Power-aware distributed protocol for a connectivity problem in wireless sensor networks

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Abstract. We consider the problem of assigning transmission powers to the nodes of a wireless network in such a way that all the nodes are connected by bidirectional links and the total power consumption is minimized.

We present a distributed protocol, obtained by extending a connectivity protocol recently appeared in the literature. The new extended protocol is obtained by using in a local, distributed fashion, well-known centralized techniques for power minimization. The result is a self-organization framework where a set of rules, implemented locally at each node, guarantees global properties, i.e. connectivity and power expenditure minimization.

Preliminary computational results are finally presented. They show that the new extended protocol guarantees a substantial saving in the total transmission power.

1 Introduction

Wireless sensor networks have received significant attention in recent years due to their potential applications in battlefield, emergency disasters relief, and other application scenarios (see, for example, Blough et al. [2], Chu and Nikolaidis [3], Kirousis et al. [6], Lloyd et al. [8], Ramanathan and Rosales-Hain [13], Singh et al. [15], Wan et al. [16] and Wieselthier et al. [17]). Unlike wired networks of cellular networks, no wired backbone infrastructure is installed in wireless sensor networks. A communication session is achieved either through single-hop transmission if the recipient is within the transmission range of the source node, or by relaying through intermediate nodes otherwise.

We consider wireless networks where individual nodes are equipped with omnidirectional antennae. Typically these nodes are also equipped with limited capacity batteries and have a restricted communication radius. Topology control is one of the most fundamental and critical issues in multi-hop wireless networks which directly affects the network performance. In wireless networks, topology control essentially involves choosing the right set of transmitter power to maintain adequate network connectivity. Incorrectly designed topologies can lead to higher end-to-end delays and reduced throughput in error-prone channels. In energy-constrained networks where replacement or periodic maintenance of node batteries is not feasible, the issue is all the more critical since it directly impacts the network lifetime.

In a seminal paper on topology control using transmission power control in wireless networks, Ramanathan and Rosales-Hain [13] approached the problem from an optimization viewpoint and showed that a network topology which minimizes the maximum transmitter power allocated to any node can be constructed in polynomial time in a centralized fashion, i.e. with the assumption that the network is fully known to the centralized optimizer. This is a critical criterion in battlefield applications since using higher transmitter power increases the probability of detection by enemy radar. In this paper, we attempt to solve the minimum power topology problem in wireless networks. Minimizing the total transmit power has the effect of limiting the total interference in the network.

For a given set of transmitters spatially located in the network's area (nodes), the *minimum power topology (MPT) problem*, sometimes also referred to as the *min-power symmetric connectivity problem*, is to assign transmission powers to the nodes of the network in such a way that all the nodes are connected by bidirectional links and the total power consumption over the network is minimized. Having bidirectional links simplifies one-hop transmission protocols by allowing acknowledgement messages to be sent back for every packet (see Althaus et al. [1]). It is assumed that no power expenditure is involved in reception/processing activities.

Unlike in wired networks, where a transmission from i to m generally reaches only node m, in wireless sensor networks with omnidirectional antennae it is possible to reach several nodes with a single transmission (this is the so-called *wireless multi-cast advantage*, see Wieselthier et al. [17]). In the example of Figure 1 nodes j and k receive the signal originated from node i and directed to node m because j and k are closer to i than m, i.e. they are within the transmission range of a communication from i to m. This property is used to minimize the total transmission power required to connect all the nodes of the network.

Althaus et al. [1], Das et. al [4] and Montemanni and Gambardella [9], [10] proposed exact algorithms for the problem. We refer the interested reader to Montemanni et al. [12] for an overview, comprehensive of theoretical and experimental comparison, of these methods. All of these approaches are based on mixed integer programming models, and all of them are designed to be run in a centralized fashion, on a single computer with full knowledge of the network, i.e. all the information about the network are assumed to be available at a central processor (e.g. power required by each node to reach every other node of the network). Turning into real world, it is very unlikely that all this knowledge is available at the central processor, and even if this is true, there would be the practical problem of transmitting optimal transmission powers to the nodes. For these reasons distributed protocols, i.e. protocols that run at each node of the network, with a partial knowledge of the network - namely the set of neighbors of each node - have to be developed.

