A Survey on Cellular Automata*

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Abstract

A *cellular automaton* is a decentralized computing model providing an excellent platform for performing complex computation with the help of only local information. Researchers, scientists and practitioners from different fields have exploited the CA paradigm of local information, decentralized control and universal computation for modeling different applications. This article provides a survey of available literature of some of the methodologies employed by researchers to utilize cellular automata for modeling purposes. The survey introduces the different types of cellular automata being used for modeling and the analytical methods used to predict its global behavior from its local configurations. It further gives a detailed sketch of the efforts undertaken to configure the local settings of CA from a given global situation; the problem which has been traditionally termed as the *inverse problem*. Finally, it presents the different fields in which CA have been applied. The extensive bibliography provided with the article will be of help to the new entrant as well as researchers working in this field.

I. Introduction

From the days of Von Neumann and Ulam who first proposed the concept of cellular automata (CA), to the recent book of Wolfram 'A New Kind of Science' [262], the simple structure of CA has attracted researchers from various disciplines. It has been subjected to rigorous mathematical and physical analysis for the last fifty years and its application has been proposed in different branches of science - both physical and social. A large number of research papers are published every year. Specialized conferences and special issues of various journals on CA have been initiated in the last decades. Several universities

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have also started offering courses on cellular automata. Furthermore, as many as sixty-four books are found to exist on cellular automata when we visit the web-site *www.amazon.com*.

The reason behind the popularity of cellular automata can be traced to their simplicity, and to the enormous potential they hold in modeling complex systems, in spite of their simplicity. Cellular automata can be viewed as a simple model of a spatially extended decentralized system made up of a number of individual components (cells). The communication between constituent cells is limited to local interaction. Each individual cell is in a specific state which changes over time depending on the states of its local neighbors. The overall structure can be viewed as a parallel processing device. However, this simple structure when iterated several times produces complex patterns displaying the potential to simulate different sophisticated natural phenomena.

The concept of CA was initiated in the early 1950's by J. Von Neumann and Stan Ulam [168]. Von Neumann showed that a cellular automaton can be universal. He devised a CA, each cell of which has a state space of 29 states, and showed that the devised CA can execute any computable operation. However, due to its complexity, Von Neumann rules were never implemented on a computer. Von Neumann's research pointed to a dichotomy in CA research. On one hand, it was proven that a decentralized machine can be designed to simulate any arbitrary function. On the other hand, the machine (CA) becomes as complex as the function it tries to simulate. This very theoretical dichotomy has since driven research on CA [17], [29], [46], [129], [149], [259], [261].

Based on the theoretical concept of universality, researchers have tried to develop simpler and more practical architectures of CA which can be used to model widely divergent application areas. In this respect, two notable developments can be credited to Conway and Wolfram. In the 1970, the mathematician John Conway proposed his now famous *game of life* [96] which received widespread interest among researchers. In the beginning of the eighties, Stephen Wolfram has studied in much detail a family of simple one-dimensional cellular automata rules (now famous Wolfram rules [259]) and showed that even these simplest rules are capable of emulating complex behavior.

This survey seeks to present the basic research directions followed by researchers to make the computing model (CA) more practically oriented. To achieve this goal, researchers should be able to predict the global behavior from the local CA rules. Once this goal is achieved, one should be able to design the local rules/initial conditions from a given prescribed global behavior. Good historical overviews highlighting works to achieve this basic goal up to the late 1990s are available in [46], [147], [148], [200], [232], [253]. In line with such surveys, we outline a concise up-to-date survey of the theory and applications of this computing model in different disciplines. We try to bring out the rich diversity in concepts and ideas proposed by the researchers while portraying the underlying unified approach.

The survey has been laid out as follows. The next section presents a survey of different types of cellular automata structures proposed over the years. A historical perspective regarding the efforts undertaken to characterize CA rule space, that is trying to understand the global dynamics from the local rules, is presented next. The fourth section highlights the inverse problem, particularly the evolutionary methodology employed for generating local rules of CA for different prescribed global behavior. While presenting the survey of these efforts of characterizing global dynamics from CA rules and vice versa, we generally restrict ourself to the Wolfram Class of CA, or slight variations of it. Finally, in the last section we take a look at the wide variety of applications of cellular automata.

II. Types of Cellular Automata

Since its inception, different structural variations of CA have been proposed to ease the design and behavioral analysis of the CA as well as make it versatile for modeling purposes. The CA structure introduced by Von Neumann uses 29 states per cell. Codd [59] introduced a machine with 8 states per cell. Arbib provided a simple description of self-reproducing CA in [8], whereas Banks worked with a CA having 4 states per cell [18]. All these two-dimensional CA are assumed to have a five-cell neighborhood (self and four orthogonal neighbors). The nine-cell neighborhood CA, with two states per cell and appropriate rules, has been shown to be capable of universal computation [232]. This structure has been utilized with a specified set of local rules to create the *game of life* [96]. The two variations of neighborhood configurations (five and nine) are termed as *Von Neumann* and *Moore* neighborhood, respectively. There are extended generalizations of these two neighborhoods configurations - the R-radial and R-axial neighborhoods respectively [100], [257], [268]. (For both *Von Neumann* and *Moore* neighborhood, $R = 1$.)

