The algebra and geometry of commitment

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Setting the (logical and computational) context

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Lorenzen dialogues

Setting the (logical and computational) context

Lorenzen dialogues

Justification of procedural rules:

1. the Proponent may only assert an atomic formula after the Opponent has asserted it
2. if one responds to an attack, this has to be the latest open attack
3. an attack may be answered at most once
4. an assertion made by P may be attacked at most once.

Setting the (logical and computational) context

Lorenzen dialogues

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why?

Setting the (logical and computational) context

Lorenzen dialogues

The Dummett-Brandom theory of assertion:

The speech act of asserting arises in a particular, socially instituted, autonomous structure of responsibility and authority. In asserting a sentence one both commits oneself to it and endorses it.

(Brandom, *Asserting*, 1983)

Setting the (logical and computational) context

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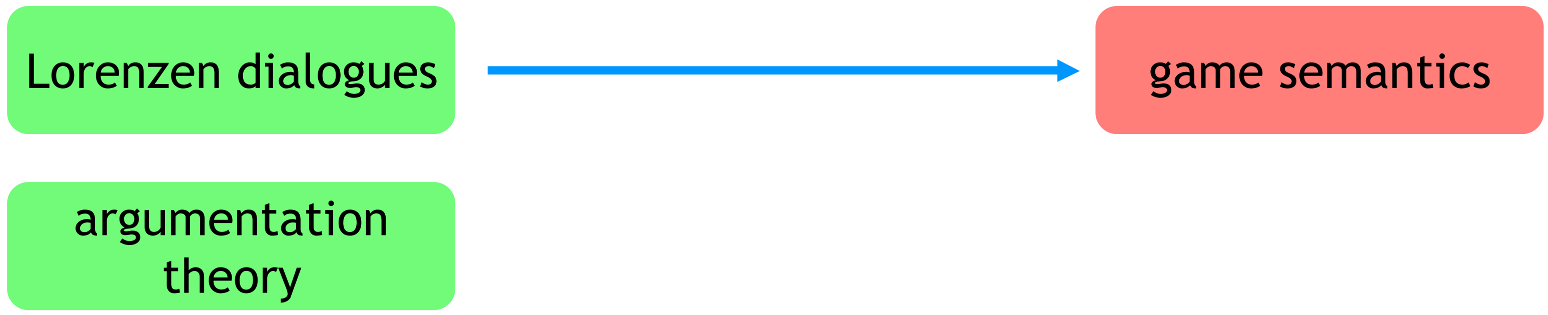
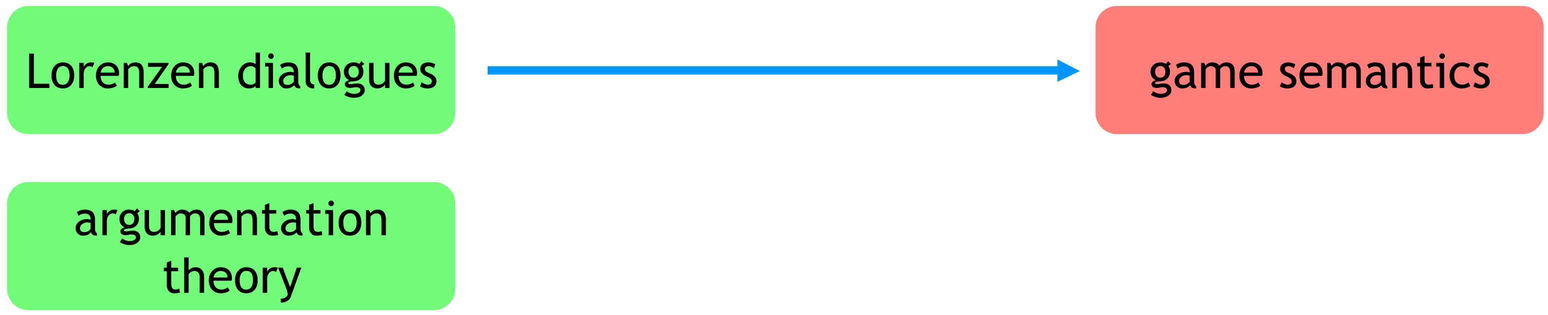


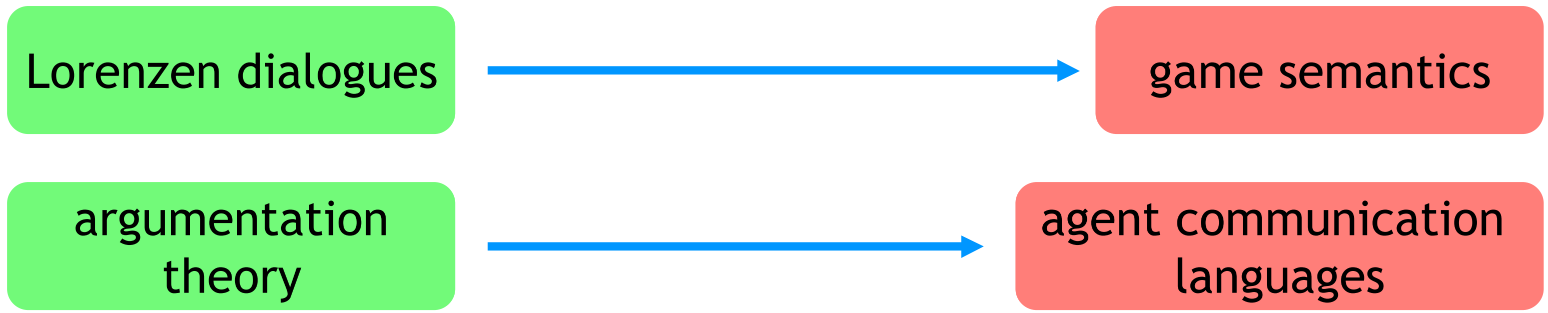
Setting the (logical and computational) context



Hamblin, *Fallacies*, 1970: the idea of a commitment store

A speaker who is obliged to maintain consistency needs to keep a store of statements representing his previous commitments, and require of each new statement he makes that it may be added without inconsistency to this store.

Setting the (logical and computational) context



Setting the (logical and computational) context

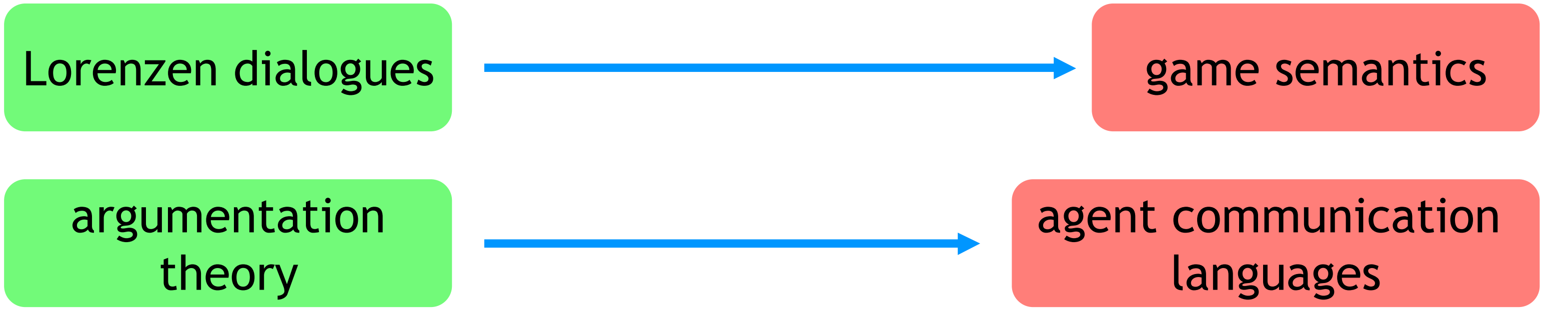
Lorenzen dialogues → game semantics

argumentation theory → agent communication languages

Walton and Krabbe, *Commitment in Dialogue*, 1995

distinction between claims (assertions) and concessions as different types of commitment in dialogue, a distinction Lorenzen never stressed. Barth and Krabbe (1982) made a clear distinction between roles in dialogue, but still spoke indiscriminately of attacks and defences. Different types of commitment were, however, a main theme in Walton and Krabbe.

Setting the (logical and computational) context



Singh (~1998): commitment as a key notion in the social semantics for agent communication languages, following ideas of Habermas

Question:

What are the formal structures underlying the complex networks of commitments that bind together interacting (logical, computational) agents?

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I look for geometric and algebraic accounts of these structures

