Interactive Generalized Semi-Markov Processes

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Abstract. We have previously introduced Generalized Semi-Markovian Process Algebra (GSMPA), a process algebra based on ST semantics which is capable of expressing durational actions, where durations are expressed by general probability distributions. In this paper we introduce the calculus of Interactive Generalized Semi-Markov Processes (IGSMP), a variant of GSMPA where synchronizable actions are restricted to have zero duration and which introduces the new possibility of expressing priority and non-determinism. The restriction to IGSMP allows us to introduce a notion of observational congruence that abstracts from internal actions with zero duration. Moreover we use a new technique for expressing ST semantics that allows to define observational congruence over IGSMP as a combination of standard observational congruence and probabilistic bisimulation. Finally we present a complete axiomatization for observational congruence over IGSMP.

1 Introduction

In [3] we have introduced Generalized Semi-Markovian Process Algebra (GSMPA), a process algebra that allows to represent actions whose durations are expressed through general probability distributions. The semantics of GSMPA is defined as a variant of ST semantics [8, 2, 6, 5]. According to ST semantics, the evolution of an action is represented in semantic models as a combination of the two events of action start and action termination, where the termination of an action is uniquely related to its start. This approach is very natural for expressing durational actions, especially when duration is expressed through general probability distributions. [1] Moreover the use of ST semantics leads to a notion of choice among durational actions which is based on *preselection policy*. A choice among alternative actions is resolved by first performing a probabilistic choice among the possible actions, and then executing the selected action. Therefore the choice of an action is naturally represented in semantic models as a probabilistic choice among transitions representing action starts. As we also show in [3] this method of solving choices is very adequate and simple when dealing with

[1] Also other languages with generally distributed durations (e.g. SPADES [7]) express timed actions as a combination of action start and action termination. Conceptually the difference with our approach is that we generate events of action start and termination at the semantic level while in SPADES this is done in the syntax.

generally distributed durations. From the semantic model of a term of GSMPA (specifying a concurrent system) it is possible to derive a performance model of the system in the form of a Generalized Semi-Markov Process [11] (GSMP). A GSMP is a stochastic process which can be analyzed through mathematical or simulative techniques in order to obtain performance measures of the system.

In this paper we consider a variant of GSMPA that allows us to define a notion of observational congruence which abstracts from standard τ actions. This is desirable because it may lead to a tremendous state space reduction of semantic models. Technically this is obtained by restricting the possible durations of synchronizable actions to zero durations only. Moreover we introduce the new possibility of expressing priority and non-determinism.

More precisely, following an approach which is quite usual in real-time process algebras (see e.g. [14]), we distinguish between actions f representing a delay whose duration is given by the probability distribution f (itself) and standard actions a of CCS/CSP (including internal τ actions) with zero duration. In analogy to [9] we call the resulting algebra: calculus of *interactive* Generalized Semi-Markov Processes. The name reflects the separated orthogonal treatment of delays and standard actions.

As for the durational actions of GSMPA, the operational semantics of a delay f in IGSMP is defined through ST semantics. Hence a delay is represented in semantic models as a combination of the event of start of the delay f^+ and the event of termination of the delay f^-. Moreover we have to include enough information in semantic models, so that the event of a delay start uniquely determines to which event of a delay termination it refers to, even in the situation of *autoconcurrency*: multiple delays with distribution f being in execution at the same time.

In this paper we employ the new technique for expressing ST semantics that we have introduced in [5]. As opposed to the technique employed in [3], the technique we use here allows to establish ST bisimulation of systems via the standard notion of bisimulation, nevertheless preserving the possibility of obtaining finite ST semantic models in the presence of recursion. While the technique employed in [3] expresses ST semantics by assigning static names (locations) to timed actions, the technique of [5] is based on dynamic names. This means that during the evolution of the system we dinamically assign, at the semantic level, a new index to each delay f that starts execution, on the basis of the indexes assigned to the delays f already started. The fact that we employ a fixed rule for computing new indexes guarantees that ST bisimulation of systems can be established via the standard notion of bisimulation. Moreover, since this rule is such that indexes of terminated delays are reused, we obtain finite ST semantic models for all processes that possess a finite state interleaving semantics as in [3]. The technique introduced in [5] relies on the idea of *levelwise renaming* in order to dynamically assign names in a compositional way, thus allowing to define ST semantics through Structural Operational Semantics (SOS) and to produce a complete axiomatization for ST bisimulation over finite state processes.

As for the durational actions of GSMPA, we resolve choices among several delays by means of preselection policy. As in GSMPA we associate with each delay a weight w: in a choice a delay is selected with probability proportional to its weight. For instance $<f, w>.\underline{0}+<g, w'>.\underline{0}$ represents a system which performs a delay of distribution f with probability $w/(w+w')$ and a delay of distribution g with probability $w'/(w+w')$. Choices are expressed in semantic models by associating weights to transitions f^+ representing the starting of a delay.

The semantics of standard actions a (including internal actions τ) in IGSMP is just the standard interleaving semantics. This reflects the fact that these actions have zero duration and can be considered as being executed atomically. We assume the so-called *maximal progress*: τ actions have priority over delays, thus expressing that the system cannot wait if it has something internal to do, i.e. $\tau.P + <f, w>.Q = \tau.P$.