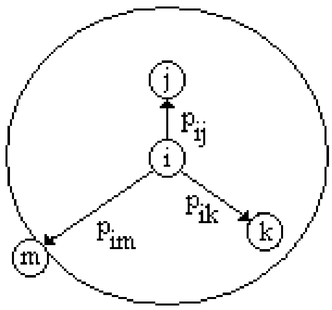

Fig. 1. Communication model.

Some distributed protocols aiming to guarantee connectivity while minimizing the number of neighbors of each node (an indirect measure of the required transmission power) have been proposed in the literature (see Glauche et al [5] and Krause et al [7]). The aim of this paper is to extend these protocols (that will be briefly described in Section 4.1) in order to preserve connectivity, while directly minimizing the total transmission power over the network.

The MPT problem is formally described in Section 2. Section 3 summarizes an efficient method to solve the problem in a centralized fashion. This method is embedded in the distributed protocol proposed in Section 4. This new protocol can be seen as the power-aware extension of the protocol described in Glauche et al. [5]. Experimental comparison of the original and extended versions of the protocol can be found in Section 5, while Section 6 contains conclusions and future work.

2 Problem description

To represent the MPT problem in mathematical terms, a model for signal propagation has to be selected. We adopt the model presented in Rappaport [14], and used in most of the papers appeared in the literature (see, for example, Wieselthier et al. [17] and Montemanni et al. [11], [12]). According to this model, signal power falls as $\frac{1}{d^\kappa}$, where d is the distance from the transmitter to the receiver and κ is a environment-dependent coefficient, typically between 2 and 4. Under this model, and adopting the usual convention (see, for example, Althaus et al. [1]) that every node has the same transmission efficiency and the same detection sensitivity threshold, the power requirement for supporting a link from node i to node j, separated by a distance d_{ij}, is then given by

$$p_{ij} = (d_{ij})^\kappa \tag{1}$$

Technological constraints on minimum and maximum transmission powers of each node are usually present. In particular they state that for each node i, its transmission power must be within the interval $[P_i^{min}, P_i^{max}]$.

The MPT problem can be formally described as follows. Given the set V of the nodes of the network, a *range assignment* is a function $r : V \to \mathcal{R}^+$. A *bidirectional link* between nodes i and j is said to be established under the range assignment r if $r(i) \geq p_{ij}$ and $r(j) \geq p_{ij}$. Let now $B(r)$ denote the set of all bidirectional links established under the range assignment r. The MPT problem is the problem of finding a range assignment r minimizing $\sum_{i \in V} r(i)$, subject to constraints on minimum and maximum transmission powers and to the constraint that the graph $(V, B(r))$ must be connected.

As suggested in Althaus et al. [1], a graph theoretical description of the MPT problem can be given as follows. Let $G = (V, E, p)$ be an edge-weighted graph, where V is the set of vertices corresponding to the set of nodes of the network and E is the set of edges containing all the possible pairs $\{i, j\}$, with $i, j \in V$, $i \neq j$, that do not violate technological constraints on transmission powers. A cost p_{ij} is associated with each edge $\{i, j\}$. It corresponds to the power requirement defined by equation (1).

For a node i and a spanning tree T of G, let $\{i, i_T\}$ be the maximum cost edge incident to i in T, i.e. $\{i, i_T\} \in T$ and $p_{ii_T} \geq p_{ij}$ $\forall \{i, j\} \in T$. The *power cost* of a spanning tree T is then $c(T) = \sum_{i \in V} p_{ii_T}$. Since a spanning tree is contained in any connected graph, the MPT problem can be described as the problem of finding the spanning tree T with minimum power cost $c(T)$.

3 Centralized approach

The approach discussed in this section aims to solve the MPT problem in a centralized fashion, i.e. the full network is supposed to be known. When the problem has been solved, the results (and the respective transmission powers for all the nodes) would have to be communicated all around the network. This is clearly impractical.