I. Accounting from first principles

The forms of a commitment

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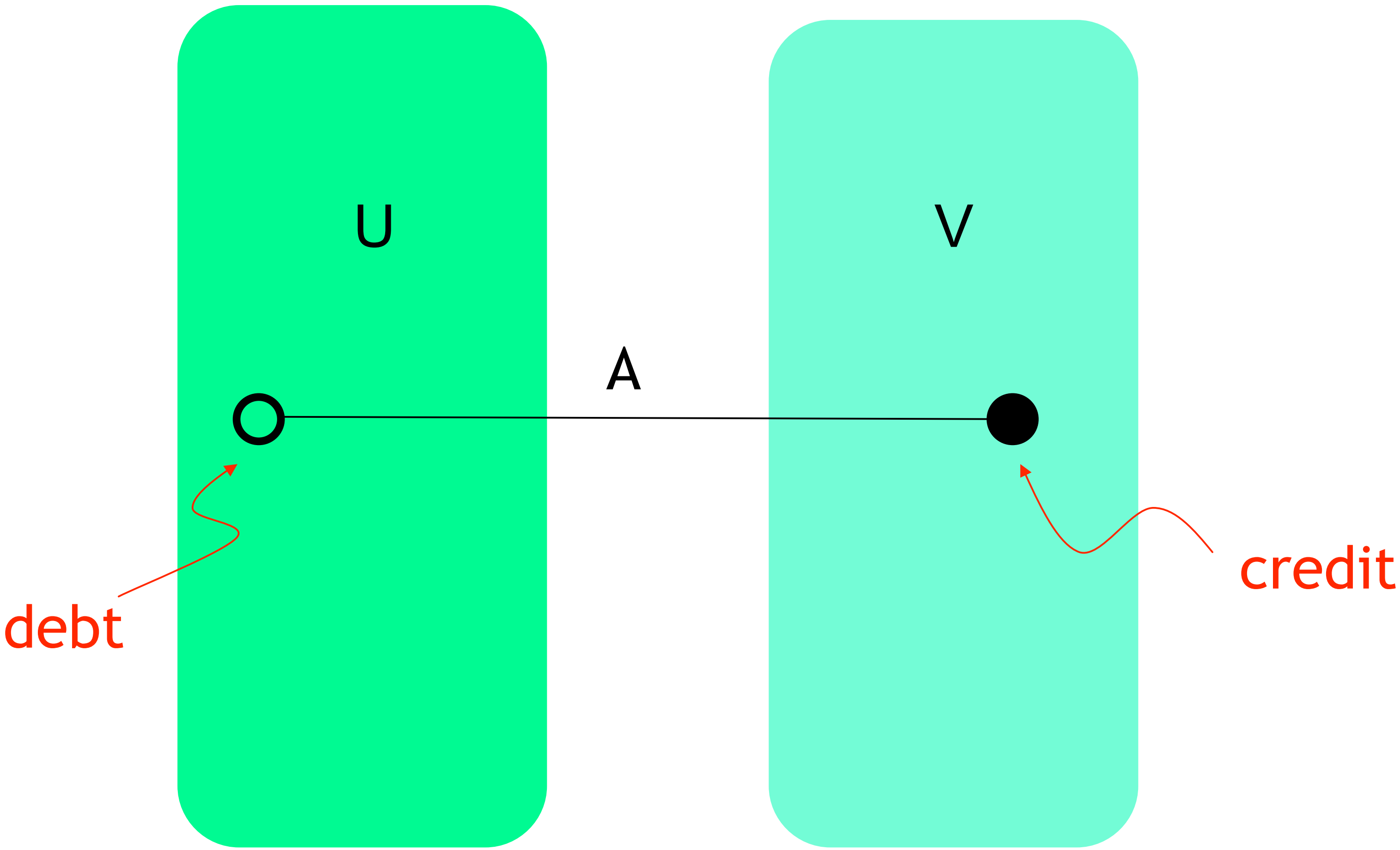

The forms of a commitment: accounts

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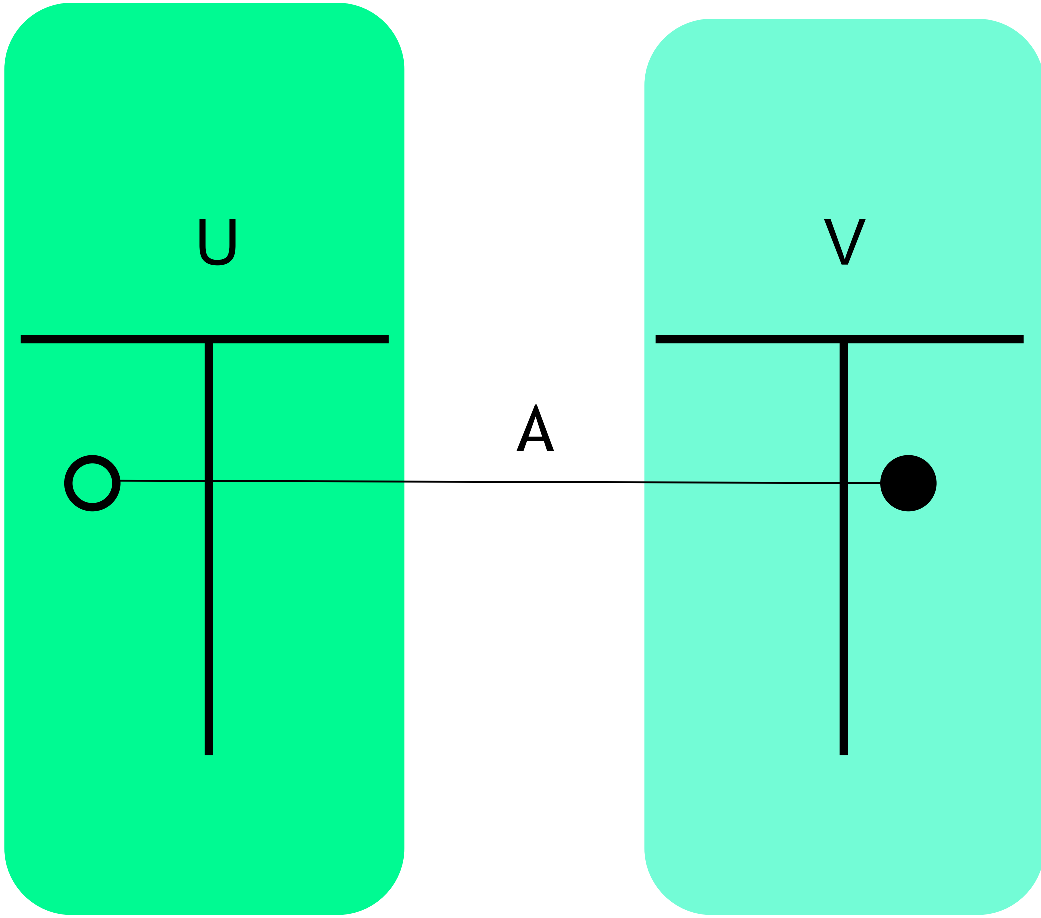
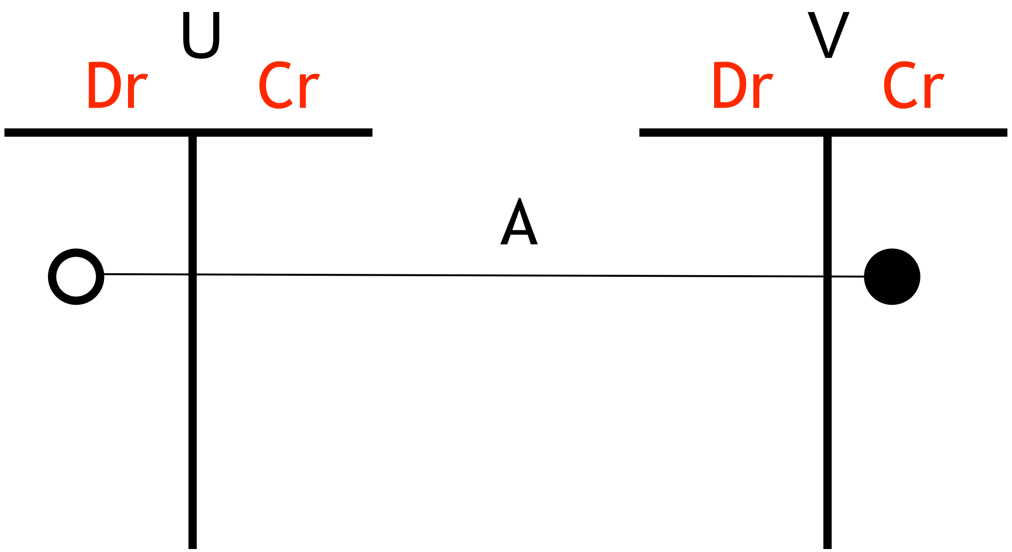


a pair of accounts…

The forms of a commitment: accounts



...with double-entry accounting

The forms of a commitment: one-liners

The forms of a commitment: one-liners

[A*U,AV]

where

- A,B,C,... are (positive) commitment types,
- U,V,W,... are places,
- ( )* is a fixed-point free involution of types (positive ⇆ negative)

A system of accounts is a string of (type,place) pairs

The social life of commitments

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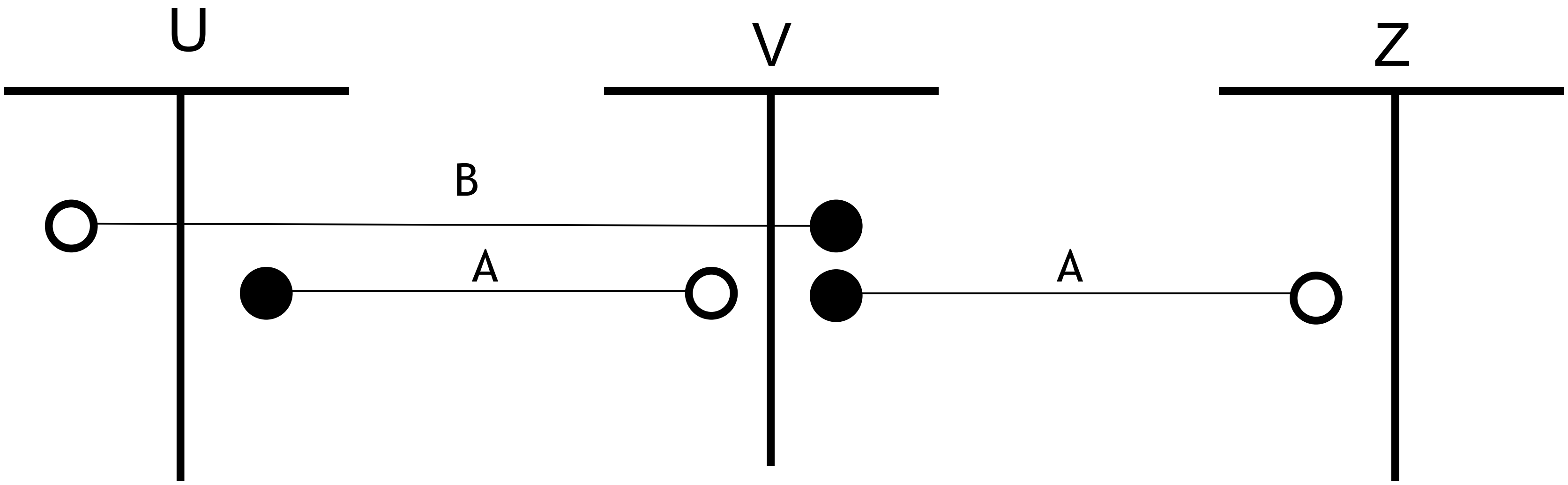
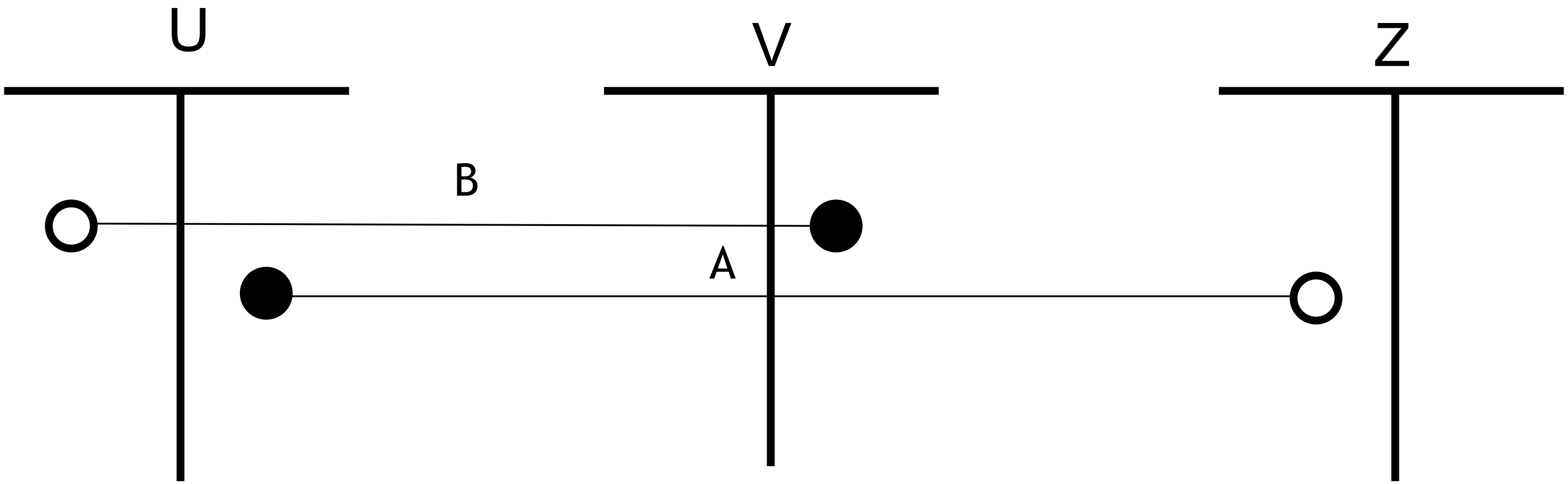