In [9] the priority of τ actions over delays is captured in the definition of observational congruence. In our approach we capture the priority of τ actions over delays by cutting transitions which cannot be performed directly in semantic models. This allows us to have smaller semantic models and to define observational congruence more simply, without having to discard delay transitions when establishing bisimulation. As opposed to [9] in this paper we do not consider processes with weakly guarded recursion. The reason for this is that here we consider a language with static operators, such as parallel composition, relabelling, and hiding operators. To the best of our knowledge, the problem of finding an axiomatization for weakly guarded recursion which is complete over finite-state processes with static operators is open also for the classical process algebras CCS/CSP. The problem is that there is a simple way to turn a finite-state process with static operators into a sequential process only if the process is strongly guarded. On the other hand the classical procedure for turning weakly guarded processes into strongly guarded ones, which is based on the three axioms developed by Milner [12], works only for sequential processes. In this context a process that is forced in a τ loop can be seen as a Zeno process, i.e. a processes which performs infinite computations without going beyond a certain point in time. Discarding weakly guarded processes allows to avoid the technical complications deriving from the treatment of Zenoness (see, e.g., the stability condition in the definition of bisimulation of [9] and the role of $\perp$ in the related axiomatization) and on the other hand seems not to be so restrictive.

Summing up, the contribution of this work is a weak semantics for a language expressing generally distributed durations, probabilistic choices (preselection policy), non-determinism and priority. The use of the technique of [5] for expressing ST bisimulation allows us to define observational congruence for IGSMP simply as a combination of the standard notion of observational congruence and probabilistic bisimulation [10] and to produce a complete axiomatization for this equivalence over IGSMP.

The paper is structured as follows. In Sect. 2 we present the calculus of interactive GSMP, its operational semantics and the notion of observational congruence. In Sect. 3 we present a complete axiomatization for IGSMP. Finally in Sect. 4 we report some concluding remarks.

2 The Calculus of Interactive GSMPs

2.1 Syntax of Terms and Informal Semantics of Operators

The calculus of interactive GSMPs is an extension of a standard process algebra with operators of CCS/CSP, which allows to express priority, probabilistic choices and probabilistic delays with arbitrary distributions. This is done by including into the calculus, in addition to standard actions, a special kind of actions representing delays. These "timed actions", represented as $<f, w>$, are characterized by a *weight* w and a *duration distribution* f. The weight w determines the probability of choosing the action in a choice among several timed actions. The set of weights is $\mathbb{R}^+$, ranged over by $w, w', \ldots$. The duration distribution f denotes the probability distribution of the action duration. The set of duration probability distribution functions is PDF^+, i.e. the set of probability distribution functions f such that $f(x) = 0$ for $x < 0$, ranged over by f, g, h. The possibility of expressing priority derives from the interrelation of timed actions and standard τ actions. In particular we make the maximal progress assumption: the system cannot wait if it has something internal to do. Therefore we assume that, in a choice, τ actions have priority over timed actions, i.e. $\tau.P + <f, w>.Q = \tau.P$.

Let Act be the set of action types containing a distinguished type τ representing an internal computation. Act is ranged over by $a, b, c, \ldots$. Let $TAct = \{<f, w> \mid f \in PDF^+ \wedge w \in \mathbb{R}^+\}$ be the set of timed actions. Let Var be a set of process variables ranged over by X, Y, Z. Let $ARFun = \{\varphi : Act \longrightarrow Act \mid \varphi(\tau) = \tau \wedge \varphi(Act - \{\tau\}) \subseteq Act - \{\tau\}\}$ be a set of *action relabeling functions*, ranged over by φ.

Definition 1. *We define the language IGSMP as the set of terms generated by the following syntax*

$$P ::= \underline{0} \mid X \mid <f, w>.P \mid a.P \mid P + P \mid P/L \mid P[\varphi] \mid P \parallel_S P \mid recX.P$$

where $L, S \subseteq Act - \{\tau\}$. An IGSMP process is a closed term of IGSMP. We denote by $IGSMP_g$ the set of strongly guarded terms of IGSMP. [2] ■

"$\underline{0}$" denotes a process that cannot move. The operators "." and "+" are the CCS prefix and choice. The choice among timed actions is carried out through the preselection policy by giving each of them a probability proportional to its weight. Note that alternative timed actions are not executed concurrently, first one of them is chosen probabilistically and then the selected action is executed. Moreover τ actions have priority over timed actions in a choice. "$/L$" is the hiding operator which turns into τ the actions in L, "$[\varphi]$" is the relabeling

[2] We consider the timed action $<f, w>$ as being a guard in the definition of strong guardedness.

operator which relabels visible actions according to φ. "$\|_S$" is the CSP parallel operator, where synchronization over actions in S is required. Finally "$recX$" denotes recursion in the usual way.

2.2 Operational Semantics

The semantic model of an *IGSMP* process is a labeled transition system. A transition may represent: the execution of a standard action, the starting of a timed action or the termination of a timed action.

The execution of a timed action is represented in semantic models by the two events of action start and action termination and enough information is provided in order to ensure that each event of action termination is uniquely related to the corresponding event of action start. This corresponds to representing the execution of durational actions with ST semantics [8, 2, 6, 5]. In [3] we show that this semantics is just what we need for representing durational actions, especially if the duration is expressed with general probability distributions. In [5] we have introduced a technique based on dynamic names for generating ST semantic models, which allows to establish ST bisimulation of systems via the standard notion of observational congruence of [12] and to preserve the finiteness of ST semantic models even in the presence of recursion. In *IGSMP* this technique is employed for giving semantics to timed actions. The "type" of a timed action is simply its duration distribution and what we observe of a system is its ability of performing timed actions of certain types $f \in PDF^+$. The problem of preserving the relationship between starts and terminations of timed actions arises, like in the ST semantics of standard process algebras, when several timed actions of the same type f are being executed in parallel. When an action f terminates (event f^-) we need some information for establishing which event of action start (f^+) it refers to.