Notwithstanding the assumption about the full network knowledge, that can appear very strong, and somehow unrealistic, the method remains of our interest since it will be embedded within the distributed protocol described in Section 4.1.

3.1 An integer programming formulation

A weighted, directed graph $G' = (V, A, p)$ is derived from G by defining $A = \{(i, j), (j, i) | \{i, j\} \in E\} \cup \{(i, i) | i \in V\}$, i.e. for each edge in E there are the respective two (oriented) arcs in A, and a dummy arc (i, i) with $p_{ii} = 0$ is inserted for each $i \in V$. p_{ij} is defined by equation (1) when $i \neq j$. In order to describe the new integer programming formulation for MPC, we also need the following definition.

Given $(i,j) \in A$, we define the *ancestor* of (i,j) as

$$a^i_j = \begin{cases} i & \text{if } p_{ij} = \min_{\{i,k\}\in E}\{p_{ik}\} \\ arg\max_{k\in V}\{p_{ik}|p_{ik} < p_{ij}\} & \text{otherwise} \end{cases} \tag{2}$$

According to this definition, (i, a^i_j) is the arc originated in node i with the highest cost such that $p_{ia^i_j} < p_{ij}$. In case an *ancestor* does not exist for arc (i,j), vertex i is returned, i.e. the dummy arc (i,i) is addressed.

In formulation IP a spanning tree (eventually augmented) is defined by z variables: $z_{ij} = 1$ if edge $\{i,j\}$ is on the spanning tree, $z_{ij} = 0$ otherwise. Variable y_{ij} is 1 when node i has a transmission power which allows it to reach node j, $y_{ij} = 0$ otherwise.

$$(IP) \quad \text{Min} \sum_{(i,j)\in A} c_{ij} y_{ij} \tag{3}$$

$$\text{s.t. } y_{ij} \le y_{ia^i_j} \qquad \forall (i,j) \in A, a^i_j \ne i \tag{4}$$

$$z_{ij} \le y_{ij} \qquad \forall \{i,j\} \in E \tag{5}$$

$$z_{ij} \le y_{ji} \qquad \forall \{i,j\} \in E \tag{6}$$

$$\sum_{i\in S, j\in V\setminus S, \{i,j\}\in E} z_{ij} \ge 1 \qquad \forall S \subset V \tag{7}$$

$$y_{ij} = 1 \qquad \forall (i,j) \in A \text{ s.t. } p_{ij} \le P_{min} \tag{8}$$

$$y_{ij} = 0 \qquad \forall (i,j) \in A \text{ s.t. } p_{ij} \ge P_{max} \tag{9}$$

$$z_{ij} \in \{0,1\} \qquad \forall \{i,j\} \in E \tag{10}$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{11}$$

In formulation IP an incremental mechanism is established over y variables (i.e. transmission powers). The costs associated with y variables in the objective function (3) are given by the following formula:

$$c_{ij} = p_{ij} - p_{ia^i_j} \quad \forall (i,j) \in A \tag{12}$$

c_{ij} is equal to the power required to establish a transmission from node i to node j (p_{ij}) minus the power required by node i to reach node a^i_j ($p_{ia^i_j}$). In Figure 2 a pictorial representation of the costs arising from the example of Figure 1 is given.

Constraints (4) realize the incremental mechanism by forcing the variable associated with arc (i, a^i_j) to assume value 1 when the variable associated with arc (i,j) has value 1, i.e. the arcs originated in the same node are activated in increasing order of p. Inequalities (5) and (6) connect the spanning tree variables z to transmission power variables y. Basically, given edge $\{i,j\} \in E$, z_{ij} can assume value 1 if and only if both y_{ij} and y_{ji} have value 1. Equations (7) state that all the vertices have to be mutually connected in the subgraph induced by z

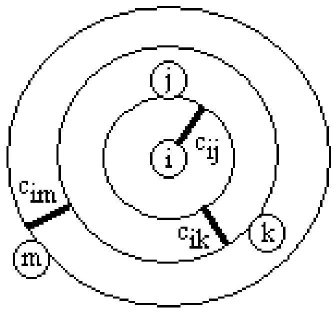

Fig. 2. Costs for the mathematical formulation IP.

variables, i.e. the (eventually augmented) spanning tree. Constraints (8) and (9) model minimum and maximum possible transmission powers. Constraints (10) and (11) define variable domains.