[B*U,AU, A*V,BV,AV, A*Z]

The social life of commitments

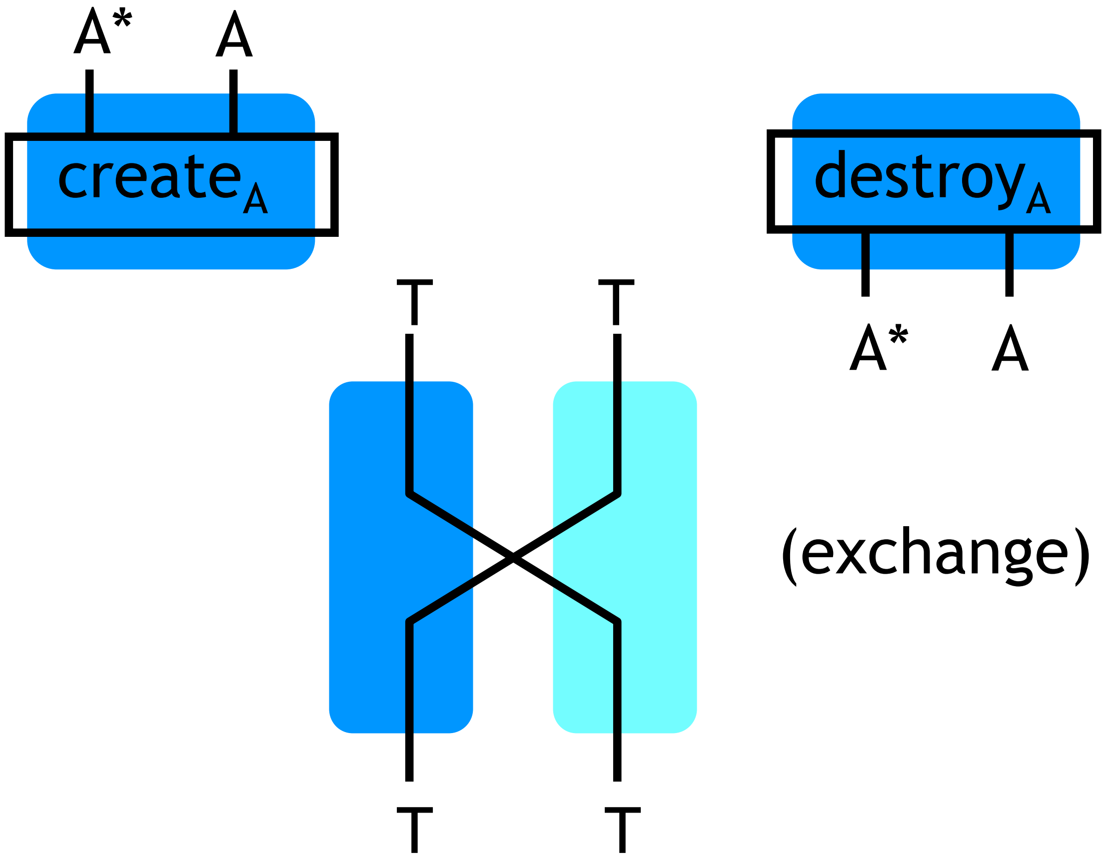


[B*U,AU, BV, A*Z]

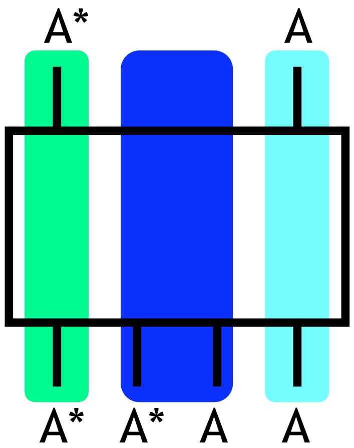
II. The geometry of commitment

Space-time diagrams

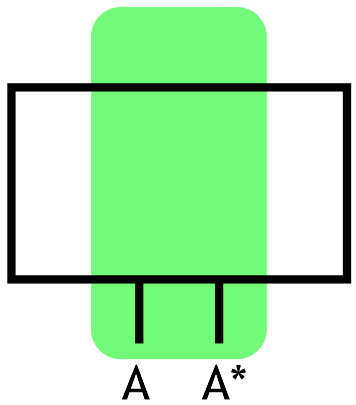
Transform commitments by composing nodes of three kinds, getting space-time diagrams



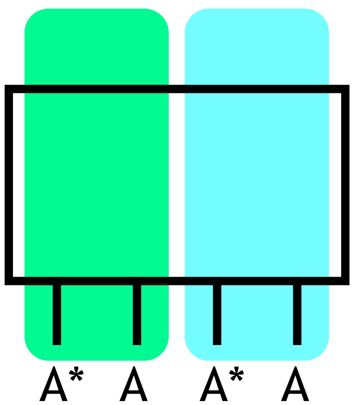
Transforming commitments (Justinian *Digesta*)



delegation



confusion



compensation

Tally sticks and their uses

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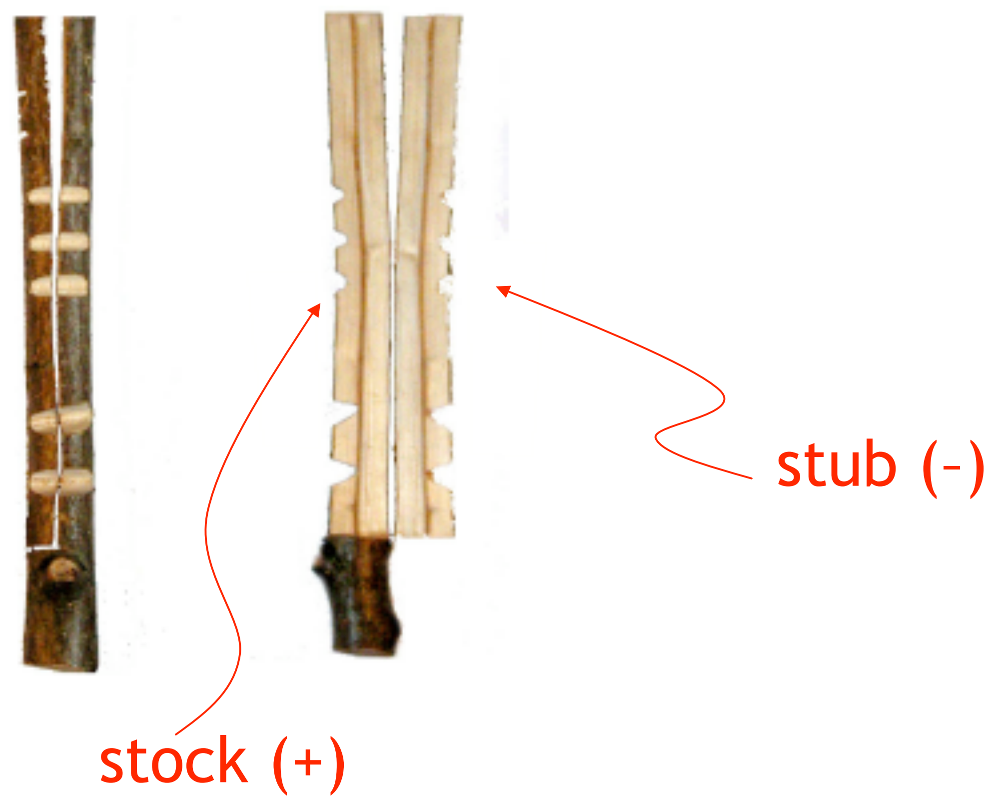
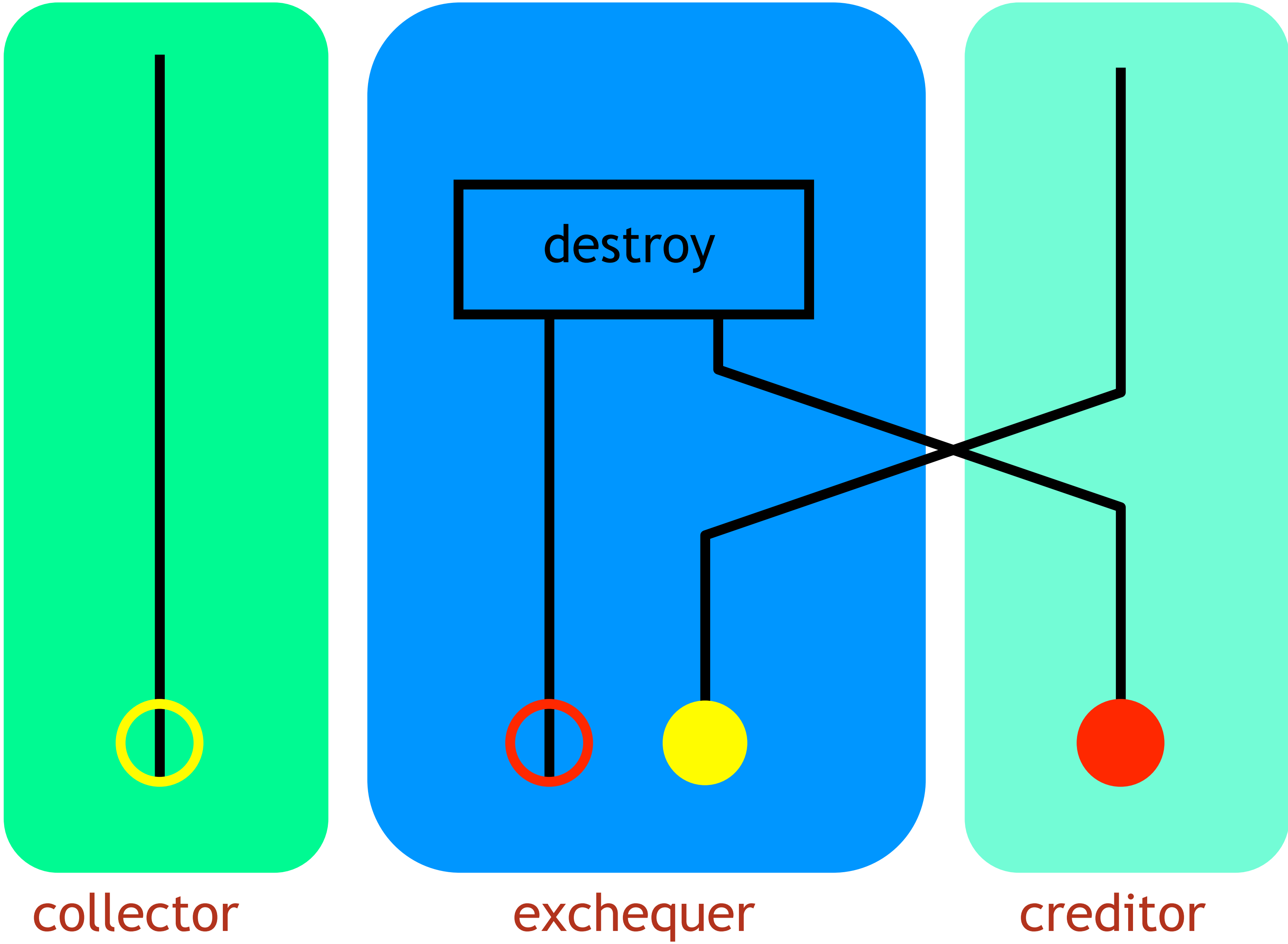


