Complex Systems and Network Science:

Cooperation and Competition

Ozalp Babaoglu
Dipartimento di Informatica — Scienza e Ingegneria
Università di Bologna
www.cs.unibo.it/babaoglu/

Introduction

- Agents need to choose among several options
- Agents do not choose in isolation but the outcome of their decisions (actions) depends on the choices made by other agents they are interacting with
  - pricing a new product in a competitive market
  - biding in an auction
  - choosing a route in a data network
  - choosing a stance in international relations
  - deciding to resort to doping or not
- Want to study notions like "cooperation" in a world where agents are in perpetual competition

Exam or project

- Student needs to decide whether to study for exam or prepare project (cannot do both)
- Project prepared jointly with a partner
- Exam:
  - if you study, expected grade for the exam is 92
  - if you do not study, expected grade for the exam is 80
- Project:
  - if both of you work on it, expected joint grade for the project is 100
  - if only one of you works on it, expected joint grade for the project is 92
  - if no one works on it, expected joint grade is 84
- Your grade for the course is the *average* of exam and project
- Each of you needs to decide independently, knowing that the other will also be making a decision

Ingredients of a game

- A set of participants called *players*
- Each player has a set of options for behavior called *strategies*
- For each choice of strategies, each player receives a *payoff* that may depend on the strategies selected by other players
- Summarized in the form of a *payoff matrix*

Exam or project

- Payoff matrix in terms of grades for the course

Your partner

You	Project	Exam
Project	90, 90	86, 92
Exam	92, 86	88, 88

Considerations for games

- How many players?
  - For now, consider only *two-player* games
- How many encounters?
  - For now, consider only *one-shot* games (as opposed to *dynamic* or *iterated* games)

Considerations for games

- What do the players know?
- For now, assume each players knows everything about the structure of the game:
  - who the other players are,
  - the set of strategies,
  - the payoff matrix (but not the strategies of the other players)
- Each players tries to maximize her own payoff, given her beliefs about the strategies used by other players — *rational* players

Back to “Exam or project”

Your partner

You	Project	Exam
Project	90, 90	86, 92
Exam	92, 86	88, 88

- Consider what you should do for each possible choice of strategy by your partner:
  - if you knew that she was going to study, you should study for the exam as well
  - if you knew that she was going to work on the project, you should still study for the exam
- *Strictly Dominant Strategy*: strategy that is the best choice regardless what the other player does

Back to “Exam or project”

- “Study for exam” is a strictly dominant strategy for both players, meaning each will get an average grade of 88
- Yet, there is an outcome that is better for both (both worked on project and obtain an average grade of 90) that cannot be achieved by rational players

Dilemmas

- *Dilemma games* are those where no matter what choice a player makes, there exists the possibility that she feels “regret” — given the possibility, she would have acted differently

Player 2

Player 1	*Cooperate*	*Defect*
Cooperate	*R, R*	*S, T*
Defect	*T, S*	*P, P*

- Generic two-player dilemma game where *R* is the ***reward*** for mutual cooperation, *S* is the ***sucker’s*** payoff, *T* is the ***temptation*** to defect and *P* is the ***punishment*** for mutual defection

Dilemmas

- With four different payoffs (*R, S, T, P*), there are 4! = 24 possible orderings
- Yet, most of these orderings correspond to games with no dilemma (e.g., when *R*>*S*>*T*>*P* always cooperate, never regret)
- In order for a dilemma to exist, players must have a reason to both cooperate and defect
- Conditions necessary for dilemma
  - (*R*>*S*) and (*T*>*P*)
  - (*T*>*R*) or (*P*>*S*)
  - *R*>*P*
- Each player wants the other to cooperate but both are tempted to defect

Dilemmas

- Out of the 24 possible games, only three are left as dilemmas
- Chicken (*T*>*R*>*S*>*P*): Two cars heading towards each other
  - Should you “Swerve (Cooperate)” or “Don’t swerve (Defect)”?
  - (“Swerve”, “Swerve”) and (“Don’t swerve”, “Don’t swerve”) both lead to “regret”
- Stag Hunt (*R*>*T*>*P*>*S*): 2-player soccer tournament game night
  - Should you “Go (Cooperate)” or “Stay (Defect)”?
  - (“Go”, “Stay”) and (“Stay”, “Go”) both lead to “regret”
- Prisoner’s Dilemma (*T*>*R*>*P*>*S*)

