A Complete Axiomatization for Observational Congruence of Prioritized Finite-State Behaviors

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Abstract. Milner's complete proof system for observational congruence is crucially based on the possibility to equate τ divergent expressions to non-divergent ones by means of the axiom $recX.(\tau.X + E) = recX.\tau.E$. In the presence of a notion of priority, where e.g. actions of type δ have a lower priority than silent τ actions, this axiom is no longer sound because a δ action performable by E is pre-empted in the left-hand term but not in the right-hand term. The problem of axiomatizing priority using the standard observational congruence has been open for a long time. Here we show that this can be done by introducing an auxiliary operator $pri(E)$, by suitably modifying the axiom above and by introducing some new axioms. Our technique provides a complete axiomatization for Milner's observational congruence over finite-state terms of a process algebra with priority and recursion.

1 Introduction

In the last years the expressiveness of classical process algebras has been extended in several ways. Often such extensions have led to the necessity of introducing a notion of priority among actions which is useful, e.g., to model mechanisms of pre-emption. The problems arising from expressing priority have been previously studied in the context of prioritized process algebras, see e.g. [5, 10, 3] and [4] for a survey. One of the open questions in this context (see [4]) is finding a complete axiomatization for observational congruence in the presence of recursion.

In [4], where process algebras with priority are studied in full generality, the authors show that it is necessary to consider rather complex equivalence notions, which deviate from Milner's standard notion of observational equivalence, in order to get the congruence property. Here, we consider a less general form of priority for which the equivalences presented in [4] reduce to Milner's standard notion of observational congruence. This allows us to focus on the problem of finding a complete axiomatization for observational congruence in the presence of priority and recursion. In particular the process algebra we consider is the algebra of finite-state agents used by Milner in [9] for axiomatizing observational congruence in presence of recursion, extended with δ prefixing, where δ actions have lower priority than silent τ actions. A language like this has been studied also in [6], where δ actions represent non-zero time delays, classical actions of

CCS are executed in zero time, and the priority of τ actions over δ actions derives from the maximal progress assumption (i.e. the system cannot wait if it has something internal to do). As in [5, 10, 3, 4, 6] we assume that visible actions never have pre-emptive power over lower priority δ actions, because we see visible actions as indicating only the potential for execution. On the other hand, as observed e.g. in [4], in the presence of a restriction operator this assumption is necessary to get the congruence property.

In the literature complete axiomatizations of equivalences for algebras with priority have been presented for recursion free processes (e.g. in [10]). For this class of processes a notion of priority of τ actions (silent actions) over actions of a type δ is simply axiomatized by adding the axiom $\tau.E + \delta.F = \tau.E$ to the standard Milner's complete proof system for observational congruence [8, 9].

When we consider also recursive processes, if we simply try to extend Milner's proof system for observational congruence [8] with the axiom above, we do not obtain a correct axiomatization. This because the axiom:

$$recX.(\tau.X + E) = recX.\tau.E \qquad (*)$$

which is part of the proof system of [9], is no longer sound. For example if $E \equiv \delta.E'$, the right-hand term can (weakly) perform δ, whilst the left-hand one cannot because of the priority of τ actions over δ actions. The axiom above is crucial for the completeness of the axiomatization in that it allows to "escape" τ divergence in weakly guarded recursive terms, during the process of turning them into strongly guarded ones. The axiom reflects the possibility of observational congruence to always equate τ divergent expressions to non-divergent ones.

The effects of priority on the possibility of escaping divergence have been previously studied in [6], where the problem of finding a complete axiomatization for recursive processes is tackled by considering a variant of Milner's standard notion of observational congruence, that makes it not always possible to escape τ divergence. In particular Milner's observational congruence is modified with the additional requirement that two bisimilar terms must have the same opportunity to silently become stable terms, i.e. terms that cannot perform τ actions. Adopting this additional constraint, which now makes observational congruence certainly sensitive to τ divergence, provides a simple necessary and sufficient condition for the possibility of escaping divergence and for applying the axiom $(*)$. Divergence can be escaped in $recX.(\tau.X + E)$ if and only if E may perform a silent move.

