, 0
A	South	0, 0	2, 2

- External or social factors may influence which equilibrium is preferred

More Coordination Games

- Battle of the Sexes

		Your Partner: Action	Your Partner: Romance
You	Action	1, 2	0, 0
You	Romance	0, 0	2, 1

Multiple Equilibria

- Multiple Nash equilibria arise in other games where players engage in a “anti-coordination” activity
- Hawks (aggressive) versus Doves (passive)

		B: Dove	B: Hawk
A	Dove	3, 3	1, 5
A	Hawk	5, 1	0, 0

Multiple Equilibria

- Has two Nash equilibria (Dove, Hawk) and (Hawk, Dove)
- Can be viewed also as the Game of Chicken by interpreting the strategies as “Swerve” and “Do not swerve”