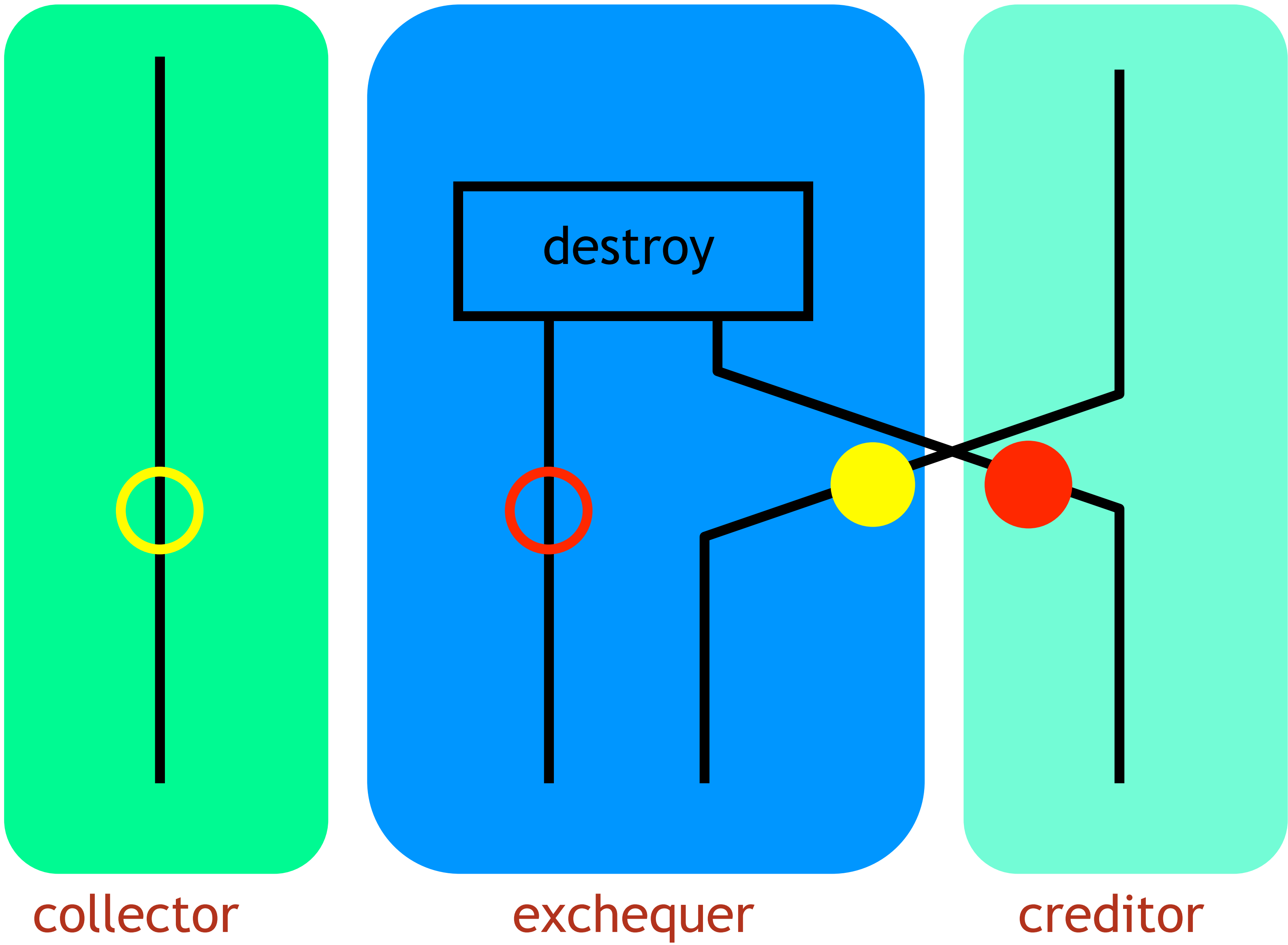
the medieval tally was split into two bits of unequal length. The stock was kept as a receipt by the person who handed over goods or money. The stub was kept by the receiver

Delegation with tally sticks

if the exchequer E was short of funds, it would cajole creditor B into taking not cash but a tally addressed to some tax collector A. The tally purported to be a receipt by the exchequer for such-and-such a sum, paid in by the collector A out of such-and-such type of revenue. Armed with this tally of assignment, creditor B presented himself to the collector, and — if all went smoothly — exchanged it for cash. The tally would afterwards serve the collector as his acquittance at the exchequer.

(Baxter, *Early accounting: The tally and the checkerboard*, The Accounting Historians Journal, 1989)