In this paper we show that priority and Milner's standard notion of observational congruence are compatible: we obtain a complete axiomatization for an algebra with priority and recursion even if it is always possible to escape τ divergence. This is done by replacing E in the right-hand term of axiom $(*)$ with $pri(E)$, where pri is an auxiliary operator. The behavior of $pri(E)$ is obtained from that of E by removing all transitions δ (and subsequent behaviors) performable in state E. Therefore the new axiom is:

$$recX.(\tau.X + E) = recX.\tau.pri(E)$$

Since E may be a term including free variables (e.g. in the case of nested recursions), in our axiomatization an important role in the process of turning unguarded recursive terms into strongly guarded ones is played by terms $pri(X)$. In particular we introduce the new axiom:

$$recX.(\tau.(pri(X) + E) + F) = recX.(\tau.X + E + F)$$

which is needed to remove unnecessary occurrences of terms $pri(X)$ in weakly guarded recursions. Finally we add some basic axioms that capture the δ eliminating behavior of the new auxiliary operator.

In this way we provide a complete axiomatization for observational congruence over processes of the process algebra with priority.

The paper is structured as follows. In Sect. 2 we present the algebra with priority and its operational semantics. In Sect. 3 we present a complete axiomatization for observational congruence over processes of the algebra. Finally in Sect. 4 we report some concluding remarks.

2 Prioritized Algebra

The prioritized algebra that we consider is just CCS without static operators [9] extended with δ prefixing, where δ actions have lower priority than silent τ actions. The set of prioritized actions, which includes the silent action τ denoting an internal move, is denoted by $PAct$, ranged over by $a, b, c, \ldots$. The set of all actions is defined by $Act = PAct \cup \{\delta\}$, ranged over by $\alpha, \alpha', \ldots$. The set of term variables is Var, ranged over by $X, Y, \ldots$. The set $\mathcal{E}$ of behavior expressions, ranged over by $E, F, G, \ldots$ is defined by the following syntax.

$$E ::= \underline{0} \mid X \mid \alpha.E \mid E + E \mid recX.E$$

The meaning of the operators is the standard one of [8, 9], where $recX$ denotes recursion in the usual way.

As in [9] we take the liberty of identifying expressions which differ only by a change of bound variables (hence we do not need to deal with α-conversion explicitly). We will write $E\{F/X\}$ for the result of substituting F for each occurrence of X in E, renaming bound variables as necessary.

We adopt the standard definitions of [8, 9] for *free* variable, and *open* and *closed* term. The set of processes, i.e. closed terms, is denoted by $\mathcal{P}$, ranged over by $P, Q, R, \ldots$.

We adopt the following standard definitions concerning guardedness of variables.

Definition 1. X is *weakly guarded* in E if each occurrence of X is within some subexpression of E of the form $\alpha.F$. X is *(strongly) guarded* in E if we additionally require that $\alpha \neq \tau$. X is *unguarded* in E if X is not (strongly) guarded in E. X is *fully unguarded* in E if X is neither strongly nor weakly guarded in E. ■

$$\alpha.E \xrightarrow{\alpha} E$$

$$\frac{E \xrightarrow{a} E'}{E + F \xrightarrow{a} E'} \qquad \frac{F \xrightarrow{a} F'}{E + F \xrightarrow{a} F'}$$

$$\frac{E \xrightarrow{\delta} E' \quad F \not\xrightarrow{\tau}}{E + F \xrightarrow{\delta} E'} \qquad \frac{F \xrightarrow{\delta} F' \quad E \not\xrightarrow{\tau}}{E + F \xrightarrow{\delta} F'}$$

$$\frac{E \xrightarrow{\alpha} E'}{recX.E \xrightarrow{\alpha} E'\{recX.E/X\}}$$

Table 1. Operational Semantics

A recursion $recX.E$ is weakly guarded, (strongly) guarded, unguarded, fully unguarded, if X is weakly guarded, (strongly) guarded, unguarded, fully unguarded in E, respectively. As in [9] we say that an expression E is guarded (unguarded) if every (some) subexpression of E which is a recursion is guarded (unguarded).

The operational semantics of the algebra terms is given as a relation $\longrightarrow \subseteq \mathcal{E} \times Act \times \mathcal{E}$. We write $E \xrightarrow{\alpha} F$ for $(E, \alpha, F) \in \longrightarrow$, $E \xrightarrow{\alpha}$ for $\exists F : (E, \alpha, F) \in \longrightarrow$ and $E \not\xrightarrow{\alpha}$ for $\nexists F : (E, \alpha, F) \in \longrightarrow$. $\longrightarrow$ is defined as the least relation satisfying the operational rules in Table 1.