Because of its inherent simplicity, the one-dimensional CA with two states per cell became the most studied variant of CA [256]. The neighborhood generally varies from three [46] to five [131] or seven cells [149]. In another type of CA, the states are assumed to be a string of elements in a *Galois field* GF(q), where q is the number of states of a CA cell [140] . Additive and linear CA gained popularity in the $VLSI$ era, due to local interaction of simple cells, each having two states '0' or '1' - the elements of the field GF(2). The next state logic of linear and additive CA is expressed in terms of *xor* and *xnor* logic gates. Recently, Paul has introduced the theory of GF(2^p) cellular automata over Galois extension field GF(2^p) [182], [186]. A cell of the GF(2^p) CA consists of p memory elements and can store an element of GF(2^p). The GF(2^p) CA provides the required structure for hierarchical modeling of different physical systems [182]. For example, with the same CA configuration, a circuit can be analysed from the gate level as well as the transistor level.

Cellular automata on multi-dimensional grids have also been proposed [140], [201]. The grids have either null or periodic boundary. In null boundary configurations the boundary cells are assumed to have 'null' (logic '0') dependency. A variation of the null boundary configuration is the fixed boundary configuration in which the boundary cells instead of being considered '0' are replaced by a fixed value [205]. A periodic boundary is one in which the grid is considered to be folded [19], [165]. That is, for one dimension, the right most cell is the neighbor of the left most one and vice versa. The concept of intermediate boundary CA has been proposed in which an intermediate cell acts as the right(left) neighbor of the rightmost (leftmost) cell of the grid. Intermediate boundary CA are found to generate better *pseudo-random patterns* [46], [165].

The local rules applied to each cell can be either identical or different. These two different possibilities are termed as uniform and hybrid CA respectively [46]. The hybrid CA has been especially applied in a linear/additive variant in which the rule set can be analyzed through matrix algebra [70], [202]. In [68], [70], Das has shown that a three-neighborhood additive CA can be represented by a tridiagonal matrix - a matrix which has the elements of its diagonal and two off-diagonals as non-zero. The properties of CA with varying (non-uniform) neighborhoods for the cells have been also studied in [117], [267].

While the next state function (rule) in general is deterministic in nature, there are variations in which the rule sets are probabilistic [22], [25], [104], [121], [136], [243], or fuzzy [35], [86], [263]. The nature of next state functions also varies significantly; researchers have defined the rule set according to the design requirements of the applications. Also there are some standard rule sets which have been used across different applications - Wolfram rules [259], linear rules [46], diffusion rules [49] etc. The next state, in almost all cases, depends upon the output of the previous state. However, there are some time-dependent rules, for example in the problem of directed percolation Chopard and Droz in [49] use two alternate rules at even and odd time steps. Similarly, to describe simultaneous random walk of many particles [78], time and state dependent local rules have been formulated by Toffoli and Margolus. A few interesting works on asynchronous CA have been published recently [78], [208], [246].

The many different CA types reviewed in this section contribute to the modeling power of the tool. In order to gain insight into the modeling capacity of CA based simulation tools, characterization of CA state transition behavior is of great importance. The next section presents an overview of the methodologies developed to analyze global state transition behavior of a CA.

III. Cellular Automata (CA) Characterization - Local to Global Mapping

Despite the simple construction of cellular automata, they are capable of highly complex behavior. For most cellular automata models, the only general method to determine the qualitative (average) dynamics of the system is to run *simulations* on a computer for various initial global configurations [114], [257]. Hence, one principal direction for research has been to study the CA dynamics as it evolves in successive time steps. A detailed analysis of CA dynamics enables us to understand the emergent behavior and computational capacity of the system [65], [102]. CA classification based on the study of its dynamics has been a major focus for the researchers. Borrowing concepts from the field of continuous dynamical systems, $Wolfram$ [257] first classified CA into four broad categories - (i) $Class$ 1: CA which evolve to a homogeneous state; (ii) $Class$ 2: displaying simple separated periodic structures; (iii) $Class$ 3: which exhibit chaotic or pseudo-random behavior, and (iv) $Class$ 4: which yield complex patterns of localized structures and are capable of universal computation [258].

Based upon the four broad classes of Wolfram, detailed categorization of different classes has been proposed by a number of researchers, notable among them are Li et al. [131], and Gutowitz [106]. On the other hand, Walker [254] has examined a family of sparsely connected Boolean nets to characterize the CA machines. A classification of CA into five disjoint groups based on the structure of their attractors was proposed by Kurka [127].

Various order/chaos measures are used to globally characterize CA dynamics. The topology of CA state space has played a very important role for this analysis. For example, a characterization has been proposed with reference to the 'Garden of Eden' (that is, non-reachable states), attractor basins, entropy of the evolved patterns etc. Investigation based on 'Garden of Eden' states was initiated in the 1970's [7], [162]; further developments in this direction are described in [121], [122]. Kaneko [119] introduced an information theoretic approach to characterize the complexity of 'Garden of Eden' states in terms of their volumes, stability against noise, information storage capacity etc. Recent work by Wuensche [266] shows that CA can be classified into ordered, complex or chaotic based on the parameters - G-Density (that is, bushiness of Garden of Eden), In-Degree Frequency etc.