In Montemanni and Gambardella [10] a set of facet defining valid inequalities is presented. These inequalities, that are strongly based on the incremental mechanism described by equations (2), (3) and constraints (4), are able to better define the polytope associated with the linear relaxation of IP, which is obtained by substituting constraints (10) and (11) with the following ones:

$$0 \leq z_{ij} \leq 1 \qquad \forall \{i, j\} \in E \qquad (13)$$

$$0 \leq y_{ij} \leq 1 \qquad \forall (i, j) \in A \qquad (14)$$

Since methods to solve integer programs are based on the iterative refinement of the solution of the linear relaxation, a tighter relaxation usually produces a speed up. In Montemanni and Gambardella [10] it is shown that for the MPT problem the speed up factor can reach 1200. For this reason it is convenient to incorporate these extra inequalities into formulation (IP).

3.2 The exact algorithm *IEX*

In this section we describe an algorithm which efficiently solves to optimality the integer program IP (i.e. the minimum power symmetric connectivity problem).

It is very difficult to deal with constraints (7) of formulation IP, because they are in a huge number. For this reason some techniques which leave some of them out have to be considered. We present an *iterative exact algorithm* (IEX) which in the beginning does not consider constraints (7) at all, and then adds them step by step only in case they are violated.

In order to speed up the approach, the following inequality should also be added to the initial integer problem IP:

$$\sum_{\{i,j\} \in E} z_{ij} \geq |V| - 1 \tag{15}$$

Inequality (15) forces the number of active z variables to be at least $|V| - 1$ - this condition is necessary in order to have a spanning tree - already at the very first iterations of the algorithm.

The integer program defined as IP without constraints (7) but with inequality (15), is solved and the values of the z variables in the solution are examined. If the edges corresponding to z variables with value 1 form a spanning tree then the problem has been solved to optimality, otherwise constraints (16), described below, are added to the integer program and the process is repeated.

At the end of each iteration, the last available solution is examined and, if edges corresponding to z variables with value 1 generate a set $\mathcal{CC}$ of connected components with $|\mathcal{CC}| > 1$, then the following inequalities are added to the formulation:

$$\sum_{i \in C, j \in V \setminus C,\ \{i,j\} \in E} z_{ij} \geq 1 \quad \forall C \in \mathcal{CC} \tag{16}$$

Inequalities (16) force z variables with value 1 to connect the (elsewhere disjoint) connected components in $\mathcal{CC}$ to each other.

The IEX algorithm is summarized by the pseudo-code presented in Figure 3:

```
IEX()
Build integer program IP;
sol := optimal solution of IP;
CC := connected components defined by
    variables z of sol;
While (|CC| > 1)
    Add inequalities (16) to IP;
    sol := optimal solution of IP;
    CC := connected components defined by
        variables z of sol;
EndWhile
return sol.
```

Fig. 3. Pseudo-code for the centralized exact algorithm IEX.

It is important to observe that the exact method discussed in this section is able to solve to optimality, in reasonable time, problems with up to 50 nodes. When the method is used in a distributed fashion - e.g. when it is used within the protocol we will describe in Section 4.2 - the practical problems of interests are sensibly smaller.

4 Distributed protocols

Glauche et al. [5] conducted a detailed study showing that there is a very close correlation between the (minimum) number of neighbors of the nodes of a network and the probability of the network to be fully connected. In particular they observed that this indicator (number of neighbors) is more interesting than transmission power when connectivity issues are studied. Following this observation they propose a simple protocol able to provide full connectivity (with high probability) with a much smaller total transmission power expenditure than methods working directly on power.

This protocol will be extended in order to locally optimize transmission powers while maintaining the good theoretical properties of the original protocol. The original protocol is sketched in Section 4.1, while the new extended version is presented in Section 4.2.