The technique introduced in [5] is based on the idea of dinamically assigning, during the evolution of the system, a new name to each timed action that starts execution, on the basis of the names assigned to the timed actions already started. Names consist of indexes that distinguish timed actions with the same duration distribution. In particular the event of starting of a timed action $<f, w>$ is represented in semantic models by a transition labeled with $<f_i^+, w>$ where i is the minimum index not already used by the other timed actions with distribution f that have started but not yet terminated. This rule for computing indexes guarantees that names are reused and that finite models can be obtained also in the presence of recursion. The termination of the action is simply represented by a transition labeled with f_i^-, where the "identifier" i uniquely determines which delay f is terminating.

Note that this technique is based on identification of actions similarly to [2, 3], but here the names for actions are not assigned statically according to their syntactical position in the initial term, they are instead computed dynamically while the system evolves. Since the method to compute the index for a starting action is fixed, it turns out that actions of processes that perform the same execution traces of visible actions get the same names. As a consequence, contrary to [2, 3], ST bisimilarity can simply be checked by applying standard bisimilarity to the semantic models of processes.

The technique introduced in [5] allows to dynamically assign names to delays (according to the rule formerly described) via SOS semantics through the idea of *levelwise renaming*. In order to obtain structural compositionality it is necessary to determine, e.g. in the case of the parallel composition operator, the computations of $P \,||\, Q$ from the computations of P and Q. As a consequence we must associate in some way the name f_i generated, according to the fixed rule for creating new names, for identifying a delay starting in P (or Q), to the name f_j generated, according to the fixed rule, for identifying the same starting delay at the level of $P \,||\, Q$. In this way when, afterwards, the delay f_i terminates inside P (or Q) the reference f_i^- can be re-mapped to the correct reference f_j^- at the level of $P \,||\, Q$. This is done by parametrizing in state terms each parallel operator with a mapping M which records such associations while new names are generated. In M a unique index is assigned to each delay f in execution at the level of $P \,||\, Q$, by singling out the delay with the unique index it gets in P or in Q and the "location" of the process that is executing it (*left* if P or *right* if Q).

The weight w associated with the starting of a timed action determines, as already explained, the probability that the action is chosen in spite of other timed actions and therefore the probability that the action starts.

As natural in the context of a stochastic process algebra, we assume timed actions to be *urgent*. Therefore we have that, if a system state can perform a timed action start, it does not let time pass, so possible timed actions in execution cannot terminate in that state. This causes another form of priority in our language: the priority of timed action starts over timed action terminations, i.e. $<f_i^+, w>.P + g_j^-.Q = <f_i^+, w>.P$.

In order to define the operational semantics for the processes of *IGSMP*, we need a richer syntax to represent states. Let $TAct^+ = \{<f_i^+, w> \mid f \in PDF^+ \wedge i \in \mathbb{N} \wedge w \in \mathbb{R}^+\}$ be the set of timed action starts, where $<f_i^+, w>$ represents the beginning of the timed action $<f, w>$ identified with i. Besides let $TAct^- = \{f_i^- \mid f \in PDF^+ \wedge i \in \mathbb{N}\}$ be the set of timed action terminations, where f_i^-, represents the termination of the timed action with duration distribution f identified with i. θ ranges over $TAct^+ \cup TAct^-$, η ranges over $TAct^+ \cup TAct^- \cup TAct$. We denote with $Loc = \{l_i \mid i \in \mathbb{N}^+\} \cup \{r_i \mid i \in \mathbb{N}^+\}$ the set of locations singling out different timed actions with a given duration distribution in the left-hand term (l) or right-hand term (r) of a parallel composition operator. Let $IAssoc$ be the set of partial bijections from $\mathbb{N}^+$ to Loc, i.e. associations mapping the index of a timed action with a given duration distribution to the index the same action gets in the left-hand or right-hand term of a parallel composition operator. M ranges over the set $\{M \subseteq PDF^+ \times (\mathbb{N}^+ \times Loc) \mid \forall f \in PDF^+ .\, M_f \in IAssoc\}$ [3] of mappings, i.e sets including the associations for all the timed actions (with any duration distribution) currently in execution. In the following we use the shorthand $f : (i, loc)$ for $(f, (i, loc)) \in M$.

[3] Given $M \subseteq A \times B$ we denote with M_a the set $\{b \in B \mid (a, b) \in M\}$.

The set $IGSMP_s$ of state terms of $IGSMP$ is generated by the following syntax:

$$P ::= \underline{0} \mid X \mid \eta.P \mid a.P \mid P + P \mid P/L \mid P[\varphi] \mid P \parallel_{S,M} P \mid recX.P$$

We denote by $IGSMP_{sg}$ the set of strongly guarded terms of $IGSMP_s$. We consider the operators "$\parallel_S$" occurring in a $IGSMP$ term P as being "$\parallel_{S,\emptyset}$" when P is regarded as a state.