As in [10] we capture the priority of τ actions over δ actions by cutting transitions which cannot be performed directly in semantic models (and not by discarding them at the level of bisimulation definition as done in [6]) so that we can just apply the ordinary notion of observational congruence [8].

Note that the rule for recursion is not the standard one used by Milner in [8, 9], in that it defines the moves of $recX.E$ in a pure structural way, starting from those of E (where X occurs free), instead of $E\{recX.E/X\}$. This rule, which was proven to be equivalent to that of Milner in [1], gives the possibility to simplify the proofs of some results. In particular such proofs can be made by simply inducing on the structure of terms instead of inducing on the depth of the inference of term transitions.

The equivalence notion we consider over the terms of our prioritized process algebra is the standard notion of observational congruence extended to open terms [8, 9]. In particular we consider here the following characterization of observational congruence given in [9].

As in [9] we use $\xrightarrow{\alpha}{}^{+}$ to denote computations composed of all $\xrightarrow{\alpha}$ transitions whose length is at least one and $\xrightarrow{\alpha}{}^{*}$ to denote computations composed of all $\xrightarrow{\alpha}$ transitions whose length is possibly zero. Let $\stackrel{\alpha}{\Longrightarrow}$ denote

$\xrightarrow{\tau}^* \xrightarrow{\alpha} \xrightarrow{\tau}^*$. Moreover we define $\stackrel{\hat{\alpha}}{\Longrightarrow} = \stackrel{\alpha}{\Longrightarrow}$ if $\alpha \neq \tau$ and $\stackrel{\hat{\tau}}{\Longrightarrow} = \xrightarrow{\tau}^*$. Let $E \rhd X$ denote that E contains a free unguarded occurrence of X.

Definition 2. A relation $\beta \subseteq \mathcal{E} \times \mathcal{E}$ is a weak bisimulation if, whenever $(E, F) \in \beta$:

- If $E \xrightarrow{\alpha} E'$ then, for some F', $F \stackrel{\hat{\alpha}}{\Longrightarrow} F'$ and $(E', F') \in \beta$.
- If $F \xrightarrow{\alpha} F'$ then, for some E', $E \stackrel{\hat{\alpha}}{\Longrightarrow} E'$ and $(E', F') \in \beta$.
- $E \rhd X$ iff $F \rhd X$.

Two expressions E, F are weakly bisimilar, written $E \approx F$, iff (E, F) is included in some weak bisimulation. ■

Definition 3. Two expressions E, F are observational congruent, written $E \simeq F$, iff:

- If $E \xrightarrow{\alpha} E'$ then, for some F', $F \stackrel{\alpha}{\Longrightarrow} F'$ and $E' \approx F'$.
- If $F \xrightarrow{\alpha} F'$ then, for some E', $E \stackrel{\alpha}{\Longrightarrow} E'$ and $E' \approx F'$.
- $E \rhd X$ iff $F \rhd X$. ■

Corollary 4. *If $E \simeq F$ then:*

$$E \xrightarrow{\tau} \quad \Leftrightarrow \quad F \xrightarrow{\tau}$$ ■

The following theorem shows that the presence of priority preserves the congruence property of observational congruence w.r.t. the operators of the algebra.

Theorem 5. *$\simeq$ is a congruence w.r.t. prefix, choice and recursion operators.*

Proof In the case of the prefix operator the proof is trivial. As far as the choice operator is concerned, the only problematic case arises when, given $E \simeq F$, we have a move $E + G \xrightarrow{\delta} H$ derived from $G \xrightarrow{\delta} H$. In this case we can conclude that $F + G \xrightarrow{\delta} H$ by using Corollary 4. See [2] for a detailed proof which also includes the case of the recursion operator. ■

3 Axiomatization

We now present an axiomatization of $\simeq$ which is complete over processes $P \in \mathcal{P}$ of our algebra.

As we already explained in the introduction, the law of ordinary CCS which allows to escape τ divergence:

$$recX.(\tau.X + E) = recX.\tau.E$$

is not sound in a calculus with this kind of priority. Consider for instance the divergent term $F \equiv recX.(\tau.X + \delta.\underline{0})$. Because of priority of "τ" actions over "δ" actions the operational semantics of F is isomorphic to that of $recX.\tau.X$. Hence F is an infinitely looping term which can never escape from divergence by executing the action "δ". If the cited law were sound, we would obtain $F = recX.\tau.\delta.\underline{0}$ and this is certainly not the case.