destroy
collector
exchequer
creditor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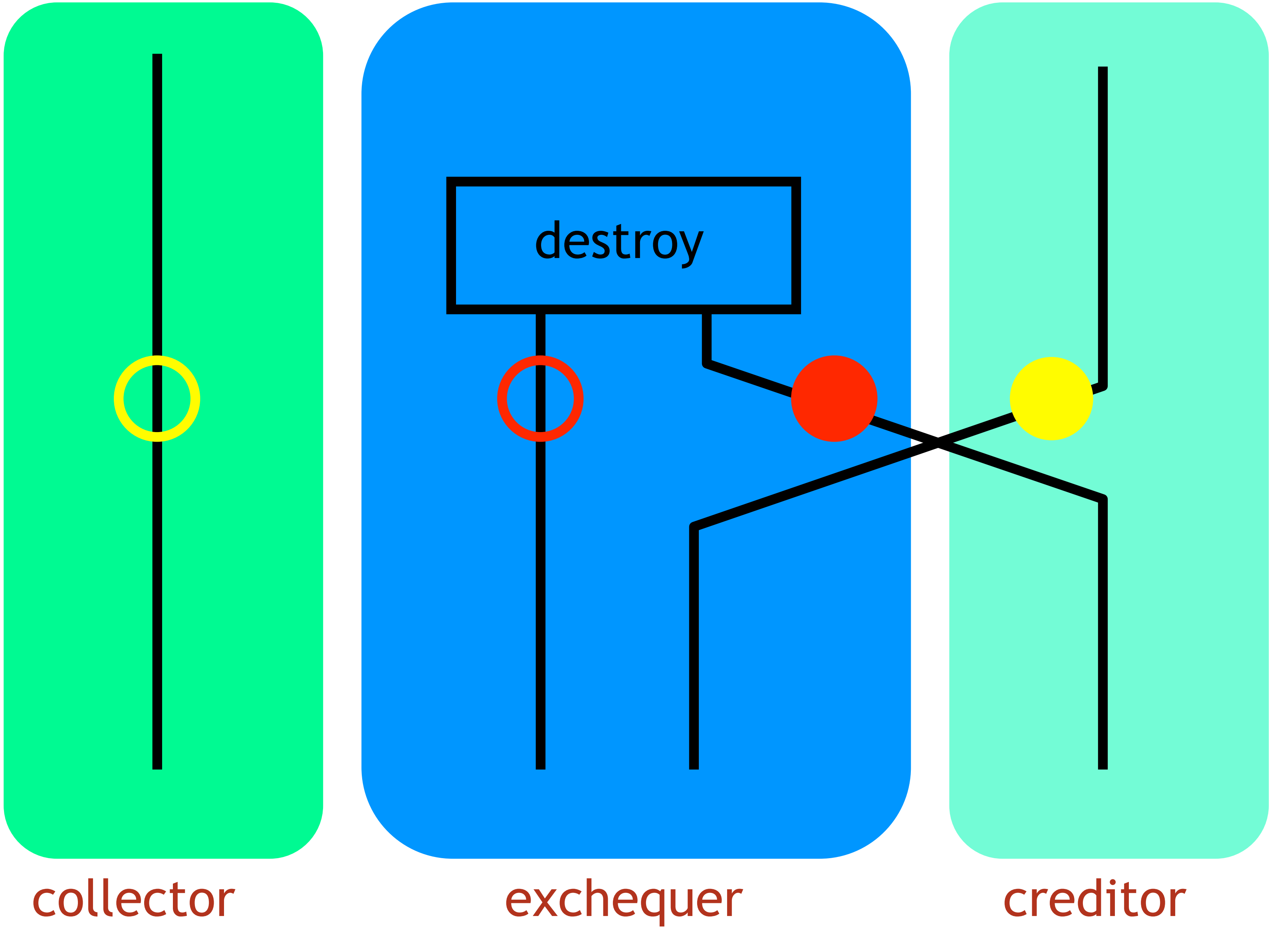
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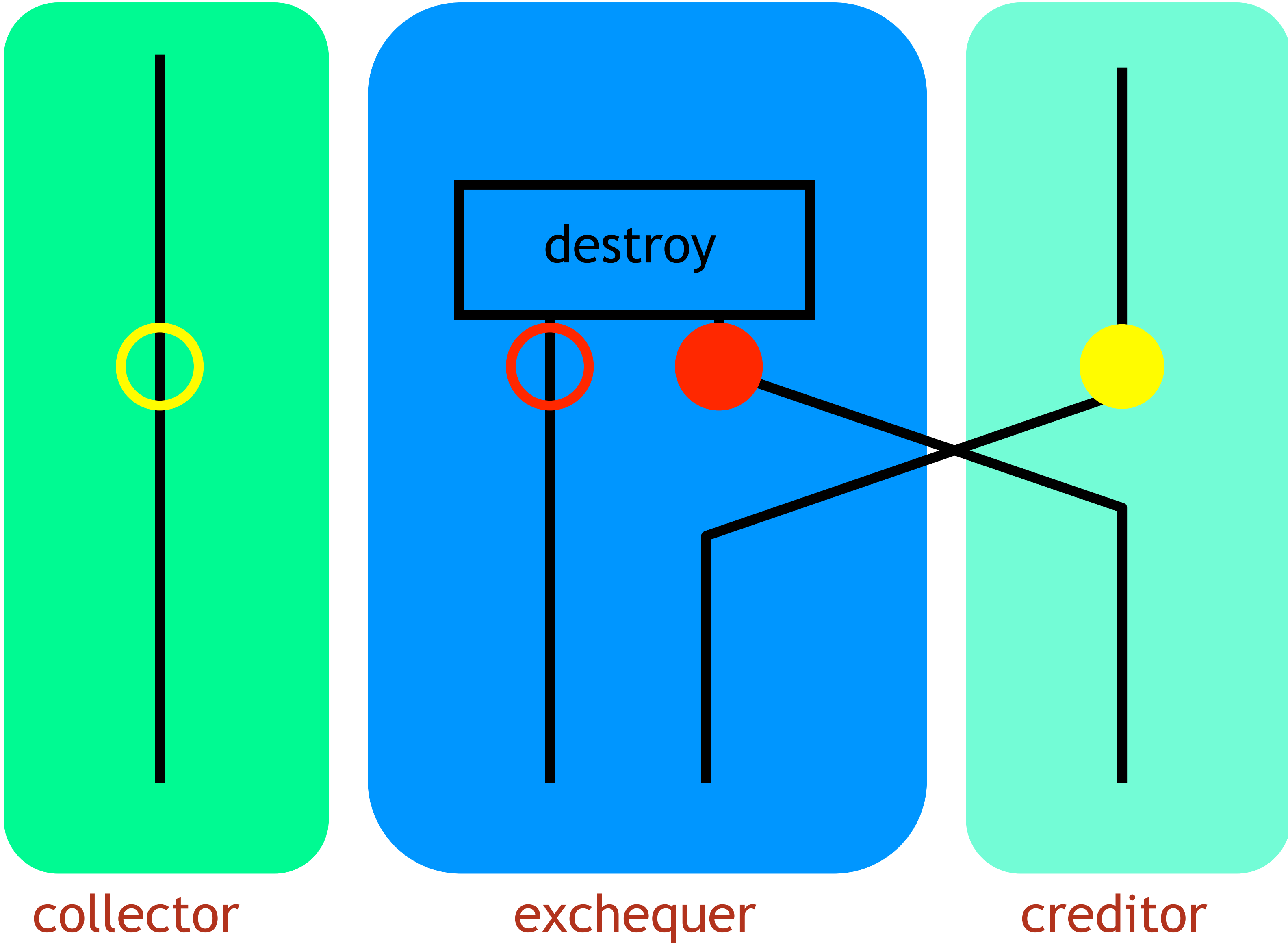
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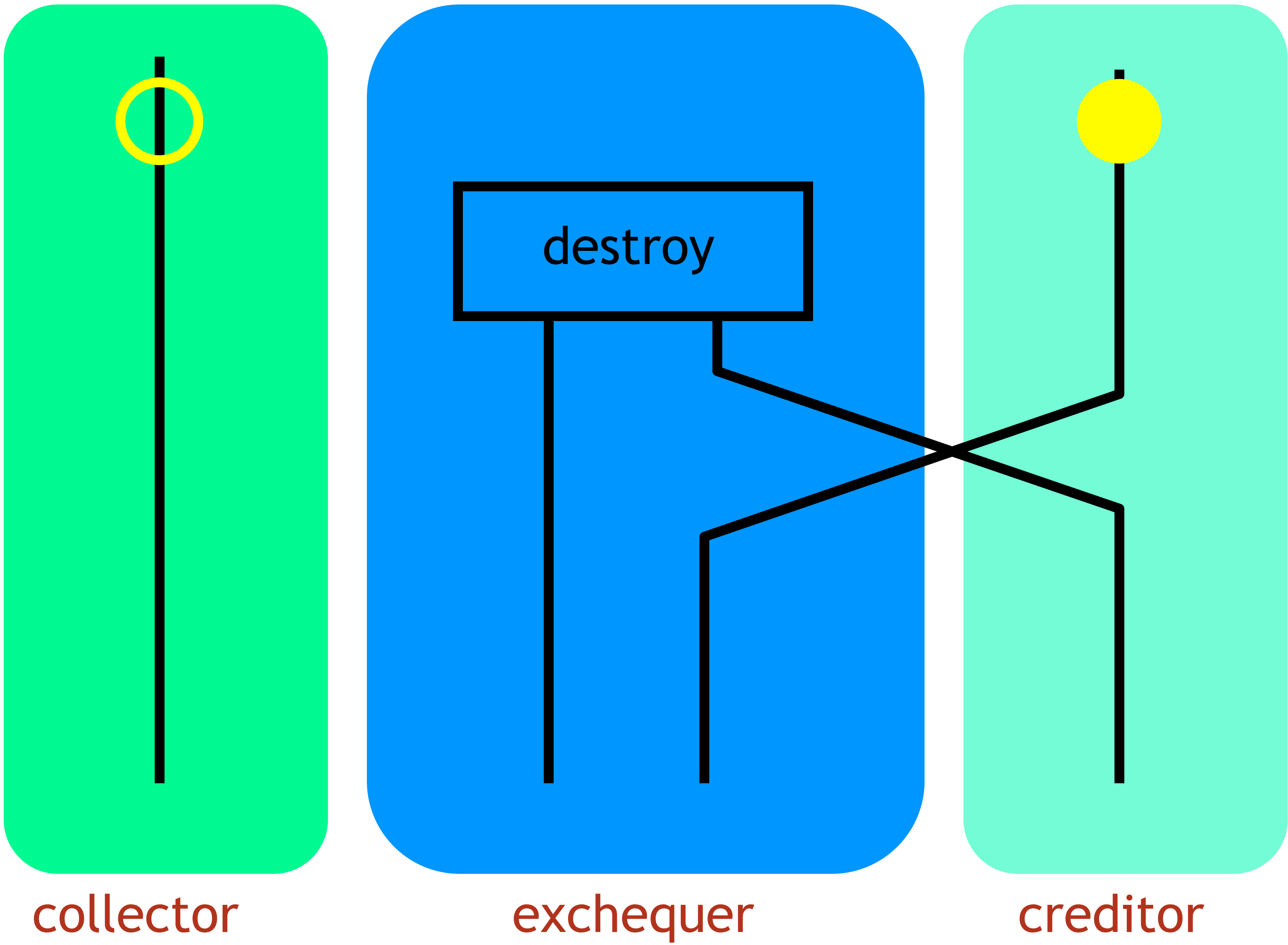
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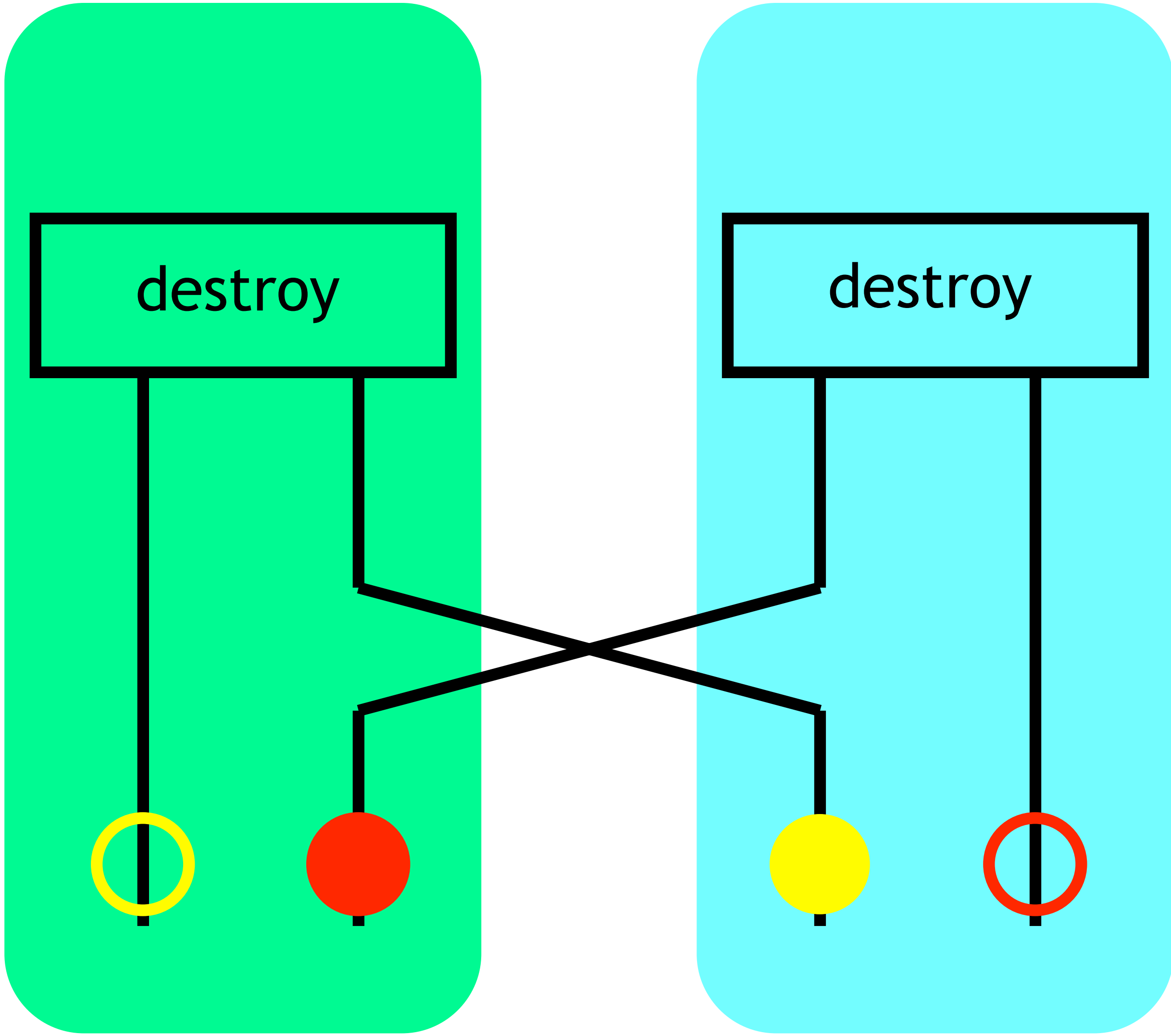