The semantics of state terms produces a transition system labeled over $Act \cup TAct^+ \cup TAct^-$. The transition relation is defined by the standard operational rules of Table 1 and by the two operational rules in the first part of Table 2 and of Table 3.

$$a.P \xrightarrow{a} P$$

$$\frac{P \xrightarrow{a} P'}{P + Q \xrightarrow{a} P'} \qquad \frac{Q \xrightarrow{a} Q'}{P + Q \xrightarrow{a} Q'}$$

$$\frac{P \xrightarrow{a} P'}{P/L \xrightarrow{a} P'/L}\ a \notin L \qquad \frac{P \xrightarrow{a} P'}{P/L \xrightarrow{\tau} P'/L}\ a \in L$$

$$\frac{P \xrightarrow{a} P'}{P[\varphi] \xrightarrow{\varphi(a)} P'[\varphi]}$$

$$\frac{P \xrightarrow{a} P'}{P \parallel_{S,M} Q \xrightarrow{a} P' \parallel_{S,M} Q}\ a \notin S \qquad \frac{Q \xrightarrow{a} Q'}{P \parallel_{S,M} Q \xrightarrow{a} P \parallel_{S,M} Q'}\ a \notin S$$

$$\frac{P \xrightarrow{a} P' \quad Q \xrightarrow{a} Q'}{P \parallel_{S,M} Q \xrightarrow{a} P' \parallel_{S,M} Q'}\ a \in S$$

$$\frac{P\{recX.P/X\} \xrightarrow{a} P'}{recX.P \xrightarrow{a} P'}$$

Table 1. Standard Rules

The rule of Table 2 defines the transitions representing the start of a timed action, by taking into account the priority of "τ" actions over timed actions and by employing the function SM. $SM(P)$ evaluates the multiset of start moves leaving state P, represented as pairs $(<f_i^+, w>, P')$. [4] This multiset is defined by structural induction as the least element of $\mathcal{M}u_{fin}(TAct^+ \times IGSMP_{sg})$ satisfying

[4] By using multisets we take into account several occurrence of the same weight w.

$$\frac{(<f_i^+, w>, P') \in melt(SM(P)) \wedge \nexists Q : P \xrightarrow{\tau} Q}{P \xrightarrow{<f_i^+, w>} P'}$$
$SM(\underline{0}) = SM(a.P) = SM(f_i^-.P) = \emptyset$ $\quad$ $SM(<f, w>.P) = \{\!\mid (<f_1^+, w>, f_1^-.P) \mid\!\}$ $SM(P/L) = SM(P[\varphi]) = SM(P)$ $\quad$ $SM(P + Q) = SM(P) \oplus SM(Q)$ $SM(P \parallel_{S,M} Q) =$ $\{\!\mid (<f_{n(M_f)}^+, w>, P' \parallel_{S,M \cup \{f:(n(M_f), l_i)\}} Q) \mid (<f_i^+, w>, P') \in SM(P) \mid\!\} \oplus$ $\{\!\mid (<f_{n(M_f)}^+, w>, P \parallel_{S,M \cup \{f:(n(M_f), r_i)\}} Q') \mid (<f_i^+, w>, Q') \in SM(Q) \mid\!\}$ $SM(recX.P) = SM(P\{recX.P/X\})$
$melt(SM) = \{(<f_i^+, w>, P) \mid \exists w' : (<f_i^+, w'>, P) \in SM \wedge$ $w = \sum \{\!\mid w'' \mid (<f_i^+, w''>, P) \in SM \mid\!\}\}$

Table 2. Rules for Start Moves

the rules in the second part of Table 2. [5] The meaning of the rule for "$P \parallel_{S,M} Q$" is the following. When P performs f_i^+ then a new index $n(M_f)$ is determined for identifying the action f at the level of "$\parallel_{S,M}$" and the new association $f : (n(M_f), l_i)$ is added to M. The function $n : IAssoc \longrightarrow \mathbb{N}^+$ computes the new index to be used for identifying the delay f that is starting execution by choosing the minimum index not used by the other delays f already in execution: $n(iassoc) = min\{k \mid k \notin dom(iassoc)\}$. Simmetrically for a move f_i^+ of Q. The function $Melt : \mathcal{M}u_{fin}(TAct^+ \times IGSMP_{sg}) \longrightarrow \mathcal{P}_{fin}(TAct^+ \times IGSMP_{sg})$, defined in the third part of Table 2, merges the start moves with the same derivative state by summing their weights.

The rule of Table 3 defines the transitions representing the termination of a timed action, by taking into account the priority of "τ" actions over timed actions and the priority of timed action starts over timed action terminations, and by employing the auxiliary transition $\rightarrowtail$. The transition relation $\rightarrowtail$, labeled over $TAct^-$, is defined in the second part of Table 3. The meaning of the operational rules for "$P \parallel_{S,M} Q$" is the following. When P performs f_i^- the timed action f with index j associated to l_i in M terminates at the level of the parallel operator. Simmetrically for a move f_i^- of Q.

[5] We denote by $\mathcal{M}u_{fin}(S)$ the set of finite multiset over S, we use $\{\!\mid$ and $\mid\!\}$ as multiset parentheses, and we use $\oplus$ to denote multiset union.

$$\frac{P \overset{f_i^-}{\rightarrowtail} P' \wedge \nexists Q : P \xrightarrow{\tau} Q \wedge \nexists f, w, i, Q : P \xrightarrow{<f_i^+, w>} Q}{P \xrightarrow{f_i^-} P'}$$

$$f_i^-.P \overset{f_i^-}{\rightarrowtail} P$$

$$\frac{P \overset{f_i^-}{\rightarrowtail} P'}{P + Q \overset{f_i^-}{\rightarrowtail} P'} \qquad \frac{Q \overset{f_i^-}{\rightarrowtail} Q'}{P + Q \overset{f_i^-}{\rightarrowtail} Q'} \qquad \frac{P \overset{f_i^-}{\rightarrowtail} P'}{P/L \overset{f_i^-}{\rightarrowtail} P'/L} \qquad \frac{P \overset{f_i^-}{\rightarrowtail} P'}{P[\varphi] \overset{f_i^-}{\rightarrowtail} P'[\varphi]}$$

$$\frac{P \overset{f_i^-}{\rightarrowtail} P'}{P \parallel_{S,M \cup \{f:(j,l_i)\}} Q \overset{f_j^-}{\rightarrowtail} P' \parallel_{S,M} Q} \qquad \frac{Q \overset{f_i^-}{\rightarrowtail} Q'}{P \parallel_{S,M \cup \{f:(j,r_i)\}} Q \overset{f_j^-}{\rightarrowtail} P \parallel_{S,M} Q'}$$

$$\frac{P\{recX.P/X\} \overset{f_i^-}{\rightarrowtail} P'}{recX.P \overset{f_i^-}{\rightarrowtail} P'}$$