In order to accurately model discrete dynamical systems, Lyapunov exponents have been defined for dynamical system lattices in [120], [220]. The Lyapunov exponent indicates whether the dynamic system is independent of the initial condition. An interesting variation of Lyapunov Exponent has also been employed to characterize cellular automata which measures the divergence of trajectories based on hamming distance [77].

Besides characterizing CA through its global dynamics, understanding the global dynamics from its local rules has been one of the driving forces. To present an overview of the methodologies developed by researchers, we broadly divide CA into two categories: additive/linear CA - that is, CA following the constraints of $xor/xnor$ logic for its next state function; and non-linear CA - CA which do not possess such constraints.

A. Linear/Additive CA

Analysis of linear/additive CA is amenable to algebraic methods. Since the next state function applied at each cell follows the operations of a Galois field, the properties of the field can be applied to characterize its state transition behavior. Consequently, the linear/additive CA are also termed as GF(q) CA where q is a prime number. The GF(2) CA - the most popular variant of GF(q) CA [71], [72] has attracted considerable attention in recent years since it can be applied in the field of $VLSI$ design and test. There have been several attempts to characterize GF(q) CA through a suitable algebraic tool. It has been characterized using dipolynomials [123], [140]; but hybrid CA cannot be represented by dipolynomials. CA state transition behavior has also been represented on arbitrary graphs and its behavior has been studied based on graph-theoretic properties [237] . There are some abstract realizations of linear CA in which the cell space is termed as an Abelian group and the state space is represented by a commutative ring [10]. However, after this initial phase, characterization of GF(q) CA by a characteristic matrix became a de facto standard [68], [72].

The matrix algebraic tool employing minimal and characteristic polynomials of the characteristic matrix showed various interesting features of CA behavior. The first important finding is the categorization of additive CA into group and non-group CA. In a group CA each of the states has a single predecessor which is not true for non-group CA. However, it is found that the non-group CA show uniform behavior in which trees rooted at any cycle state are isomorphic [140]. A detailed review regarding analysis of group and non-group CA follows.

Group CA : The most effective application of null boundary group CA has been proposed in the field of pseudo-random pattern generation. Serra et al. [217] showed that maximum length CA - group CA with all non-zero states lying in a single cycle - produces high quality pseudo-random patterns. It has been established that the maximum length cycle can be produced only if the characteristic polynomial is primitive as well as only if rule 90 and/or rule 150 is used to construct the CA [36] (rule 90 = xor(left neighbor, right neighbor); rule 150 = xor(left neighbor, self, right neighbor)). A maximum length CA cannot be generated by periodic boundary CA since its characteristic polynomial can be factorized [20]; a formal proof of this was provided by Nandi [165]. The next important theoretical hurdle was the synthesis of a CA with rule 90/150 from a given irreducible/primitive polynomial. Serra et al. used a version of

Lanczos tridiagonalization method over GF(2) to solve this problem [216]. Simplified versions are reported in [36], [239]. The synthesis of irreducible polynomial was generalized for GF(q) CA by Muzio et al [161]. Further, there has been work by Makato that shows for certain lattice sizes, maximum length CA can be generated only through rule 90 [141]. The phase-shift properties of CA-cells important for analyzing pseudo-random patterns are studied in [157], [164]. Some interesting characterizations of group CA are also reported in [45], [202]. Similar to the results of one-dimensional null boundary CA, work on two-dimensional CA has been reported [37], [53], [56]. Recently, Tomassini has reported a characterization of the pseudo-randomness of the patterns generated by two-dimensional CA [245]. Ganguly developed a unique algorithm through which the rules of the CA can be synthesized given the cycle structure of a group CA [89]. He also characterizes the relationship between the cycle structure of additive CA - CA which use *xor* and *xnor* logic as a next state function - and linear CA - CA which use only *xor* logic as next state function.

Non-Group CA : The non-group CA initially received less attention under the assumption that it is a *degenerate case of a nonsingular (group) machine* [236]. In recent times, the trend has been reversed with a large number of publications exploring this area [24], [40], [42], [48], [90], [195]. The isomorphism of tree structures of non-group CA brought forward two important results [42], [46], [140]. First of all, the non-group CA can be mapped to a table structure with its cyclic states producing the address of the table [41]. Secondly, the linear and complemented variant of a non-group CA produces interesting symmetry within themselves [163]. To formalize the behavior of this symmetry, Chakraborty et al. brought forward the concept of CA and dual CA [40] pair. These two results opened up a number of new avenues and researchers realized that non-group CA have more potential applications than group CA. Interesting classes of non-group CA which have been extensively studied are - *multiple attractor cellular automata* ($MACA$) [42], *depth-1* cellular automata* ($D1^*$ CA) [53] and *single attractor cellular automata* ($SACA$) [75]. They have been used in a wide range of functions like hashing [94], classification [187], designing easy and fully testable FSM [55], authentication [75] etc. Chattopadhyay et al. presented some interesting results showing that *boolean decision diagrams* [28] can be used for efficient characterization of non-group CA [43], [44]. Ganguly et. al. [90] showed that $MACA$ - a special class of non-group CA - model a hash family termed as *hamming hash family* for which the probability of collision between a pair of patterns varies with their hamming distance. Recently, Cho has further characterized the dual property of non-group CA [47].