Prisoner's Dilemma

- Two robbery suspects apprehended by police, being interrogated in separate rooms
- There isn't enough evidence to convict either one, but each can be charged with a lesser crime (resisting arrest)
- Each suspect needs to decide unilaterally (no talking, collusion) whether to "Confess" (*C*) or "Deny" (*D*)

Suspect 2 (columns); Suspect 1 (rows)

	D	*C*
D	−2, −2	−10, −1
C	−1, −10	−8, −8

- Confession leads to higher individual payoff — selfishness
- Denial leads to higher global payoff — cooperation

Prisoner's Dilemma

- Note that (*C*, *C*) represents an *equilibrium* state — neither prisoner can improve her payoff by changing her strategy
- No other pair of strategies is an equilibrium — some prisoner is always better off by changing her strategy
- "Dilemma" because both prisoners would have been better off if both had chosen "Deny"
- But (*D*, *D*) is *not* an equilibrium state
- PD captures the conflict between *individual rationality* (selfishness) and *common good* (cooperation):
  - Doping among athletes
  - Arms race, nuclear disarmament
  - Trade wars, trade embargo among nations
  - Breakaway in bike races

Best responses

- Let *S* be the strategy chosen by Player 1 and *T* be the strategy chosen by Player 2
- Let P1(*S*,*T*) denote the payoff to Player 1
- Strategy *S* for Player 1 is a ***best response*** *to a strategy T* for Player 2 if *S* produces at least as good a payoff as any other strategy paired with *T*:
  - $P1(S,T) \geq P1(S',T)$ for all other strategies S' of Player 1

Best responses

- ***Strict best response***: $P1(S,T) > P1(S',T)$
- ***Dominant strategy*** for Player 1 is a strategy that is a best response to *every* strategy of Player 2
- ***Strictly dominant strategy*** for Player 1 is a strategy that is a strict best response to every strategy of Player 2

Nash equilibrium

- Even in games where there are no dominant strategies, we should expect players to use strategies that are best responses to each other
- If players chose strategies that are best responses to each other, then no player will have an incentive to deviate to an alternative strategy and the system will remain in an "equilibrium"

Nash equilibrium

- Pair of strategies (*S*,*T*), *S* for Player 1 and *T* for Player 2, is a ***Nash equilibrium*** if *S* is a best response to *T* and *T* is a best response to *S*
- John Nash shared the 1994 Nobel Prize in Economics for this idea that he developed in 1950

Coordination games

- Two possible behaviors: *A* and *B*
- Players have an incentive to adopt the same behavior
  - Units of measure (Metric vs Imperial)
  - Sports (Basketball vs Soccer)
  - Social networking (Instagram vs TikTok)
- Payoff matrix

Player 2

Player 1	*A*	*B*
A	a, a	$0, 0$
B	$0, 0$	b, b

Coordination games

- Balanced Coordination Game (equal payoffs: $a=b$)
- Two individuals are trying to meet at a shopping mall with two entrances, a *North* entrance and a *South* entrance

Player 2

Player 1	*North*	*South*
North	1, 1	0, 0
South	0, 0	1, 1

- Reasonable to expect that players will play strategies in the Nash equilibrium
- Two Nash equilibria: (*North*, *North*) and (*South*, *South*)

More coordination games

- Unbalanced Coordination Game

Player 2

Player 1	North	South
North	1, 1	0, 0
South	0, 0	2, 2

- External or social factors may influence which equilibrium is preferred (South entrance has a nice coffee shop ideal for passing time while waiting)

More coordination games

- A couple trying to pick a movie for viewing together
- *Battle of the Sexes*

Your Partner

You	Action	Romance
Action	1, 2	0, 0
Romance	0, 0	2, 1

Multiple equilibria

- Multiple Nash equilibria arise also in other games where players engage in an “anti-coordination” activity
- Two strategies: Aggressive “*Hawk*” versus Passive “*Dove*”

Player 2

Player 1	Dove	Hawk
Dove	3, 3	1, 5
Hawk	5, 1	0, 0

- Has two Nash equilibria (*Dove*, *Hawk*) and (*Hawk*, *Dove*)
- Similar to Chicken if we interpret the strategies “Dove” and “Hawk” as “Swerve” and “Don’t swerve”