Table 3. Rules for Termination Moves

Example 1. In Fig. 1 we depict the semantic model of $recX.f.X \parallel_\emptyset recX.f.X$. In Fig. 1, we let f_i^+ stand for $<f_i^+, 1>$ ■

Note that the semantic model of an *IGSMP* process can simply be turned into a Generalized Semi-Markov Process [11] whose elements are the "identified" timed actions f_i labeling the transitions in the semantic model.

Example 2. In Fig. 2 we depict the minimum semantic model for the recursive system of Fig. 1, which is obtained by merging bisimilar states (the notion of bisimilarity that we employ will be made precise in the following section 2.3). In Fig. 2, as in Fig. 1, we let f_i^+ stand for $<f_i^+, 1>$. The weight 2 of the initial transition derives from the aggregation of the weights of the two initial transitions in the model of Fig. 1 as will be made clear in section 2.3. However since in the initial state there is no alternative to such transition, its weight is not relevant for the actual behavior of the system. From this model we can simply derive the GSMP of Fig. 3 through a formal translation similar to that presented in [4]. In the GSMP of Fig. 3 the states are labeled with the active elements and the transitions with the terminating elements. The elements e_1 and e_2 represent the delays f_1 and f_2 respectively, and the Probabilistic Distribution Function (PDF) of both is given by function f. ■

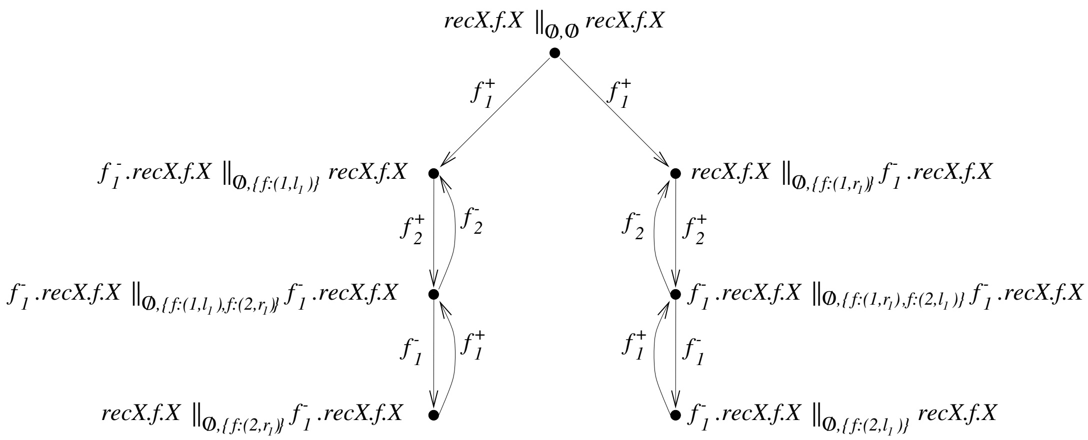


Fig. 1. Example of recursive system

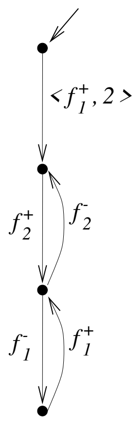
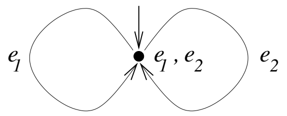


Fig. 2. Minimal semantic model



$PDF(e_1) = f, PDF(e_2) = f$

Fig. 3. GSMP derived from the semantic model

2.3 Observational congruence for IGSMP

The notion of observational congruence for IGSMP is defined as a combination of the classical notion of observational congruence [12] and the notion of probabilistic bisimulation of [10].

In our context we express cumulative probabilities by aggregating weights.

Definition 2. *The function* $w : IGSMP_{sg} \times PDF^{+} \times \mathcal{P}(IGSMP_{sg}) \longrightarrow \mathbb{R}^{+}$, *which computes the aggregated weight that a state* $P \in IGSMP_{sg}$ *reaches a set of states* $C \subseteq \mathcal{P}(IGSMP_{sg})$ *by starting a timed action with duration distribution* $f \in PDF^{+}$ *is defined as:*

$$w(P, f, C) = \sum \{\!| \, w \mid \exists i \in \mathbb{N}, Q \in C : P \xrightarrow{<f_i^{+}, w>} Q \, |\!\} \text{ [6]}$$

■

We are now in a position to define the notion of strong bisimilarity for terms of $IGSMP_{sg}$. Let $NPAct = Act \cup TAct^{-}$, ranged over by α.