In order to further extend the versatility of additive CA to analyze a physical system at different levels of hierarchy, work on GF(q) based architecture with $q > 2$ has been reported. Cattel and Muzio have provided short analysis of CA over GF(q) in [161]. Considerable interest has been generated in extending CA research based on the theory of finite field in GF(m), where m is a positive integral power of 2 [176], [177]. The above scenario has motivated the researchers at Bengal Engineering College to start investigation to develop a hierarchical modeling tool with cellular automata. Paul has introduced the theory of GF(2^p) cellular automata designed over Galois extension field GF(2^p) [182]. A cell of the GF(2^p) CA consists of p memory elements and can store values in the extension field GF(2^p) [186], [222], [227]. The GF(2^p) CA is further extended to GF(2^{p^q}) CA to arrive at a hierarchical structure that can be employed for studying the hierarchical structure of a $VLSI$ circuit [223].

B. Non-Linear CA

A detailed overview of the study of the nature of the CA's emergent global behavior based upon the rule configuration of the CA cells for non-linear CA is provided next. The most important parameter derived out of the rule structure is Langton's λ parameter [130]. If the CA consists of two states 0 and 1, then the λ parameter is defined as the probability that a particular CA cell will have its next state as 1, that is, it indicates the fraction of 1's in the binary rule configuration (table) of a CA cell. The λ parameter can be accordingly defined if the CA has more than 2 states. λ parameter has been the most important parameter used to characterize CA dynamics. It has been shown by Langton that with the increase of the λ value, the CA changes from 'order to chaos'. There have been several interesting works and polemics [149], [179] regarding the critical value of λ, termed as λ_c - the value of λ around which the CA behavior changes from 'order' to 'chaos' [130]. Besides λ, several other local parameters have been proposed. Notable among these is the Z parameter [265], [267] defined in terms of the distribution of 1's and 0's in the rule table. Further, a parameter referred to as P parameter [265] has been suggested to characterize the behavior of hybrid CA. The P parameter is the larger fraction of 0's or 1's in the look-up table enlisting binary patterns of a CA rule.

In probabilistic cellular automata, *stationary Markov-chains* have been used for analysis of global behavior. The *Chapman Kolmogorav Equation* which is derived by using the concept of *stationary Markov-chain* predicts the probability distribution of CA states from CA rules [95]. Moreover, *mean field approximation*, which is based upon the assumption that at any time the states of sites are independent of states of other sites in the lattice [209], [256] has been recently used by scientists [15], [31] to understand the emergent pattern formation, local sensitivity and phase transform of the CA states. Stability analysis of *mean field approximation* [80] further clarifies the idea of emergent pattern formation.

IV. Evolutionary Algorithm and The Inverse Problem - Global to Local Mapping

The inverse problem addresses the following questions - *Find a cellular automata rule that will have some preselected global properties.* The inverse problem of deducing the local rules from a given global behavior is extremely difficult [33], [118]. There have been some efforts, with limited success, to build the attractor basin according to a given design specification. Notable among these are the works reported by Wuensche [264], Askenazi [9], Meyer [144]. However, most popular methodology to address the inverse problem of mapping the global behavior to local CA rules are based on evolutionary computation techniques namely genetic algorithms and simulated annealing.

The initial work on CA evolution was reported by Packard and his colleagues [179], [194]. Koza [126] also applied genetic algorithms to generate simple random numbers. The first major publication responsible for making genetic algorithms a popular tool for evolving CA was due to Mitchell et al. [149]. In this paper, the authors portrayed the detailed phase transformation that a CA population undergoes during the lifetime of an evolution algorithm. The paper emphatically established the viability of using the evolving cellular automata model in solving the computation task of density classification. Subsequently, this concept has been refined and reaffirmed by a series of works on Density Classification [1] and Synchronization [2] by their group - the EVCA (*evolutionary algorithm cellular automata*) group of Santa Fe Institute [64], [73], [74].

The sampling error which arises from random selection of an initial configuration often reduces efficiency of the evolutionary process. In order to circumvent this, Paredis [181] proposed the co-evolution process in which both the CA and the initial configuration (IC) are simultaneously evolved. Juille and Pollack changed the co-evolutionary setup by introducing a limit on the selection of ICs [116]. Pagie and Hogeweg embedded the co-evolutionary model in a 2D grid and introduced an extension on the fitness function used to evaluate the ICs [180]. The evolutionary process on two-dimensional CA to perform the density classification task is currently pursued [154].