Mixed strategy games

- There exist games that have no Nash equilibria when restricted to ***pure strategies*** (always play the *same* strategy)
- Need to enlarge the set of strategies to include the possibility of randomization
- *Matching pennies*

Player 2

Player 1	H	T
H	−1, +1	+1, −1
T	+1, −1	−1, +1

- Example of *attack-defense* or *zero-sum* games
- Note that there is no pair of strategies that are best responses to each other

Mixed strategy games

- If we limit the two players as having the two strategies H or T, there exists no Nash equilibrium
- Do not treat strategies as simply H and T, but ways of *randomizing* one's behavior between H and T
- Strategy becomes choosing a *probability* with which to play H
- Strategy p for Player 1: play H with probability p, play T with probability $(1-p)$
- Strategy q for Player 2: play H with probability q, play T with probability $(1-q)$
- Payoffs for these *mixed strategy games* become random quantities

Mixed strategy games

- Compute payoffs in terms of *expected values*
- Suppose Player 1 chooses pure strategy H while Player 2 chooses probability q for playing H
- Expected payoff to Player 1 is $(-1)q+(1)(1-q)=1-2q$
- Suppose Player 1 chooses pure strategy T while Player 2 chooses probability q for playing H
- Expected payoff to Player 1 is $(1)q+(-1)(1-q)=2q-1$

Equilibrium with mixed strategies

- *Nash equilibrium* for a mixed strategy game is a pair of strategies (now probabilities) such that each is a best response to the other
- *Observe*: No pure strategy can be part of a Nash equilibrium
- Suppose pure strategy H by Player 1 were part of Nash equilibrium
- Player 2's best response would be H (pennies match)
- But H by Player 1 is *not* a best response to H by Player 2 — contradiction
- Reason analogously for other possible pure strategies
- So, at Nash equilibrium, both players must be using probabilities *strictly* between 0 and 1

Equilibrium with mixed strategies

- What is Player 1's best response to strategy q used by Player 2?
- Expected payoffs to Player 1 from pure strategies H and T are $1-2q$ and $2q-1$
- If $1-2q \neq 2q-1$, one of pure strategies H or T must be a unique best response by Player 1 to strategy q by Player 2 — if the two expected payoffs are not equal, then one of them must be larger
- But this contradicts our conclusion that pure strategies cannot be part of any Nash equilibrium
- Neither can probabilities that make these expectations unequal be part of any Nash equilibrium

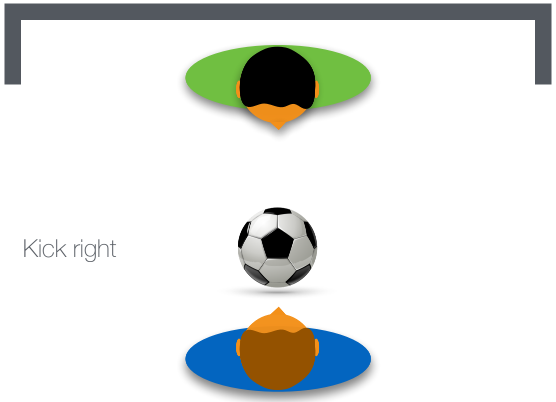
Equilibrium with mixed strategies

- Thus we must have $1-2q = 2q-1$, or $q = 1/2$
- Symmetric reasoning from Player 2's point of view leads us to conclude that $p = 1/2$, so (½, ½) is a Nash equilibrium for Matching Pennies
- The choice of $q = 1/2$ by Player 2 makes Player 1 *indifferent* between playing H or T and we say that "strategy $q = 1/2$ by Player 2 is *non-exploitable* by Player 1"

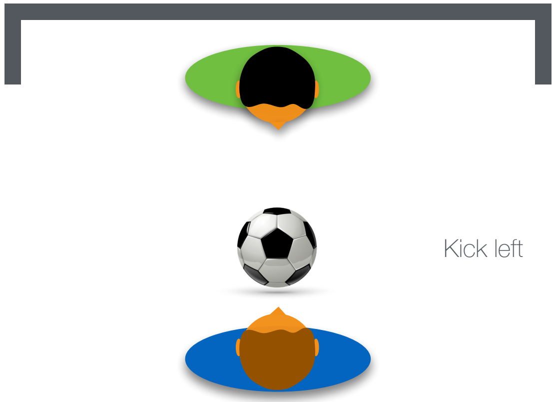
Penalty kick game

- Two players — *goalie* and *kicker* are facing each other
- Kicker has to pick the direction of *kick* and the goalie has to pick the direction of *dive*
- The ball is kicked fast enough so that the two decisions are effectively simultaneous — goalie cannot wait to see which direction the ball is kicked but must react immediately