Definition 3. *An equivalence relation* β *over closed terms of* $IGSMP_{sg}$ *is a strong bisimulation iff* $P \beta Q$ *implies*

- *for every* $\alpha \in NPAct$,
  $P \xrightarrow{\alpha} P'$ *implies* $Q \xrightarrow{\alpha} Q'$ *for some* Q' *with* $P' \beta Q'$,
- *for every* $f \in PDF^{+}$ *and equivalence class* C *of* β,
  $w(P, f, C) = w(Q, f, C)$

Two closed terms P, Q *of* $IGSMP_{sg}$ *are strongly bisimilar, written* $P \sim Q$, *iff* (P, Q) *is included in some strong bisimulation.* ■

We consider $\sim$ as being defined also on the open terms of $IGSMP_{sg}$ by extending strong bisimilarity with the standard approach of [12].

The definition of weak bisimilarity is an adaptation of that presented in [9] to our context.

We let $\stackrel{\alpha}{\Longrightarrow}$ denote $\xrightarrow{\tau}{}^{*} \xrightarrow{\alpha} \xrightarrow{\tau}{}^{*}$. Moreover we define $\stackrel{\hat{\alpha}}{\Longrightarrow} \; = \; \stackrel{\alpha}{\Longrightarrow}$ if $\alpha \neq \tau$ and $\stackrel{\hat{\tau}}{\Longrightarrow} \; = \; \xrightarrow{\tau}{}^{*}$. Moreover we let C^{τ} denote the set of processes that may silently evolve into an element of C, i.e. $C^{\tau} = \{P \mid \exists Q \in C : P \stackrel{\hat{\tau}}{\Longrightarrow} Q\}$.

Definition 4. *An equivalence relation* β *over closed terms of* $IGSMP_{sg}$ *is a weak bisimulation iff* $P \beta Q$ *implies*

- *for every* $\alpha \in NPAct$,
  $P \stackrel{\hat{\alpha}}{\Longrightarrow} P'$ *implies* $Q \stackrel{\hat{\alpha}}{\Longrightarrow} Q'$ *for some* Q' *with* $P' \beta Q'$,

[6] The summation of an empty multiset is assumed to yield 0. Since the method for computing the new index of a timed action f that starts in a state P is fixed, we have that several actions f^{+} leaving P have all the same index i.

– $P \stackrel{\hat{\tau}}{\Longrightarrow} P'$ *implies* $Q \stackrel{\hat{\tau}}{\Longrightarrow} Q'$ *for some* Q' *such that, for every* $f \in PDF^+$ *and equivalence class* C *of* β,
$w(P', f, C^\tau) = w(Q', f, C^\tau)$

Two closed terms P, Q *of* $IGSMP_{sg}$ *are weakly bisimilar, written* $P \approx Q$, *iff* (P, Q) *is included in some weak bisimulation.* ■

Differently from [9] we do not need to express conditions about the stability of bisimilar processes because we consider only strongly guarded processes. As a consequence there is no process of $IGSMP_{sg}$ that is forced in a τ loop and we do not have to recognize this situation.

Similarly as in [9] it is possible to reformulate weak bisimilarity in the following way, which is simpler but less intuitive.

Lemma 1. *An equivalence relation* β *over closed terms of* $IGSMP_{sg}$ *is a weak bisimulation iff* $P \beta Q$ *implies*

– *for every* $\alpha \in NPAct$,
$P \xrightarrow{\alpha} P'$ *implies* $Q \stackrel{\hat{\alpha}}{\Longrightarrow} Q'$ *for some* Q' *with* $P' \beta Q'$,
– $Q \stackrel{\hat{\tau}}{\Longrightarrow} Q'$ *for some* Q' *such that, for every* $f \in PDF^+$ *and equivalence class* C *of* β,
$w(P, f, C) = w(Q', f, C)$ ■

The proof that this reformulation is correct derives from that given in [9], simply by substituting rates of exponential distributions with weights.

The definition of observational congruence, where again we discard the requirement about stability, is the following one.

Definition 5. *Two closed terms* P, Q *of* $IGSMP_{sg}$ *are observational congruent, written* $P \simeq Q$, *iff:*

– *for every* $\alpha \in NPAct$,
$P \xrightarrow{\alpha} P'$ *implies* $Q \stackrel{\alpha}{\Longrightarrow} Q'$ *for some* Q' *with* $P' \approx Q'$,
– *for every* $\alpha \in NPAct$,
$Q \xrightarrow{\alpha} Q'$ *implies* $P \stackrel{\alpha}{\Longrightarrow} P'$ *for some* P' *with* $P' \approx Q'$,
– *for every* $f \in PDF^+$ *and equivalence class* C *of* β,
$w(P, f, C) = w(Q, f, C)$ ■

Again we consider $\simeq$ as being defined also on the open terms of $IGSMP_{sg}$ by extending observational congruence with the standard approach of [12].

Theorem 1. $\simeq$ *is a congruence w.r.t. all the operators of* $IGSMP$, *including recursion.*

Proof The only relevant case is that of parallel composition operator. It suffice to show that $\{(P_1 \|_{S,M} Q, P_2 \|_{S,M} Q) \mid P_1 \simeq P_2\}$ is a (weak) bisimulation. ■

3 Axiomatization

In this section we present an axiom system which is complete for $\simeq$ on finite state $IGSMP_{sg}$ terms.

The axiom system $\mathcal{A}_{IGSMP}$ for $\simeq$ on $IGSMP_{sg}$ terms is formed by the axioms presented in Table 4. "$\lfloor\lfloor$" and "$|$" denote, respectively, the left merge and synchronization merge operators. γ ranges over semi-actions $<f_i^+, w>$ and f_i^-, and action types $a \in Act$.