In order to overcome this problem in [6] the distinguished symbol "$\perp$" is introduced, which represents an ill-defined term that can be removed from a summation only if a silent computation is possible. In this way by considering the rule $recX.(\tau.X + E) = recX.\tau.(E + \perp)$ the resulting term which escapes divergence can be turned into a "normal" term only if E may execute a silent move.

This law is surely sound (over terms without "$\perp$") also in our language, but is not sufficient to achieve completeness. Since, differently from [6], we do not impose conditions about stability in our definition of observational congruence, we can escape divergence not only when E includes a silent move but *for all possible* terms E. For example in our calculus (but not in [6]) the term $recX.\tau.X$ is equivalent to $\tau.\underline{0}$ (as in standard CCS), so we can escape divergence in F even if $\tau.X$ has not a silent alternative inside $recX$. In general the behavior of E' such that $recX.\tau.E' = recX.(\tau.X + E)$ is obtained from that of E by removing all "δ" actions (and subsequent behaviors) performable in state E. We denote such E', representing the "prioritized" behavior of state E, with $pri(E)$. The operational semantics of the auxiliary operator pri is just:

$$\frac{E \xrightarrow{\alpha} E'}{pri(E) \xrightarrow{\alpha} E'} \qquad \alpha \neq \delta$$

Therefore in our case τ divergence can always be escaped by turning E into $pri(E)$ and the strongly guarded terms we obtain are always "well-defined" terms.

Note that the introduction of this auxiliary operator is crucial for being able to axiomatize the priority of τ actions over δ actions in our case. Since we have to remove δ actions performable by a term E even if E does not include a silent move, we cannot do this by employing a special symbol like "$\perp$" instead of using an operator. This because $\perp$ must somehow be removed at the end of the deletion process (in [6] $\perp$ is eliminated by silent alternatives) in order to obtain a "normal" term.

The axiomatization of "$\simeq$" we propose is made over the set of terms $\mathcal{E}_{pri}$, generated by extending the syntax to include the new operator $pri(E)$. We start by noting that the congruence property trivially extends to this operator.

Theorem 6. *$\simeq$ is a congruence w.r.t. the new operator $pri(E)$.*

We adopt the following notion of *serial* variable, which is used in the axiomatization.

Definition 7. X is *serial* in $E \in \mathcal{E}_{pri}$ if every subexpression of E which contains X, apart from X itself, is of the form $\alpha.F$, $F' + F''$ or $recY.F$ for some variable Y. ∎

$(A1)$	$E + F = F + E$	
$(A2)$	$(E + F) + G = E + (F + G)$	
$(A3)$	$E + E = E$	
$(A4)$	$E + \underline{0} = E$	
$(Tau1)$	$\alpha.\tau.E = \alpha.E$	
$(Tau2)$	$E + \tau.E = \tau.E$	
$(Tau3)$	$\alpha.(E + \tau.F) + \alpha.F = \alpha.(E + \tau.F)$	
$(Pri1)$	$pri(\underline{0}) = \underline{0}$	
$(Pri2)$	$pri(a.E) = a.E$	
$(Pri3)$	$pri(\delta.E) = \underline{0}$	
$(Pri4)$	$pri(E + F) = pri(E) + pri(F)$	
$(Pri5)$	$pri(pri(E)) = pri(E)$	
$(Pri6)$	$\tau.E + F = \tau.E + pri(F)$	
$(Rec1)$	$recX.E \; = \; E\{recX.E/X\}$	
$(Rec2)$	$F = E\{F/X\} \Rightarrow F = recX.E$	provided that X is strongly guarded and serial in E
$(Ung1)$	$recX.(X + E) = recX.E$	
$(Ung2)$	$recX.(\tau.X + E) = recX.(\tau.pri(E))$	
$(Ung3)$	$recX.(\tau.(X + E) + F) = recX.(\tau.X + E + F)$	
$(Ung4)$	$recX.(\tau.(pri(X) + E) + F) = recX.(\tau.X + E + F)$	

Table 2. Axiom system $\mathcal{A}$

The axiom system $\mathcal{A}$ is formed by the axioms presented in Table 2.