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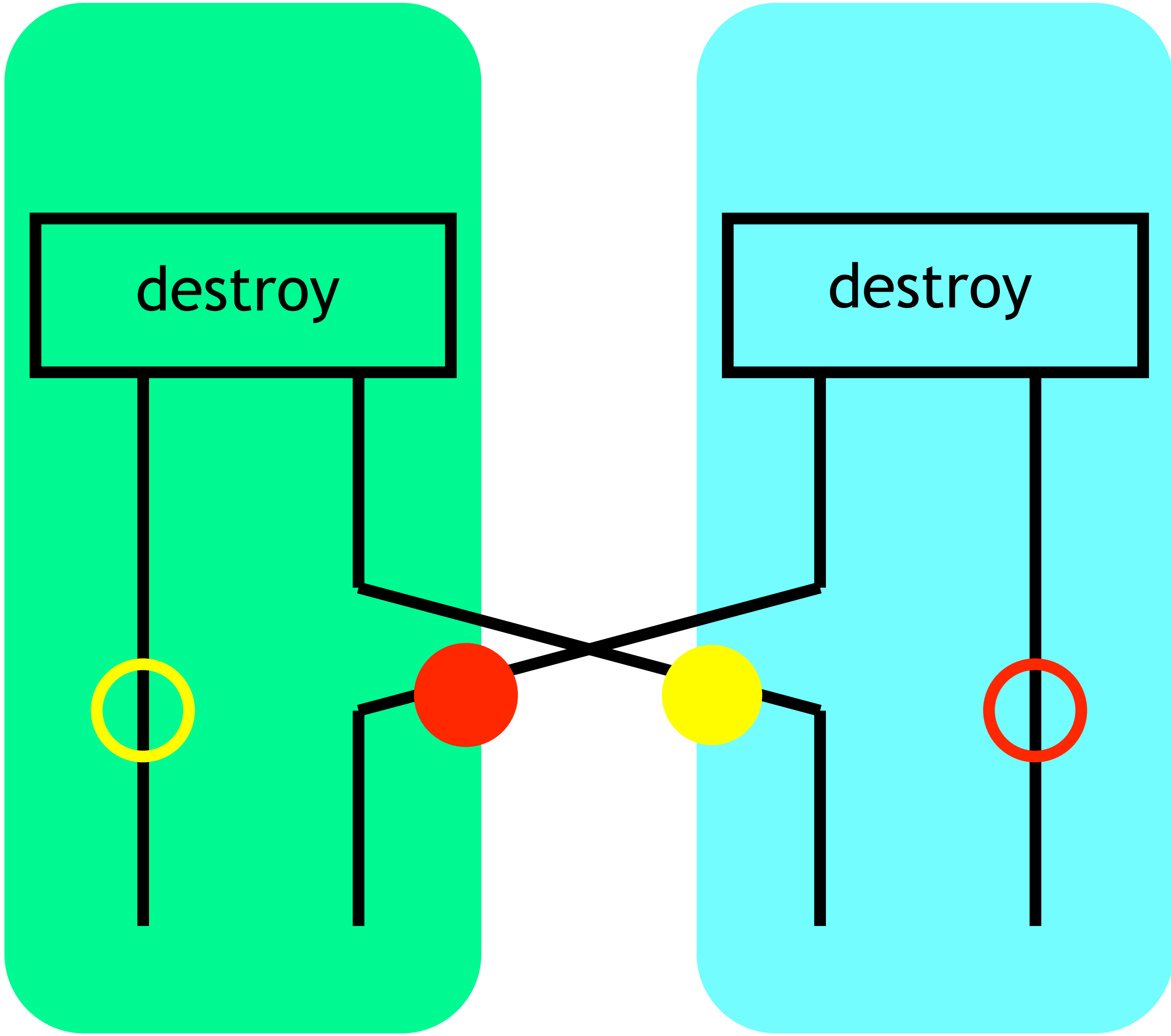

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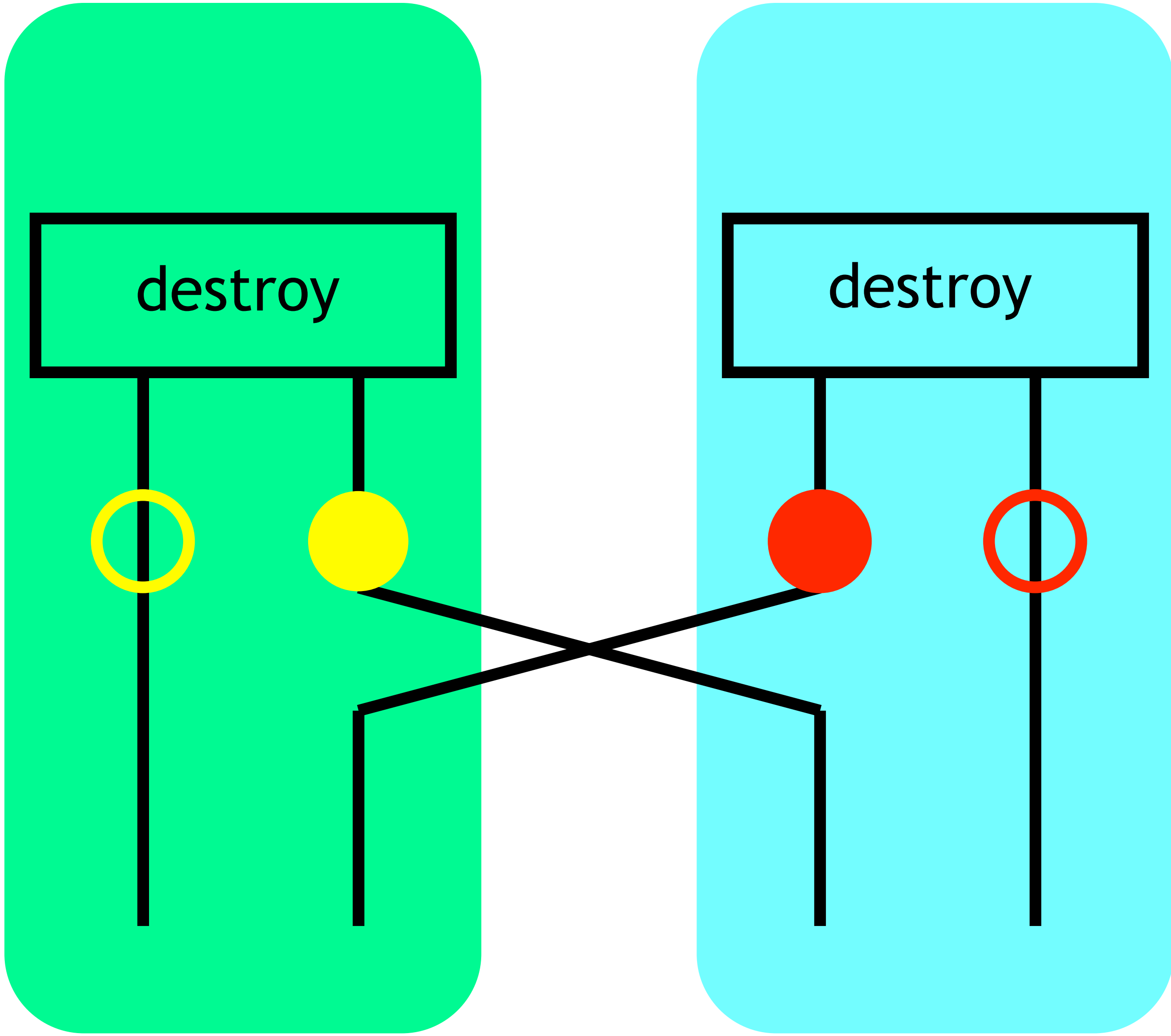