4.1 Protocol LMLD (Glauche et al. [5])

The LMLD (*Local Minimum Link Degree*) protocol has been originally proposed in Glauche et al. [5]. It has been inspired by the following observation, motivated by reasonings based on percolation theory. By exchanging so-called *hello* and *hello-reply* messages each ad hoc node is able to access direct information only from its immediate neighbors, defined by its links. The simplest local observable for a node is the number of its links, which is equal to the number of its one-hop neighbors. Based on this observable alone, a simple strategy for a node would be to decrease/increase its transmission power once it has enough neighbors. Consequently the target node degree should be defined by a parameter, that we will refer to as *ngb*. A value of the latter has to be chosen such that all nodes are part of one connected network and reflects the only external input to this otherwise fully local link rule.

The simple protocol just lined out has two main drawbacks. The first one is that the value of *ngb* must be very conservative in order to guarantee full connectivity in case of clustered networks (with an undesired high density of links in density populated areas as a side effect). The second drawback is that the protocol does not take into account that links have to be bidirectional.

The idea introduced in Glauche et al. [5] elaborates on the protocol described above, aiming to eliminate these drawbacks. In particular, upon setting up the communication links to the other nodes, a node attaches to its hello message information about its current link neighborhood list and its current transmission power. Starting with P_{min}, the node increases its transmission power by a small amount once it has not reached a minimum link degree ngb_{min}. Whenever another node, which so far does not belong to the neighborhood list, hears the hello message of the original node for the first time, it realizes that the latter has too few neighbors, either sets its power equal to the transmission power of the hello-sending node or leaves it as before, whichever is larger, and answers the hello message. Now the original and new node are able to communicate back and forth and have established a new link. The original node adds one new node to its neighborhood list. Only once the required minimum link degree is reached, the original node stops increasing its power for its hello transmissions. At the end each node has at least ngb_{min} neighbors. Some have more because they have been forced to answer nodes too low in ngb; their transmission power is larger than necessary to obtain only ngb_{min} neighbors for themselves.

In Glauche et al. [5] it is shown that small values of parameter ngb_{min} (e.g. 10) already guarantee, from a theoretical and practical point of view, full connectivity with probability almost 1 for very large networks (e.g. 1600 nodes).

4.2 Protocol LMPT

Our aim here is to enrich the LMLD protocol described in the previous section by introducing explicit transmission power minimization. In order to do this, we need a little bit more of local information about neighbors, and a slightly more articulated protocol. We will refer to this new protocol as the LMPT protocol, which stands for *Local Mimimum Power Topology* protocol.

Similarly to the LMLD protocol sketched in Section 4.1, where each node is, in turn, in charge of establishing links with ngb_{min} neighbors, here each node is, in turn, in charge of local optimization. We will refer to this node as the (temporarily) head node. It needs to know the list of neighbors (at the time of the local optimization) for each of its ngb_{min} potential neighbors. Moreover, each node has to send to the head node the power required to reach each one of its neighbors (it collected these information while it incrementally increased its power in order to reach a minimum number of neighbors or when it receives a connection request by another node).

Once the head node has collected these information for the ngb_{min} nodes (same parameter of LMLD protocol) closest to it, it solves the local optimization problem involving itself and these nodes (details about the constructions of the local problem are given below). In the meantime the nodes in its neighborhood wait for the optimization to be concluded. At these point, according to the solution of the optimization, the head node distributes the new neighbors lists and the new transmission powers for its (current) neighbors. Once they receive this information they update their state and lists.

The overhead introduced for information exchange (and for solving the local optimization problem) is justified by the efficiency gained in terms of transmission power expenditure.

It is very important to stress that when the new protocol LMPT is applied, all the theoretical results of Glauche et al, that guarantee connectivity “almost for sure” for proper values of ngb_{min}, are still completely valid, since after the local optimization has been concluded, each node is able to reach at least ngb_{min} nodes, although now a multi-hop transmission could be necessary. The power saving we guarantee is consequently directly related to the acceptance of multi-hop transmission instead of direct one-hop transmissions only.