The axiom $(Pri6)$ expresses the priority of τ actions over δ actions. Note that from $(Pri6)$ we can derive $\tau.E + \delta.E = \tau.E$ by applying $(Pri3)$. The axioms $(Rec1)$, $(Rec2)$ handle strongly guarded recursion in the standard way [9]. The axioms $(Ung1)$ and $(Ung2)$ are used to turn unguarded terms into strongly guarded ones similarly as in [9]. The axiom $(Ung3)$ and the new axiom $(Ung4)$ are used to transform weakly guarded recursions into the form required by the axiom $(Ung2)$, so that they can be turned into strongly guarded ones. In particular the role of axiom $(Ung4)$ is to remove unnecessary occurrences of terms $pri(X)$ in weakly guarded recursions.

3.1 Soundness

Theorem 8. *Given $E, F \in \mathcal{E}_{pri}$, if $\mathcal{A} \vdash E = F$ then $E \simeq F$.*

Proof The soundness of the laws $(Rec1)$ and $(Rec2)$ derives from the fact that systems of equations of the form $X_i = E_i$, where the variables X_i are guarded and serial in terms E_i and are not in the scope of any recursion, have a unique solution up to provable equality. The proof of this fact is a straightforward adaptation of the proof given in [8]. The soundness of the new equations $(Pri1) - (Pri6)$ trivially derives from the fact that the semantic models of left-hand and right-hand terms are isomorphic. The soundness of the laws $(Ung1)-(Ung4)$ is proved in [2]. ■

3.2 Completeness

Now we will show that the axiom system $\mathcal{A}$ is complete over processes of $\mathcal{P}$. In order to do this we follow the lines of [9], so we deal with systems of recursive equations.

We start by introducing the machinery necessary for proving completeness over guarded expressions $E \in \mathcal{E}$. Afterwards we will show how the axioms (Ung) can be used to turn an unguarded processes of $\mathcal{P}$ into a guarded process of $\mathcal{P}$. Note that the new operator $pri(E)$ is used only in the intermediate steps of the second procedure.

Definition 9. An *equation set* with *formal variables* $\tilde{X} = \{X_1, \ldots, X_n\}$ and *free variables* $\tilde{W} = \{W_1, \ldots, W_m\}$, where $\tilde{X}$ and $\tilde{W}$ are disjoint, is a set $S = \{X_i = H_i \mid 1 \leq i \leq n\}$ of equations such that the expressions H_i $(1 \leq i \leq n)$ have free variables in $\tilde{X} \cup \tilde{W}$. ■

We say that an expression E *provably satisfies* S if there are expressions $\tilde{E} = \{E_1, \ldots, E_n\}$ with free variables in $\tilde{W}$ such that $E_1 \equiv E$ and for $1 \leq i \leq n$ we have $\mathcal{A} \vdash E_i = H_i\{\tilde{E}/\tilde{X}\}$.

The equation sets that are actually dealt with in the proof of completeness of [9] belong to the subclass of *standard equation sets*. Here we have to slightly change the characterization of this subclass because of the presence of priority.

Definition 10. An equation set $S = \{X_i = H_i \mid 1 \leq i \leq n\}$, with formal variables $\tilde{X} = \{X_1, \ldots, X_n\}$ and free variables $\tilde{W} = \{W_1, \ldots, W_m\}$, is *standard* if each expression H_i $(1 \leq i \leq n)$ is of the form: [1]

$$H_i \equiv \sum_{j \in J_i} \alpha_{i,j}.X_{f(i,j)} + \sum_{k \in K_i} W_{g(i,k)}$$

where:

$$\exists j \in J_i : \alpha_{i,j} = \tau \;\Rightarrow\; \nexists j \in J_i : \alpha_{i,j} = \delta \;.$$

■

As in [9], for a standard equation set S we define the relations $\longrightarrow_S \; \subseteq \tilde{X} \times Act \times \tilde{X}$ and $\rhd_S \subseteq \tilde{X} \times \tilde{W}$ as follows:

$$\begin{array}{lll} X_i \xrightarrow{\alpha}_S X & \text{iff} & \alpha.X \text{ occurs in } H_i \\ X_i \rhd_S W & \text{iff} & W \text{ occurs in } H_i \end{array}$$

[1] We assume $\sum_{j \in J} E \equiv \underline{0}$ if $J = \emptyset$.