The axioms $(Pri1)$ and $(Pri2)$ express the two kind of priorities of IGSMP, respectively, priority of τ actions over (semi-)timed actions and priority of timed action starts over timed action terminations. The axiom (Par) is the standard one except that when the position of processes P and Q is exchanged we must invert left and right inside M. The inverse $\overline{M}$ of a mapping M is defined by $\overline{M} = \{f : (i, r_j) \mid f : (i, l_j) \in M\} \cup \{f : (i, l_j) \mid f : (i, r_j) \in M\}$. Axioms $(LM4)$ and $(LM5)$ just reflect the operational rules of the parallel operator for a timed move of the left-hand process. The axioms (Rec) handle strongly guarded recursion in the standard way [13].

If we consider the obvious operational rules for "$\lfloor\lfloor_{S,M}$" and "$|_{S,M}$" that derive from those we presented for the parallel operator [7] then the axioms of $\mathcal{A}_{IGSMP}$ are sound.

Defined a sequential state to be a state that includes only "$\underline{0}$", "X" and operators ".", "+", "$recX$", we have the following theorem.

Theorem 2. *If an $IGSMP_{sg}$ process P is finite state, then $\exists P' : \mathcal{A}_{IGSMP} \vdash P = P'$ with P' sequential state.*

Proof Let $s_1 \dots s_n$ be the states of the operational semantics of P, $s_n \equiv P$. It can be easily seen that for each $i \in \{1 \dots n\}$, there exist J_i; $\theta_{i,j}$, $k_{i,j}$ with $j \in J_i$ s.t. $\mathcal{A}_{IGSMP} \vdash s_i = \sum_{j \in J_i} \theta_{i,j}.s_{k_{i,j}}$ where $\sum_{\emptyset} \equiv \underline{0}$. Then for each i, from 1 to n, we do the following. If i is such that $\exists j \in J_i : k_{i,j} = i$ we have, by applying $(Rec3)$, that $s_i = recX.(\sum_{j \in J_i : k_{i,j} \neq i} \theta_{i,j}.s_{k_{i,j}} + \sum_{j \in J_i : k_{i,j} = i} \theta_{i,j}.X)$. Then we replace each subterm s_i occurring in the equations for $s_{i+1} \dots s_n$ with its equivalent term. When we have replaced s_{n-1} in the equation for $s_n \equiv P$ we are done. ■

For sequential states the axioms of $\mathcal{A}_{IGSMP}$ involved are just the standard axioms of [13], and the axioms for priority and probabilistic choice. From Theorem 2 and by resorting to arguments similar to those presented in [13] and [9] we derive the completeness of $\mathcal{A}_{IGSMP}$.

Theorem 3. *$\mathcal{A}_{IGSMP}$ is complete for $\simeq$ over finite state $IGSMP_{sg}$ processes.*

Proof The proof of this theorem closely follows the lines of the proof presented in [9]. In particular as far as strongly guarded recursion is concerned the treatment of rates of exponential distributions in [9] is mapped into the treatment of weights. ■

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[7] The definition of the operational rule for "$|_{S,M}$" must allow for actions "τ" to be skipped [1], as reflected by axiom $(SM4)$.

$(A1)$	$P + Q = Q + P$	$(A2)$	$(P + Q) + R = P + (Q + R)$
$(A3)$	$a.P + a.P = a.P$	$(A4)$	$P + \underline{0} = P$
$(Tau1)$	$\gamma.\tau.P = \gamma.P$	$(Tau2)$	$P + \tau.P = \tau.P$
$(Tau3)$	$a.(P + \tau.Q) + a.Q = a.(P + \tau.Q)$		
$(TAct)$	$<f, w>.P = <f_1^+, w>.f_1^-.P$		
$(Prob)$	$<f_i^+, w>.P + <f_i^+, w'>.P = <f_i^+, w + w'>.P$		
$(Pri1)$	$\tau.P + \theta.Q = \tau.P$	$(Pri2)$	$<f_i^+, w>.P + g_j^-.Q = <f_i^+, w>.P$
$(Hi1)$	$\underline{0}/L = \underline{0}$	$(Hi2)$	$(\gamma.P)/L = \gamma.(P/L) \quad \gamma \notin L$
$(Hi3)$	$(a.P)/L = \tau.(P/L) \; a \in L$	$(Hi4)$	$(P + Q)/L = P/L + Q/L$
$(Rel1)$	$\underline{0}[\varphi] = \underline{0}$	$(Rel2)$	$(a.P)[\varphi] = \varphi(a).(P[\varphi])$
$(Rel3)$	$(\theta.P)[\varphi] = \theta.(P[\varphi])$	$(Rel4)$	$(P + Q)[\varphi] = P[\varphi] + Q[\varphi]$
(Par)	$P \parallel_{S,M} Q = P \lfloor\!\lfloor_{S,M} Q + Q \lfloor\!\lfloor_{S,\overline{M}} P + P \,|_{S,M}\, Q$		
$(LM1)$	$\underline{0} \lfloor\!\lfloor_{S,M} P = \underline{0}$		
$(LM2)$	$(a.P) \lfloor\!\lfloor_{S,M} Q = \underline{0}$	$a \in S$	
$(LM3)$	$(a.P) \lfloor\!\lfloor_{S,M} Q = a.(P \parallel_{S,M} Q)$	$a \notin S$	
$(LM4)$	$(<f_i^+, w>.P) \lfloor\!\lfloor_{S,M} Q = <f_{n(M_f)}^+, w>.(P \parallel_{S,M \cup \{f:(n(M_f),l_i)\}} Q)$		
$(LM5)$	$(f_i^-.P) \lfloor\!\lfloor_{S,M \cup \{f:(j,l_i)\}} Q = f_j^-.(P \parallel_{S,M} Q)$		
$(LM6)$	$(P + Q) \lfloor\!\lfloor_{S,M} R = P \lfloor\!\lfloor_{S,M} R + Q \lfloor\!\lfloor_{S,M} R$		
$(SM1)$	$P \,|_{S,M}\, Q = Q \,|_{S,M}\, P$		
$(SM2)$	$\underline{0} \,|_{S,M}\, P = \underline{0}$		
$(SM3)$	$(\gamma.P) \,|_{S,M} (\gamma'.Q) = \underline{0}$	$\gamma \notin S \;\vee\; \gamma \neq \gamma'$	
$(SM4)$	$(\tau.P) \,|_{S,M}\, Q = P \,|_{S,M}\, Q$		
$(SM5)$	$(a.P) \,|_{S,M} (a.Q) = a.(P \parallel_{S,M} Q)$	$a \in S$	
$(SM6)$	$(P + Q) \,|_{S,M}\, R = P \,|_{S,M}\, R + Q \,|_{S,M}\, R$		
$(Rec1)$	$recX.P = recY.(P\{Y/X\})$	provided that Y is not free in *rec*X.P	
$(Rec2)$	$recX.P = P\{recX.P/X\}$		
$(Rec3)$	$Q = P\{Q/X\} \Rightarrow Q = recX.P$	provided that X is strongly guarded in P	