Figure 4 illustrates the algorithmic implementation of the distributed rule in more detail. Initially, all nodes come with a minimum transmission power $P_i = P_{min}$ and an empty neighborhood list $\mathcal{N}_i = \emptyset$ (with the respective list of required transmission powers $\mathcal{I}_i$ empty as well). All of them start in the receive mode. Then, at random, one of the nodes switches into the discovery mode. By subsequently sending Ask4Info messages and receiving replies, the picked node increases its power until it has discovered enough neighbors to guarantee connectivity with high probability. At this point it uses the collected information to set up the optimization problem IP (see below) and solves it.

Once IP has been solved, the optimal solution of IP is distributed to the set of neighbors that sent their information in order to set up problem IP. The head node can now set up its new transmission power P_i, its set of neighbors $\mathcal{N}_i$ with the respective required transmission powers $\mathcal{I}_i$.

The other nodes will use the information received to set up their power and their new neighbor lists. The node returns then into the receive mode.

For simplicity we assume that only one node at a time is in the discovery mode; furthermore, we assume the maximum transmission power P_{max} to be sufficiently large, so that each node is able to discover at least ngb_{min} neighbors.

In the receive mode a node listens to incoming Req4Info messages. Upon receipt of such a message, the node first checks whether it already belongs to the incoming neighborhood list. If yes, the requesting node has already asked before with a smaller discovery power and there is no need for the receiving node to react. Otherwise, it updates its transmission power to $\max(P_i, P_j)$. Then it sends back information about its neighbors and the respective transmission powers required to reach them. The node then waits for the head node j to solve IP and collects the results. These results are used to update transmission power P_i, the set of neighbors $\mathcal{N}_i$ and the respective transmission powers $\mathcal{I}_i$.

Construction of the local mixed integer program IP The set of nodes V of the local IP for head node i is given by the elements of $\mathcal{N}_i^{disc}$, while power requirements between nodes are set according to the following rule: if $j \in V \cap \mathcal{N}_i$ then p_{ij} is given by the respective power requirement (contained in $\mathcal{I}_i$), otherwise $p_{ij} := +\infty$. This last assignment is equivalent to state that node i will never reach node j in the optimal solution of IP (since they are not aware of each other and do not know the required power to reach each other). Another issue has to be taken into account while setting up problem IP. In case there exists a node $j \in \mathcal{N}_k \backslash V, k \in \mathcal{N}_i^{disc}$ (j is not a neighbor of i, but j is a neighbor of k, that in turn is a neighbor of i), we have to force k to keep transmitting to j in order to ensure global connectivity. This can happen when k has already been replying to Ask4Info messages before the current round. In the situation depicted we have to force node k to reach (at least) node j. We then add the following constraints to IP.

$$y_{kl} = 1 \qquad \forall (k,l) \in A \text{ s.t. } p_{kl} \leq p_{kj} \tag{17}$$

```
LMPT()
P_i := P_i^min;
N_i := ∅;
ReceiveMode();
DiscoveryMode();
ReceiveMode();

DiscoveryMode()
P_i^disc := P_i^min;
N_i^disc := ∅;
I_i := ∅;
While ( P_i^disc ≤ P_i^max and |N_i^disc| < ngb_min )
     P_i^disc := P_i^disc(1 + Δ );
     Ask4Info(i, P_i^disc, N_i^disc);
     (j_1, info_j1, i),... := ReceiveInfo();
     N_i^disc := N_i^disc ∪ {j_1,...} ;
     Update I_i according to info_j1, ...;
EndWhile
Create IP according to I_i;
Sol := Optimal solution of IP;
SendSol(i, N_i^disc, sol);
Set P_i, N_i and I_i according to sol;

ReceiveMode()
(j, P_j, N_j) := ReceiveReq4Info();
If ( i ∉ N_j )
     P_i := max(P_i, P_j);
     info_i := combination of N_i and I_i;
     SendInfo(i, info_i, j);
     sol := ReceiveSol();
     Set P_i, N_i and I_i according to sol;
EndIf
```

Fig. 4. Pseudo-code for the Local Minimum Power Topology (LMPT) protocol.

Constraints (17) are enough to keep the valid global properties that guarantee connectivity with high probability for appropriate values of parameter ngb_{min}. It is interesting to observe that they also reduce the complexity of IP (new facets are added), making it easier to solve.