Definition 11. A standard equation set S with formal variables $\tilde{X} = \{X_1, \ldots, X_n\}$ is *guarded* if there is no cycle $X_i \xrightarrow{\tau}_S^+ X_i$. ■

The following theorem guarantees that from a guarded expression $E \in \mathcal{E}$ we can derive a standard guarded equation set which is provably satisfied by E.

Theorem 12. *Every guarded expression $E \in \mathcal{E}$ provably satisfies a standard guarded equation set S.*

Proof The proof is as in [9], where, in addition, we modify the resulting equation set by eliminating the occurrences of $\delta.X$ (for some X) in the equations which include terms $\tau.Y$ (for some Y). This is done by using laws $(Pri6)$ and $(Pri3)$. ■

Once established the form of standard guarded equation sets S, completeness over guarded expressions is obtained by saturating equation sets S as in [9]. In particular the proofs of the following lemma and two theorems are the same as in [9].

Definition 13. A standard equation set S with formal variables $\tilde{X}$ is *saturated* if, for all $X \in \tilde{X}$:

$$
\begin{array}{lllll}
(i) & X \xrightarrow{\tau}_S^* \xrightarrow{\alpha}_S \xrightarrow{\tau}_S^* X' & \Rightarrow & X \xrightarrow{\alpha} X' \\
(ii) & X \xrightarrow{\tau}_S^* \rhd_S W & \Rightarrow & X \rhd_S W
\end{array}
$$

■

Lemma 14. *Let $E \in \mathcal{E}$ provably satisfy S, standard and guarded. Then there is a saturated, standard and guarded equation set S' provably satisfied by E.* ■

The possibility of saturating standard and guarded equation sets S leads to the following theorem.

Theorem 15. *Let $E \in \mathcal{E}$ provably satisfy S, and $F \in \mathcal{E}$ provably satisfy T, where both S and T are standard, guarded sets of equations, and let $E \simeq F$. Then there is a standard, guarded equation set U provably satisfied by both S and T.* ■

Theorem 16. *Let $E, F \in \mathcal{E}$ provably satisfy the same standard guarded equation set S, then $\mathcal{A} \vdash E = F$.* ■

Hence we have proved completeness over guarded expressions.

Theorem 17. *If E and F are guarded expressions of $\mathcal{E}$ and $E \simeq F$ then $\mathcal{A} \vdash E = F$.* ■

Now we show that each unguarded process can be turned into a guarded process of $\mathcal{P}$, so that we obtain completeness also over unguarded processes. We start with a technical lemma which, in analogy to [9], is important to obtain this result.

Lemma 18. *If X occurs free and unguarded in $E \in \mathcal{E}$, then $\mathcal{A} \vdash E = X + E$.* ■

The proof of this lemma is the same as in [9].

Theorem 19. *For each process $P \in \mathcal{P}$ there exists a guarded $P' \in \mathcal{P}$ such that $\mathcal{A} \vdash P = P'$.*

Proof We show, by structural induction, that given an expression $E \in \mathcal{E}$, it is possible to find an expression $F \in \mathcal{E}_{pri}$ such that:

1. if $pri(G)$ is a subexpression of F then $G \equiv X$ for some free variable X;
2. for any variable X, $pri(X)$ is weakly guarded in F, i.e. each occurrence of $pri(X)$ is within some subexpression of F of the form $\alpha.G$;
3. a summation cannot have both $pri(X)$ and Y as arguments, for any (possibly coincident) variables X and Y;
4. for any variable X, each subterm $recX.G$ of F is (strongly) guarded in F, i.e. each occurrence of $recX.G$ is within some subexpression of F of the form $\alpha.H$, with $\alpha \neq \tau$;
5. F is guarded;
6. $\mathcal{A} \vdash E = F$.

Note that a consequence of property 4 is that each unguarded occurrence of any free variable X of F does not lie within the scope of a subexpression $recY.G$ of F.