However, complex computation tasks such as density classification cannot be fully modeled through uniform CA [33]. To map such complex tasks to CA, use of hybrid CA is a necessity. Hybrid CA, using different rules in different cells, allows mapping of more complex behavior. But, the search space of hybrid CA is larger by several orders of magnitude than uniform CA. This results in convergence problems of

[1]Density Classification: Design a CA in which the initial state of the CA (containing 1's and 0's) will converge to all 1's state if the number of 1s in the initial configuration is large and converge to all 0's state if the number of 0s is large.

[2]Synchronization: Design a CA which will reach a final configuration after (say) M time steps that oscillates between all 0's and all 1's in successive time steps.

the genetic algorithms. In light of these problems, parallel genetic algorithms have been proposed for CA evolution [32]. Specifically, Sipper proposed schemes for evolution of hybrid CA [228], [229]. The cellular algorithm proposed by Sipper presented a novel technique of optimization by assigning fitness to each individual cell. Similar cellular programming schemes are also proposed in [238], [244]. Capcarre et al. presented a detailed study about the dynamics of evolution of hybrid CA while solving the three standard tasks - synchronization, density classification, and random number generation [34] .

Further, some interesting theoretical insights regarding evolutionary dynamics of CA are also drawn from evolving cellular automata model used in generating *deterministic test patterns* [118] and *pseudo random test pattern* for *sequential circuits* [61], [62]. A more recent work on evolving hybrid CA is proposed by Maji et al. [137]. In this work, the state transition diagram of the CA has been conceived as a graph. This graph has been optimized through evolutionary schemes like genetic algorithms and simulated annealing to arrive at the desired CA.

However, the works on evolving cellular automata suffer from an inherent problem. There is hardly any work which analytically derives a subset of the total possible CA rules to be the probable candidate for displaying a particular global behavior. This type of characterization in many cases can in fact dramatically reduce the search space. And by the same token, if genetic algorithms can be constrained to evolve only within the reduced search space, then the convergence speed and accuracy of the algorithm would be greatly enhanced. This very problem has been attacked by Ganguly et al in [91], [93]. In their work, they have developed constrained genetic algorithms, with the help of which the evolutionary process can be guided through a special class of additive or linear CA. For example, in [93], the genetic algorithms perform search only through group CA pool to identify the exact CA suitable as a *test pattern generator*. In [91], the evolutionary algorithm restricts the search through a special class of non-group CA - $MACA$ to perform the task of pattern classification.

V. CA Applications

Wolfram in his recent book 'A New Kind of Science' [262], explored the reason behind the widespread appeal of cellular automata in a large number of application domains. It is worth quoting a few lines to assess the reason of such an widespread appeal. *Traditional intuition might suggest that to do more sophisticated computations would always require more sophisticated underlying rules. But what launched the computer revolution is the remarkable fact that universal systems with fixed underlying rules can be built that can in effect perform any possible computation. The threshold for such universality has however generally been assumed to be high, and to be reached only by elaborate and special systems like typical* *electronic computers. But $\cdots\cdots$ in fact there are systems whose rules are simple enough to describe in just one sentence that are nevertheless universal. And this immediately shows that the phenomenon of universality is vastly more common and important - in both abstract systems and nature - than has been ever been imagined before.*

Consequently, researchers from diverse fields, without necessarily being aware of the above mentioned framework, have intuitively identified cellular automata dynamics with problems in their own fields. For example, CA have been used to model biological systems from the level of intracellular activity to the levels of clusters of cells, and population of organisms [6], [169]. CA have been used to model the kinetics of molecular systems and crystal growth in chemistry [178]. In physics, the applications cover the study of dynamical systems starting from the interaction of particles to the clustering of galaxies [206]. In the field of computer science, cellular automata based methods have been employed to model the Von Neumann (self-reproducing) machines as well as the parallel processing architecture [260] . Beyond the domain of natural science, CA have also been used to study other diverse fields - as diverse as whether the membership of NATO should be more restricted or not [88].

In view of such diversity, we are presenting the *main* applications that have not only taken the research on cellular automata to new heights, but also made researchers from different fields join and collectively exploit the exciting world of cellular automata. The broad application fields are presented one by one. This section is of course not exhaustive.

A. CA Games

Cellular automata have been used to model different games, the most famous one proposed by Conway and his colleagues [23]. They have illustrated how extremely simple CA rules can be used to characterize highly complex system behavior such as the *game of life*. The game was originally proposed by Conway and made popular through Martin Gardner [96], [97]. There are different variations of the *game of life* like games of proto-life which provides a model for the emergence of a crystalline precursor to life from an initial random prebiotic soup. Besides the *game of life*, there are other games which have been modeled through CA. Notable among these are the games which provide insights into the synchronization problems - for example, the *firing squad* [151], *firing mob* [66], and *queen bee* [232]. A CA simulation of the famous game of *iterated prisoners dilemma* [173], [210] has also been proposed.