Table 4. Axiomatization for IGSMP

Example 3. Let us consider the system $recX.f.X \,||_{\emptyset}\, recX.f.X$ of the previous example. In the following we show how this process can be turned into a sequential process by applying the procedure presented in the proof of Theorem 2.

In the following we let f_i^+ stand for $<f_i^+, 1>$ and we abbreviate $\mathcal{A}_{IGSMP} \vdash P = Q$ with $P = Q$. Moreover we let $P \equiv recX.f.X$ and $P' \equiv f_1^-.recX.f.X$.

Initially we note that $P = f.recX.f.X = f_1^+.f_1^-.recX.f.X$, by applying $(Rec2)$ and $(TAct)$.

We start the procedure of the proof of Theorem 2 with the initial state the system $P \,||_{\emptyset,\emptyset}\, P$. We have $P \,||_{\emptyset,\emptyset}\, P = f_1^+.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P) + f_1^+.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P)$ by applying (Par), $(LM4)$ and $(SM3)$. From this equation we derive $P \,||_{\emptyset,\emptyset}\, P = <f_1^+, 2>.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P)$ by applying $(Prob)$.

As concerns the state $P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P$ we have $P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P = f_2^+.(P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P')$ by applying (Par), $(LM4)$, $(LM5)$, $(SM3)$ and $(Pri2)$.

As concerns the state $P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P'$ we have $P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P' = f_1^-.(P \,||_{\emptyset,\{f:(2,r_1)\}}\, P') + f_2^-.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P)$ by applying (Par), $(LM5)$ and $(SM3)$.

As concerns the state $P \,||_{\emptyset,\{f:(2,r_1)\}}\, P'$ we have $P \,||_{\emptyset,\{f:(2,r_1)\}}\, P' = f_1^+.(P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P')$ by applying (Par), $(LM4)$, $(LM5)$, $(SM3)$ and $(Pri2)$.

Now we perform the second part of the procedure where we generate recursive processes and we substitute states with equivalent terms.

We start with $P \,||_{\emptyset,\{f:(2,r_1)\}}\, P'$. Since the state does not occur in its equivalent term we do not have to generate any recursion. Substituting the state with its equivalent term in the other equations generates the new equation: $P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P' = f_1^-.f_1^+.(P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P') + f_2^-.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P)$.

Then we consider the state $P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P'$. We change its equation by generating a recursion as follows. $P' \,||_{\emptyset,\{f:(1,l_1),f:(2,r_1)\}}\, P' = recY.(f_1^-.f_1^+.Y + f_2^-.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P))$ Substituting the state with its equivalent term in the remaining equations generates the new equation: $P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P = f_2^+.recY.(f_1^-.f_1^+.Y + f_2^-.(P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P))$.

Now we consider the state $P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P$. We change its equation by generating a recursion as follows. $P' \,||_{\emptyset,\{f:(1,l_1)\}}\, P = recX.(f_2^+.recY(f_1^-.f_1^+.Y + f_2^-.X))$. Substituting the state with its equivalent term in the remaining equations generates the new equation: $P \,||_{\emptyset,\emptyset}\, P = <f_1^+, 2>.recX.(f_2^+.recY(f_1^-.f_1^+.Y + f_2^-.X))$.

Therefore we have turned our initial system $recX.f.X \,||_{\emptyset}\, recX.f.X$ into the recursive sequential process $<f_1^+, 2>.recX.(f_2^+.recY(f_1^-.f_1^+.Y + f_2^-.X))$. Note that the operational semantics of this process generates the labelled transition system of Fig. 2. ■

4 Conclusion

As far as future work is concerned we are trying to extend our approach in two directions.

The capability of IGSMP to express general distributions should allow to extend the notion of observational congruence as follows. The idea is that we could be "weak" also on timed internal actions and equate a sequence of timed τ with a single τ provided that distribution of durations are in the correct relationship. For example a sequence (or a more complex pattern) of exponential τ could be equated by a phase-type distributed τ. The solution of this problem seems quite involved.

Moreover we are investigating the possibility of introducing in GSMPA an operator for action (or delay) interruption. This requires the introduction of a special event in semantic models representing "action interruption" instead of "action termination". An interruption operator would greatly enhance the expressive power of GSMPA, since a preemption mechanism is needed to model many real systems.

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