5 Preliminary experimental results

In this section we aim to compare the results obtained by the distributed protocol described in Glauche et al. [5] with those of the power-aware $LMPT$ protocol, discussed in Section 4.1.

The following three indicators are taken into account for the comparison:

- **Total transmission power:** the sum of the transmission power of all the nodes of the network;
- **Average number of neighbors:** the average number of connections each node has to maintain in the solution generated by the protocols. This indicator is important because having too many neighbors leads to problematic communications due to the resulting high noise over the network;
- **Maximum number of neighbors:** the maximum number of connections a node within the network has to maintain.

Is is important to stress that in the comparison we do not take into account the overhead generated by the extra operations carried out by the new $LMPT$ protocol. This overhead is however marginal, and can be reduced to the extra transmission power dissipated when information about (old and new) neighbors are exchanged within the local neighborhood of each node. However this overhead is very marginal, since the extra operations are carried out only once when the network is established.

The network topologies considered are those already adopted in Glauche et al. [5]. Namely, we consider *homogeneous*, *multifractal* and *Manhattan* topologies. We refer the interested reader to Glauche et al. [5] for details about these topologies and how to generate the networks. All the networks considered here have 1600 nodes, path loss exponent $\kappa = 2$ and are generated according to [5]. Parameter ngb_{min}, that defines the minimum number of neighbors of each node, has been set to 6 for homogeneous networks, to 7 for multifractal networks and to 10 for Manhattan networks. These value are those suggested in [5] and guarantee full connectivity with probability almost 1.

Average results of the indicators over 50 networks are summarized in Tables 1, 2 and 3 for the three families of networks considered. Percentage gains achieved by the extended protocol LMPT also appear in the tables.

For all the experiments reported in Tables 1, 2 and 1 the use of the extended protocol $LMPT$ brings a substantial gain over protocol $LMLD$, in terms of both the total transmission power and the number of neighbors (average and maximum).

In particular the most impressive results have been obtained on Manhattan networks (Table 3), where the gains for the three indicators are in the order of

Table 1. Homogeneous networks. Averages over 50 networks.

	LMLD ([5])	**LMPT**	**Gain (%)**
Total transmission power	2.547	1.403	44.92
Average number of neighbors	7.085	2.879	59.36
Maximum number of neighbors	12.317	7.683	37.62

Table 2. Multifractal networks. Averages over 50 networks.

	LMLD ([5])	**LMPT**	**Gain (%)**
Total transmission power	4.311	2.047	52.44
Average number of neighbors	8.320	3.393	59.22
Maximum number of neighbors	14.146	9.334	34.02

Table 3. Manhattan networks. Averages over 50 networks.

	LMLD ([5])	**LMPT**	**Gain (%)**
Total transmission power	3.417	0.618	81.91
Average number of neighbors	11.890	3.514	70.45
Maximum number of neighbors	24.789	12.684	48.83

81.91 %, 70.45 % and 48.83 % respectively. These results are due to the intrinsic characteristics of these networks, that in fact are critical cases for the original protocol presented in [5].

We can conclude that the results are indeed very encouraging and they completely justify the marginal overhead generated by the extra operations carried out by the extended protocol $LMPT$.

6 Conclusions and future work

In this paper we have considered the problem of assigning transmission powers to the nodes of a wireless network in such a way that all the nodes are connected by bidirectional links with probability almost 1 and the total power consumption is minimized.

We have presented a distributed protocol, which can be seen as the power-aware extension of a protocol recently appeared in the literature. The extended protocol uses a well-known centralized technique for power minimization in a local, distributed fashion. An important characteristic of the new protocol is that all the nice theoretical and experimental properties about connectivity of the original protocol, can be directly transferred to it.

Preliminary computational results are very encouraging, and our future work will be in the direction of assessing more in detail the potentialities of the new approach, both from a theoretical and experimental point of view. In particular it will be very interesting to compare the quality, in terms of power consumption, of the solution computed by the power-aware distributed protocol we propose, with the theoretical optimal solution, obtained by assuming full knowledge of the network at a centralized location.

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