B. CA as Parallel Computing Machine

In machine design, the application of CA was proposed for building parallel multipliers [11], [60], prime number sieves [85], parallel processing computers [139], [189], and also for sorting machines [170]. The CA

as a fault-tolerant computing machine has been projected in [167], [171]. Two-dimensional CA have been used extensively for image processing and pattern recognition [196], [235]. The MPP (*Massively Parallel Processor*) of *Goodyear Aerospace Corporation* [81], was one of the fastest computers of the early 1980s. CA based machines termed as CAMs (CA Machines) have been developed by Toffoli and others [240]. These CAMs operate in autonomous mode. The structure of such machines having a high degree of parallelism (with local and uniform interconnection) is ideally suited for simulation of complex systems [242]. A CAM can achieve simulation performance of at least several orders of magnitude higher than that can be achieved with a conventional computer at comparable cost. CAMs were developed as a result of over a decade of machine and modeling research by the Information Mechanics Group at MIT [105].

Since the publication of Von Neumann's seminal work in the late 1950s, the study of artificial self-replicating structures has produced a plethora of results [231]. The studies have raised the possibility of using such self-replicating machine to perform computations [230]. In [52], it is shown that self-replicating structures can be used to solve the NP-complete problem known as satisfiability. Recently, researchers have started exploring the cellular automaton as a typical computing device - it has been presented as a nanometer-scale classical computer in [21].

Modeling Nature and Society

Modeling different physical systems is the most widely explored application of cellular automata. The time evolutions of physical quantities are often analysed with the help of nonlinear partial differential equations. Due to the nonlinearities, the solution of these dynamical system is very complex and often lead to erroneous results due to round-off problems and selection of initial conditions. In this respect, cellular automata provide an alternative approach to study the behavior of dynamical systems. By virtue of their simplicity, they are potentially amenable to easier analysis than partial differential equations. An advantage of cellular automata with respect to systems of differential or partial differential equations is the stability of their dynamics. Adding some new feature or interactions never leads to structural instabilities. The same principles also apply to modeling social systems, where mainly sociologists and economists are trying to replace the partial differential equations by cellular automata as an analytical tool. Good theoretical overviews and insights highlighting the possibilities of CA replacing partial differential equations can be found in [14], [49], [79], [211]. The subsequent two sections provide a sketch of the models proposed by physicists, chemists, theoretical biologists, economists and sociologists.

C. CA For Modeling Physical and Biological Systems

It is from the physicists the drive for developing cellular automata as an alternative to differential equations in modeling laws of physics [241], [174] began. This has resulted in investigation of CA models for physical systems with an emphasis on spin systems [63], [134], [188], [252], models for various forms of regular, dendritic, and random growth based on two-dimensional CA [178], models for pattern formation in reaction-diffusion systems [135], [175], [255], modeling of hydrodynamical systems [87] etc. Cellular automata have been also used to model different chemical processes: the absorption-desorption phenomenon important for analyzing poisoning of a surface during heterogeneous catalysis [51], the inter-diffusion of atoms of two materials [50], the driven diffusion system where the external field biases the movement of each species in opposite direction [99], the solidification process with special analysis of the phase transformation of the substance from liquid to solid [133], alloy formation [178] etc. The phenomenon of coalescence of clouds, fog, atmospheric pollution has been an important modeling problem for CA [190]. In this respect a special mention of lattice gas automata is needed. The Lattice Gas Automaton (LGA) has been the central model for simulating hydrodynamics and reaction-diffusion processes [82]. Despite the discrete dynamics which a LGA generates, it is able to follow the behavior prescribed by Navier-Strokes equations of hydrodynamics.

The successful application of cellular automata in modeling the immune system have been explored by Celada, Seiden, and De Boer et al. [26], [38]. There has been commendable work on developing drug therapy for HIV infection [199], on developing CA models of Tumor Development [155], and on detecting genetic disorders of cancerous cells [152], [153]. In ecology, it has been used to model the predator-prey ecosystem [193], to detect the nature of fish migration in rivers [207], and to model the growth of vegetable population [16], [269]. The chemotaxis CA modeled by Resnik [193] has been used to determine the random walk of animals in response to a chemical gradient. The effect of *simple random walk* by a single individual and *multiple random walks* by a number of individuals in a system has been modeled and extensively studied [221]. A number of CA applications have been reported in the fields of DNA sequences [30], [233], [268]. A very detailed description of biological applications of cellular automata is found in [79]. In [79], the authors have developed theoretical analysis of the various types of elementary cellular interactions with the help of CA model. Each interaction represents a type of elementary biological activity. The authors have analysed the stability of cell-cell interaction which is termed as adhesive interaction without growth or loss of cells, discussed the viability of developing swarming models through CA, and modeled complex pattern formation in salamander larvae with CA. Besides this, the effect of chemotaxis and pressure on biological pattern formation is also discussed. Tumor growth models are developed based upon all the above mentioned elementary biological pattern formation steps. Further, the authors have also analyzed CA models for Turing pattern formation (eg. stripes in zebra) and excitable media based on microscopic interactions.