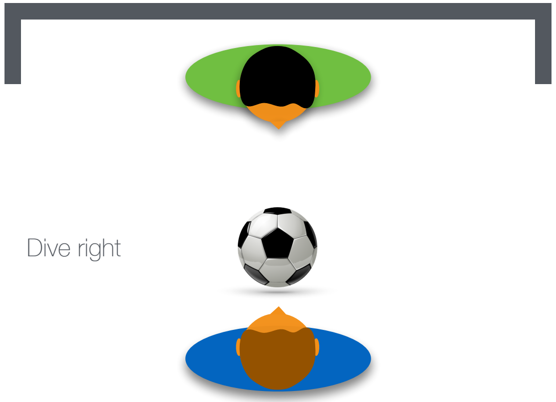
Penalty kick game
Kicker's strategy



Penalty kick game
Kicker's strategy

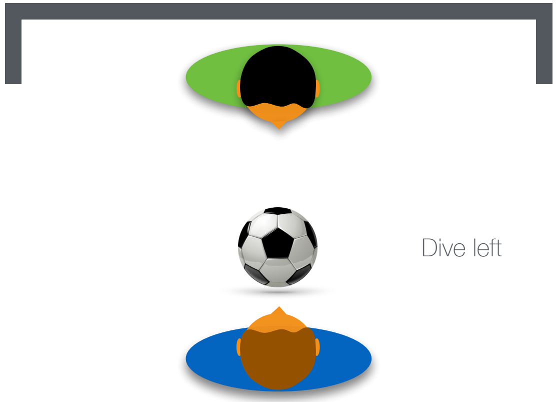


Penalty kick game
Goalie's strategy



Dive right

Penalty kick game
Goalie's strategy



Dive left

Penalty kick game

- Note that kicking and diving directions are from the goalie's perspective
- Also, note that "kick down the middle" (for kicker) and "stay put" (for goalie) are not admissible strategies for fear of shame
- This is similar to the "Matching Pennies" game — if the direction of kick and dive match, the goalie has a good chance of saving, if they differ, the kicker has a good chance of scoring
- Unlike "Matching Pennies", the "Penalty Kick" game is asymmetric

Penalty kick game

- Payoff matrix obtained from analysis of 1,417 penalty kicks in 5 years of professional soccer matches in European leagues

Kicker's payoff:

Kicker \ Goalie	*Left*	*Right*
Left	0.58, –0.58	0.95, –0.95
Right	0.93, –0.93	0.70, –0.70

Goalie's payoff:

Kicker \ Goalie	*Left*	*Right*
Left	–0.42, 0.42	–0.05, 0.05
Right	–0.07, 0.07	–0.30, 0.30

Penalty kick game

- Let p be the probability that the goalie dives to the left and $1-p$ be the probability he dives to the right
- To make the kicker *indifferent*, the goalie must pick p such that *payoff from kicking left = payoff from kicking right*

  $(0.58)(p) + (0.95)(1-p) = (0.93)(p) + (0.7)(1-p)$

  resulting in p=0.417
- In other words, the goalie should dive left roughly 42% of the time to make the kicker indifferent between kicking left and right

Penalty kick game

- Let q be the probability that the kicker kicks to the left and $1-q$ be the probability he kicks to the right
- To make the goalie *indifferent*, the goalie must pick q such that *payoff from diving left = payoff from diving right*

  $(0.42)(q) + (0.07)(1-q) = (0.05)(q) + (0.3)(1-q)$

  resulting in q=0.383
- In other words, the kicker should kick left roughly 39% of the time to make the goalie indifferent between diving left and right

Penalty kick game

- From the 1,417 observed penalty kicks, the actual behavior of kickers and goalies:
  - Portion of kicks to the left: 40% (predicted 38.3%)
  - Portion of dives to the left: 42% (predicted 41.7%)