Showing this result proves the theorem, in that if $E \in \mathcal{E}$ is a process of $\mathcal{P}$, i.e. a closed term, we have (by the properties of F above) that F is also a process of $\mathcal{P}$, it is guarded, and $\mathcal{A} \vdash E = F$. The result above is derived by structural induction on the syntax of an expression $E \in \mathcal{E}$. In particular in the case $E \equiv recX.E'$ a fundamental role is played by Lemma 18 which generates from a term E'' such that X ($pri(X)$) is unguarded in E'' an equivalent term $X + E''$ ($pri(X) + E''$) so that law ($Ung3$) (law ($Ung4$)) can be applied. See [2] for a complete proof. ■

From Theorem 17 and Theorem 19 we derive the completeness of $\mathcal{A}$ over processes of $\mathcal{P}$.

Theorem 20. *Given $P, Q \in \mathcal{P}$, if $P \simeq Q$ then $\mathcal{A} \vdash P = Q$.* ■

Note that all the axioms of $\mathcal{A}$ are actually used in the proof of completeness. In particular in the proof of completeness over guarded expressions (Theorem 17) we employ the standard axioms $(A1) - (A4)$, $(Tau1) - (Tau3)$ and $(Rec1), (Rec2)$ as in [9], plus the new axioms $(Pri3)$ and $(Pri6)$. All these axioms are necessary even if we restrict to consider completeness over guarded processes only. Moreover, proving that a process of $\mathcal{P}$ can always be turned into a guarded process (Theorem 19) requires the use of the remaining axioms $(Pri1), (Pri2), (Pri4), (Pri5)$ and $(Ung1) - (Ung4)$ [2]. This supports the claim that our axiomatization is irredundant.

4 Conclusion

The algebra presented in this paper and its axiomatization can be extended to include all the operators of CCS, by employing operational rules like those presented in [4]. This can be done easily if parallel composition is assumed to have an operational semantics implementing *local* pre-emption (and not *global* pre-emption, see [4]). This means that τ actions of a sequential process may pre-empt only actions δ of the same sequential process. For instance in $\tau.E|\delta.F$ the action δ of the righthand process is not pre-empted by the action τ of the lefthand process, as instead happens if we assume global pre-emption. On the other hand, local pre-emption seems to be natural in the context of distributed systems, where the choices of a process do not influence the choices of another process.

If global pre-emption is, instead, assumed, then Milner's standard notion of observational equivalence is not a congruence for the parallel operator (see [4]) with the usual operational rule, where $E|F$ may perform a δ action (originating from E or F) only if it cannot execute any τ action. This because, e.g., $recX.\tau.X$ is observationally congruent to $\tau.\underline{0}$, but $recX.\tau.X|\delta.\underline{0}$, whose semantics, due to global pre-emption, is that of $recX.\tau.X$, is not observationally congruent to $\tau.\underline{0}|\delta.\underline{0}$, whose semantics is that of $\tau.\delta.\underline{0}$. In this case a possibility is to resort to a finer notion of observational congruence similar to that presented in [6].

On the other hand, when global priority derives from execution times associated with actions as in [6], where δ actions represent non-zero time delays and classical actions of CCS are executed in zero time, the operational rule for the parallel operator implementing global pre-emption of [4] does not seem the most natural one. In this context a process like $recX.\tau.X$ represents a Zeno process which executes infinite τ actions in the same time point. According to the operational rule for parallel operator of [4], the semantic model of $recX.\tau.X|\delta.\underline{0}$, as already stated, is that of $recX.\tau.X$, where we observe only the τ moves of the lefthand Zeno process which are all executed in zero time. This is in contrast with the intuition that in $recX.\tau.X|\delta.\underline{0}$ the righthand process should eventually advance because, since no synchronization occurs between the two processes, the behavior of the righthand process should not be influenced from that of the lefthand process. The point is that the infinite sequence of τ moves executed by the lefthand process prevents the time from advancing and as a consequence, by employing the operational rule of [4], we do not observe the behavior of the system after the time point where the τ actions are executed. This leads to an incomplete description of the behavior of $recX.\tau.X|\delta.\underline{0}$ that makes the semantics of $recX.\tau.X|\delta.\underline{0}$ to be different from that of $\tau.\underline{0}|\delta.\underline{0}$. Therefore it seems that the real point here is not to consider a finer notion of observational congruence that makes $recX.\tau.X$ not equivalent to $\tau.\underline{0}$ (as done in [6]), but to have an operational semantics for the parallel operator that allows to observe the behavior of the system after the critical time point. The definition of such a semantics is an open problem which we are currently working on.

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