D. CA Application in Social Sciences

It appears that CA based modeling started at about the same time in social and natural sciences. James M. Sakoda was the first person to develop a CA based model in social sciences. Sakoda published the article 'The Checkerboard Model of Social Interaction' in 1971 [198]; however, the basic design of the model was already present in his unpublished dissertation of 1949. The central goal of his model was to understand group formation. Another early example of CA based modeling was provided by Thomas Schelling. Schelling [203], [204] analyzed segregation processes among individuals belonging to two different classes : black and white. Neither Sakoda nor Schelling ever referred to CA. The formal concept of CA was obviously not known to this group of researchers in early seventies. The first person who explicitly classified checkerboard models under CA framework was the economist Peter S. Albin in his book 'The Analysis of Complex Socioeconomic Systems' [4] and essays [5]. He was also the first to stress the enormous potential of CA and finite automata for understanding social dynamics.

Nevertheless, it is only in the last decade that CA based models have been used more frequently in behavioral and social sciences. In economics, Keenan and O'Brien [124] introduced a one-dimensional CA to model and analyze pricing in a spatial setting. Axelrod [13] made the first step in analyzing the dynamics of cooperation within a CA framework. Nowak and May developed the idea and studied the dynamics of cooperation using a two-dimensional CA with two-person games as building blocks [172], [173]. Bruch [27] and Kirchkamp [125] followed the same line, but applied different and more sophisticated learning rules. The same framework is used in Messick and Liebrand who analyzed the dynamics of three different decision principles [132], [143]. A substantial number of artificial societies described in Epstein and Axtell are based on CA models [84]. Gaylord and D'Andra developed a toolkit for CA based modeling of social dynamics using MATHEMATICA [98]. Oomes used CA to model the effect of economic inequality in emerging markets [103]. Cellular automata have also been used to model traffic flow [99] as well as a design tool for urban development [191], and to develop a model for voters [158]. A good theoretical analysis of CA modeling in a social perspective is reported in [107].

E. VLSI Application of CA

Because of its simplicity, regularity, modularity and cascadable structure with local neighborhood, additive CA are ideally suited for $VLSI$ implementation. Different applications ranging from $VLSI$ *test domains* to the design of a hardwired version of different CA based schemes have been proposed. Some of these applications are briefly reviewed here.

VLSI Design and Test : Based on the statistical properties of the patterns generated, Wolfram broadly classified the 3-neighborhood *CA* into 4 major categories. Out of these, *Class 3 CA* rules are found to be most suitable for pseudo random pattern generation [257]. Hortensius proposed the hybrid *CA* based *pseudo random pattern generator* (*PRPG*) for *built in self test* in *VLSI* circuits [110]. Subsequently, the performance of *CA*-based *PRPG*s has been compared with other existing *PRPG* by a number of researchers. The major contributions in this direction are reported by Serra [214], [215], [217], Chowdhury and Das [53], [68], and Tsalides [247], [248]. Chowdhury [53], Das [68], [71] and Tsalide et al. [248], [247] also proposed the *CA* as a framework for *built in self test* (*BIST*) structures.

Applications of *CA* are also investigated by Albicki et al. [1], [2], [3] and Das et al. [68], [69] for deterministic test pattern generation. The cyclic property of *CA* are utilized to generate the specific set of patterns. Subsequently, Nandi has established *CA* as a universal test pattern generator [163].

Signature analysis is the most widely used data compression technique for test response evaluation. Both additive and non-additive *CA* have been explored as efficient signature analyzers in [111]. Serra et al. have analyzed one-dimensional linear *CA* and their aliasing properties [217]. Similar works have also been reported by Das [68], and Misra [145]. They studied the application of both group and non-group *CA* as signature analyzer. One of the pioneering works on the use of *CA* for *VLSI* testing was done by McLeod et al. [142]. He proposed *CALBO* (*cellular automata logic block observer*), a structure analogous to *BILBO* (*built-in logic block observer*) used in *VLSI* circuit testing.

Mitra et al. have shown that a testable *FSM* can be designed with group *CA* [146], [150]. An elegant *synthesis for testability* (*SFT*) approach for synthesis of easily and fully testable *FSM* has been dealt with by Chowdhury [40], [55]. The scheme synthesizes a given *FSM* around a particular class of *CA* referred to as $D1 * CA$. Such a $D1 * CA$ has been also successfully employed as a low cost, non-invasive *BIST* structure for testing the experimental INTEL chip RAPPID implementing asynchronous circuit block for instruction decoder [195]. Nandi extended the scheme for synthesis of easily testable combinational logic [163].

The GF(2^p) *CA* proposed by Pal [186] and later extended by Sikdar to hierarchical *CA* (*HCA*) are increasingly becoming important in the *VLSI* test and diagnosis field [226]. They are found to be generating a better quality of pseudo-random patterns [227], and are capable of detecting higher numbers of faults in both combinational and sequential circuits [222]. In the field of diagnosis, the *HCA* provides the platform for hierarchical diagnosis of *VLSI* circuits [224].

Error correcting codes: Chowdhury et al. introduced the *CA based error correcting codes* (*CAECC*) [54]. The encoder/decoder circuit complexity for *CAECC* has been shown to be lower than that of the well known Hsiao code [112]. The CA based single byte error correcting and double byte error detecting code proposed in [54] was found to be superior to other schemes in terms of throughput and silicon area. The scheme has been further enhanced by Paul by using the concept of the extension field [183].

Design of CA based cipher system: Nandi et al. presented an elegant low cost scheme for CA based cipher system design [166]. Both *block* ciphering and *stream* ciphering strategies designed with *programmable cellular automata* (PCA) have been reported. Recently, an improved version of the cipher system has been proposed [213], [218].

Design of a CA based Authentication Scheme: Dasgupta et al. proposed an ASIC design for message authentication [75], [76]. The scheme has been refined by Mukherjee et al. by using GF(2^p) CA [160]. Mukherjee et al. have further extended the scheme for inserting invisible watermark in images [159]. The watermark can be either fragile if the watermark shall serve the purpose of authentication or it can be robust if the watermark shall serve the purpose of copyright protection. Fragile watermarking is particularly important in the domain of medical images while robust watermarking plays an important role, for example, in transfer of video/audio files over the Internet.

Cellular automata based compression: Bhattacharya et al. proposed methods to use CA to perform text compression [24]. Lafe reported a novel technique of deriving CA transform functions for compression and encryption [128]. In his work, Lafe showed that cellular automata are capable of generating billions of orthogonal, semi-orthogonal, bi-orthogonal, and non-orthogonal bases. He further devised methods to generate those transforms which have an orthogonal basis and fewer significant coefficients. These transforms are ideal for generating Walsh, Hadamard, Haar and Wavelet transforms which are used for lossy compression. CA based transforms have been investigated by Paul [184], [185] and Shaw [219] for developing efficient schemes for image compression.

Next, we explore an upcoming and highly promising application area of cellular automata. It deals with pattern recognition/classification.

F. Pattern Recognition

There have been quite a few works on pattern recognition based on syntactic approach. A finite CA can be thought of as a language acceptor by considering initial configuration as the input string and acceptance or rejection is determined by a specific cell of the CA. It has been shown that CA can accept context-free language [234], non context-free language [113], and also context-sensitive language [83]. Mahajan's thesis work provides some exciting insights into the potential of CA to act as a language recognizer [136]. She has provided several examples of language recognition by Time Varying CA ($TVCA$) - CA whose transition function at each time step is determined by some external control. She has also noted several open problems regarding CA based simulation of different types of languages.

The advent of neural net with the seminal work of Hopfield [108], [109] popularized the use of machine intelligence techniques in recognizing patterns. However, the inherent dense structure of neural networks is not suitable for $VLSI$ implementation. So, researchers in the neural network domain tried to simplify the structure of the neural network by pruning unnecessary connections [67], [212]. Simultaneously, the CA research community explored the advantages of the sparse network structure of cellular automata for relevant applications. The hybridization of cellularity and neural network has given rise to the popular concept of *cellular neural networks* [12], [57], [58].

There are some earlier theoretical works by CA researchers which have not directly dealt with pattern recognition, but gave important insights in developing the pattern recognition applications. Notable among these are the study of the capacity of CA to perform the task of density classification [149], [228], study of the nature of attractor dominance [254] of CA rules etc. Tzionas et al. proposed a hybrid scheme for multi-valued pattern classification using the parallel architecture that employs a two-dimensional additive cellular automata ($2D$-CA) combined with a single layer perceptron architecture [250], [251]. Cellular automata are used for the amplification of the discrimination sensitivity of the classifier, while the neural networks are used for development of a weighting scheme that reflects the relative bit significance of the multi-valued input patterns leading to an improved classifier performance. Tzionas et al. also presented another variation of CA based pattern classifier based on a nearest neighborhood discriminant [249].

There has been some other notable work to design CA based model of associative memory and its application for pattern recognition [39], [115], [156], [192]. A more robust model of associative memory has been reported in the thesis [89], some of whose results are published in [92], [137]. This work provides interesting design solutions to show that the memorizing capacity of a hybrid 3-neighborhood CA is better than that of Hopfield network - the model of neural network known for its association capacity. Many concepts from the discipline of biology have been borrowed to build the clustering concept through cellular automata model. One such model mimics the behavior of ants to gather and sort corpses in a self-organized way [49], [101]. Chattopadhyay et al. have recently observed that a special class of CA, referred to as $MACA$, behaves as a natural classifier [42]. Ganguly in his thesis further characterized the $MACA$ basins, he showed that $MACA$ basins form natural clusters which can be employed to the task of pattern classification and associative memory [89]. The series of publications [91], [90], [94], [197], [225] develops the applications of $MACA$ based pattern classifiers and associative model. The superiority of $MACA$ based classifier over conventional schemes like decision tree, multi-layer perceptron and its application in fields of datamining, image compression etc has been shown by Maji et al [138].

VI. Conclusion

This paper reports a detailed survey of the various modeling applications of CA. The survey also provides a vivid sketch of the different theoretical developments which have taken place over the years in the CA research field. These developments have established the immense potential of CA in modeling different applications, thus spreading the appeal of cellular automata over a wide cross-section of researchers. The extensive bibliography in support of the different developments of CA research provided with the paper should be of great help to CA researchers in the future.

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