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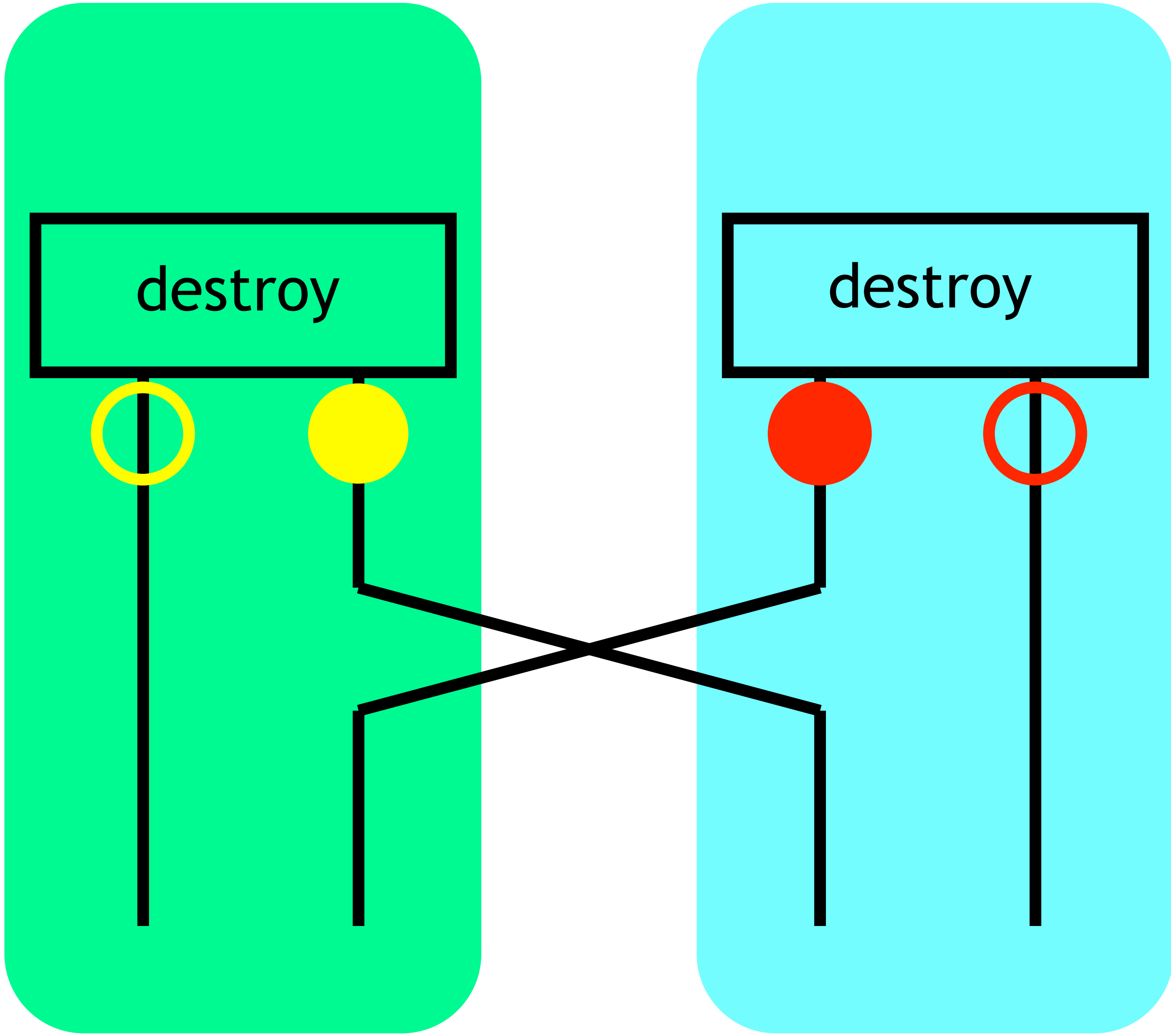

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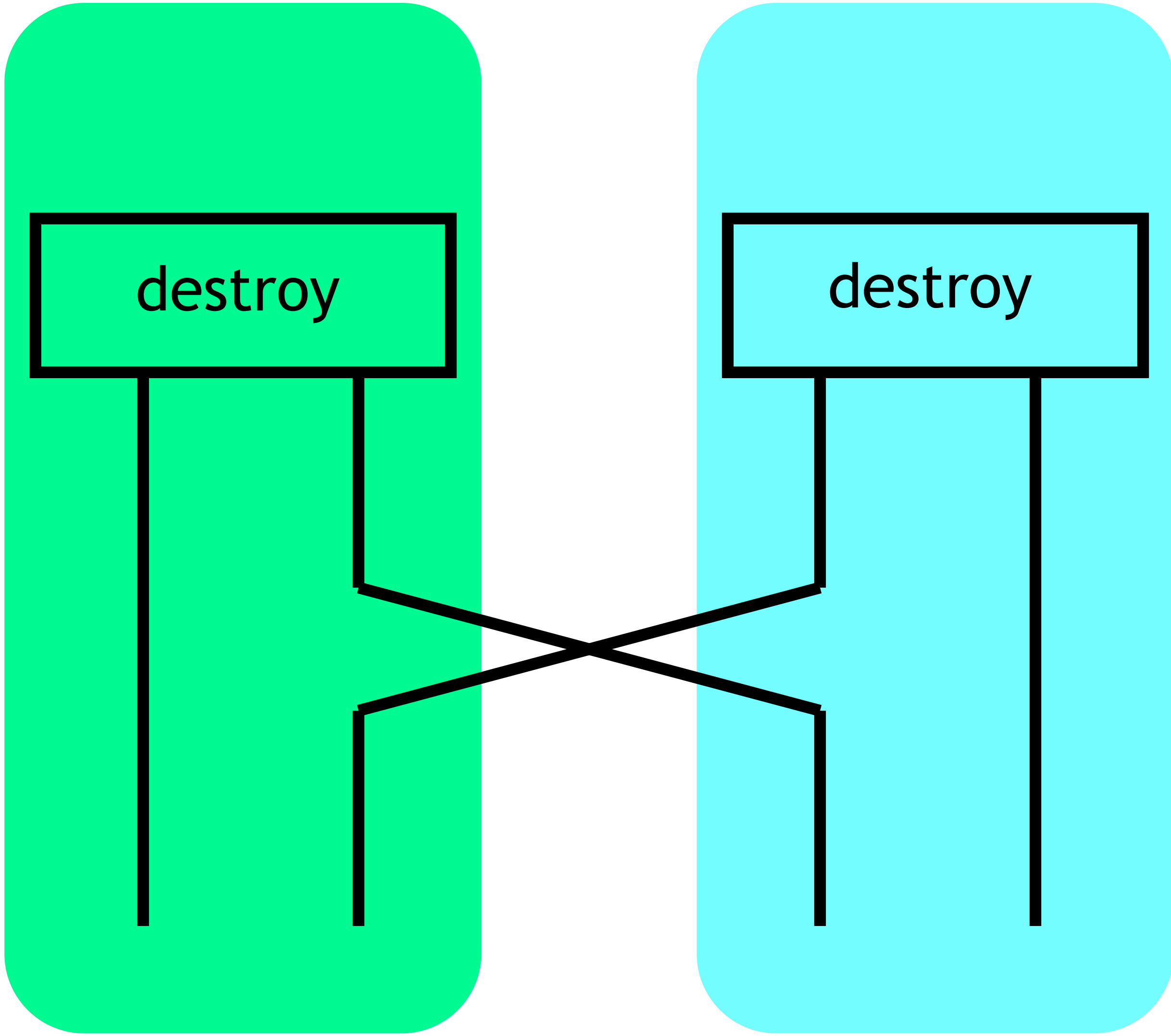
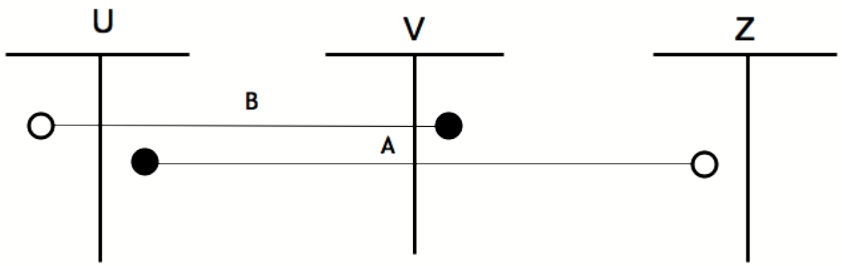


III. The algebra of commitment

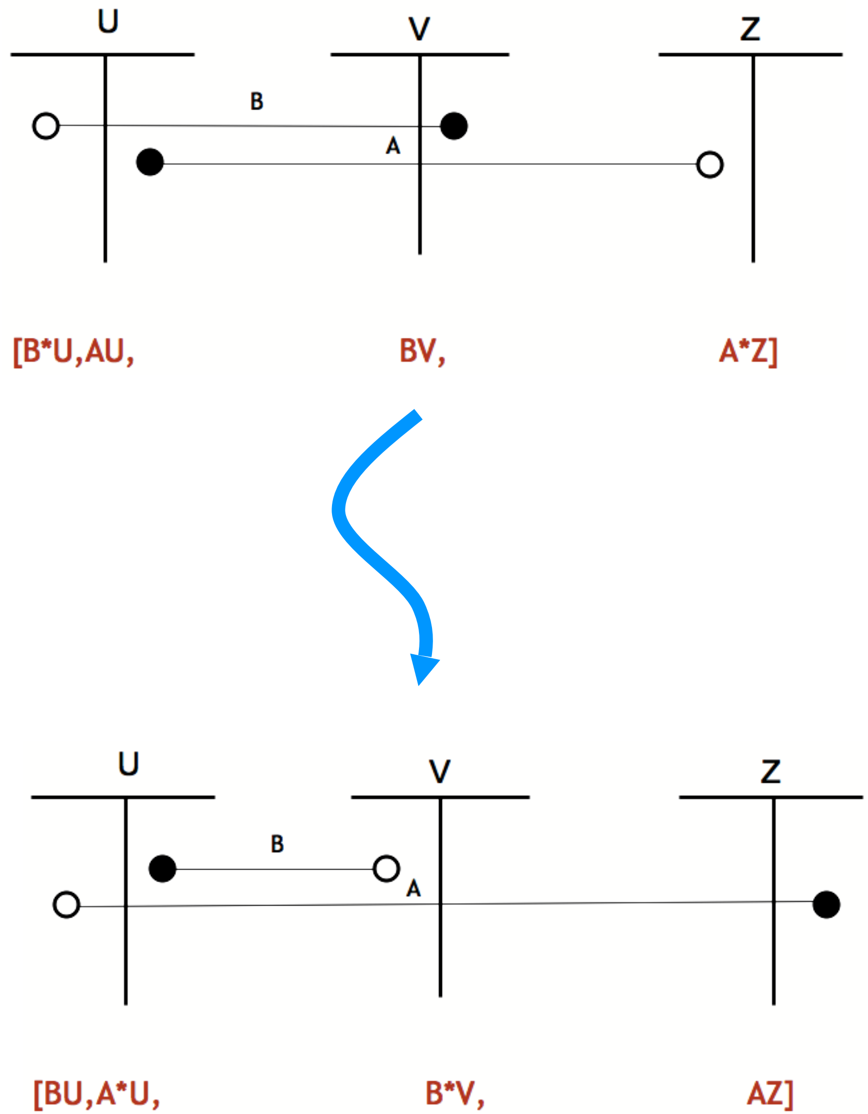
Operations on accounts

For systems of accounts (one-liners) X,Y:

- sum X + Y is concatenation
- the dual X* is defined by
  - [ ]* = [ ]
  - ([AU] + Y)* = [A*U] + Y*
  - ([A*U] + Y)* = [AU] + Y*



[B*U,AU, BV, A*Z]


U
V
Z
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A
[B*U,AU,
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A*Z]
U
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B*V,
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- the zero account 0 is [ ]

Positions

$$X = [T_1U_1, \quad \ldots \quad ,T_nU_n]$$

$$Pos(X) = \{\ 1, \quad \ldots \quad , n\ \}$$

Matchings

A matching of X with Y is a bijection

$$f : Pos(X) \rightarrow Pos(Y)$$

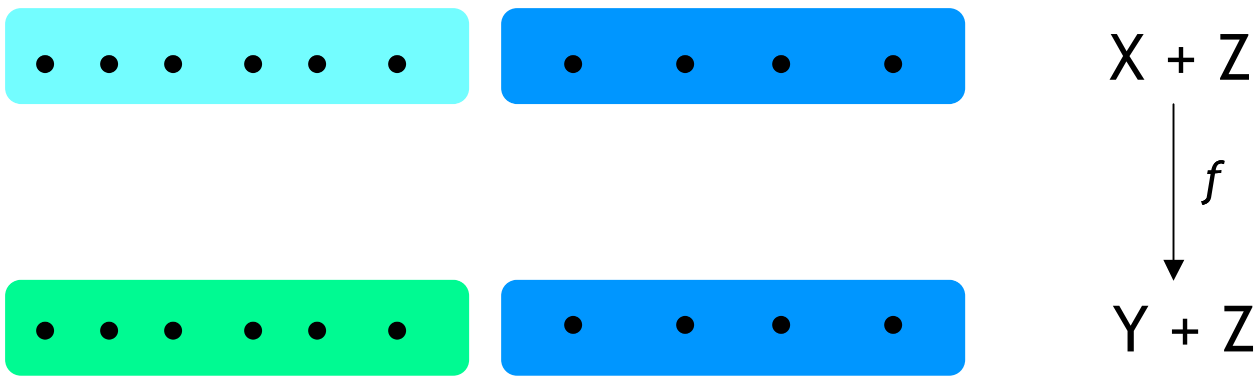
that preserves types (but not necessarily places)

A category of matchings, $\mathcal{M}$

objects: strings $[A_1U_1,\ldots,A_nU_n]$

morphisms $X \rightarrow Y$: matchings of X with Y

Fact: $\mathcal{M}$ is traced



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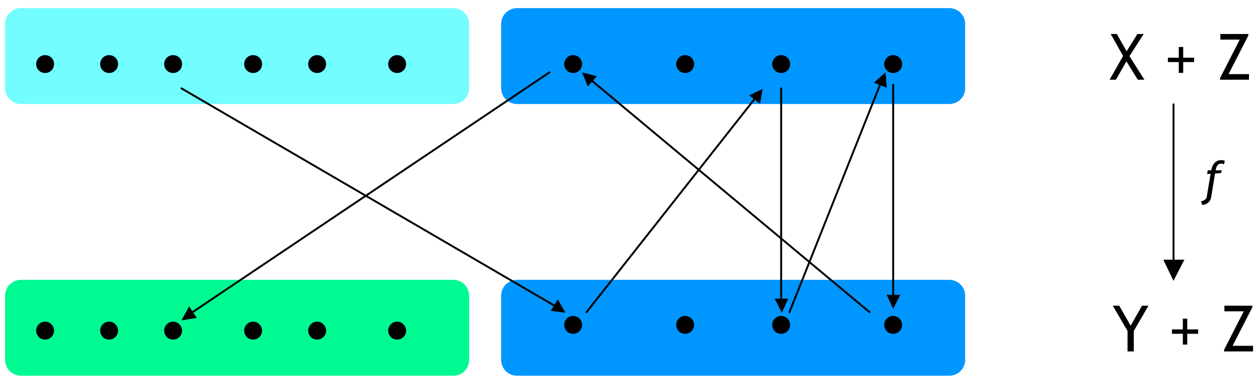

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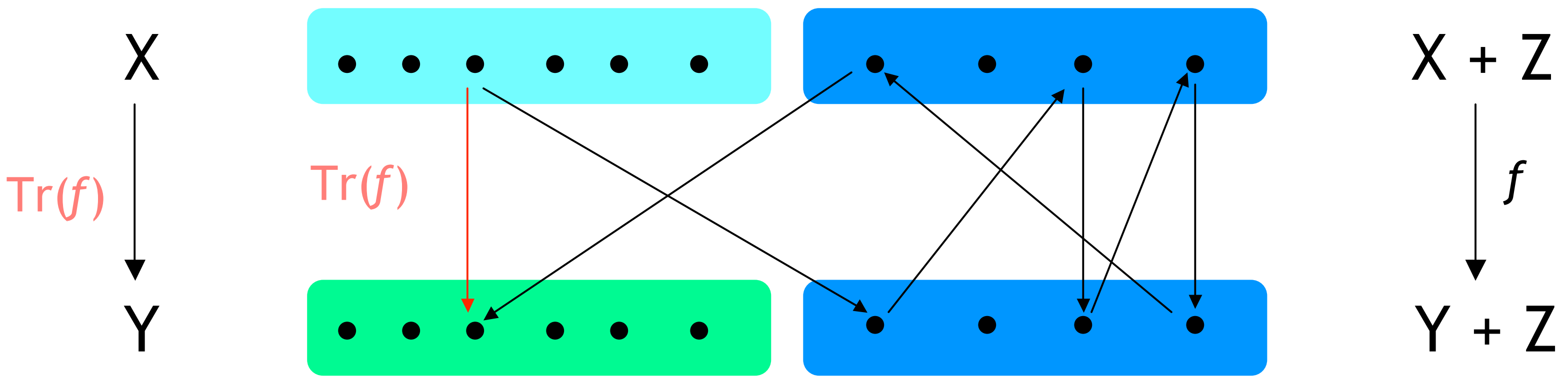

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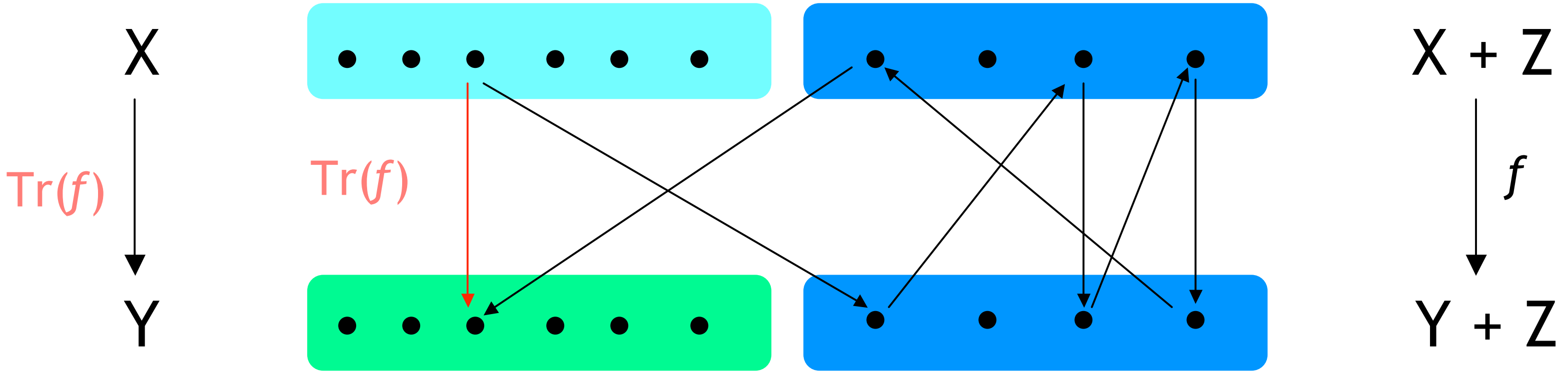

(Garsia-Milne involution principle)

Accounting and the geometry of interaction

X 0-account if there is a matching of X^+ with X^-

If $X + Y^*$ is a 0-account, a morphism $X \to Y$ in the category $\mathcal{A}cc$ is a matching of $X^+ + Y^-$ with $X^- + Y^+$

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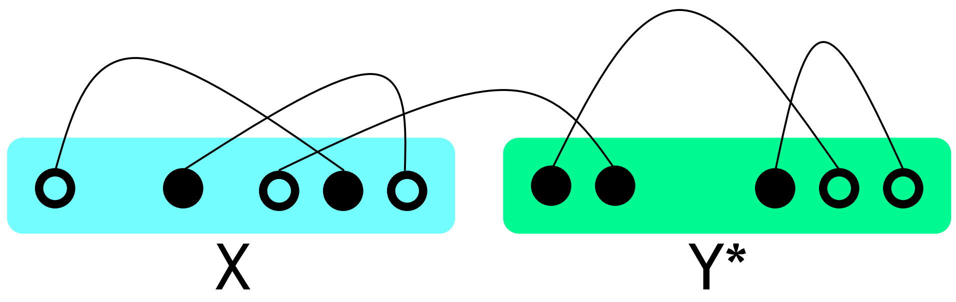

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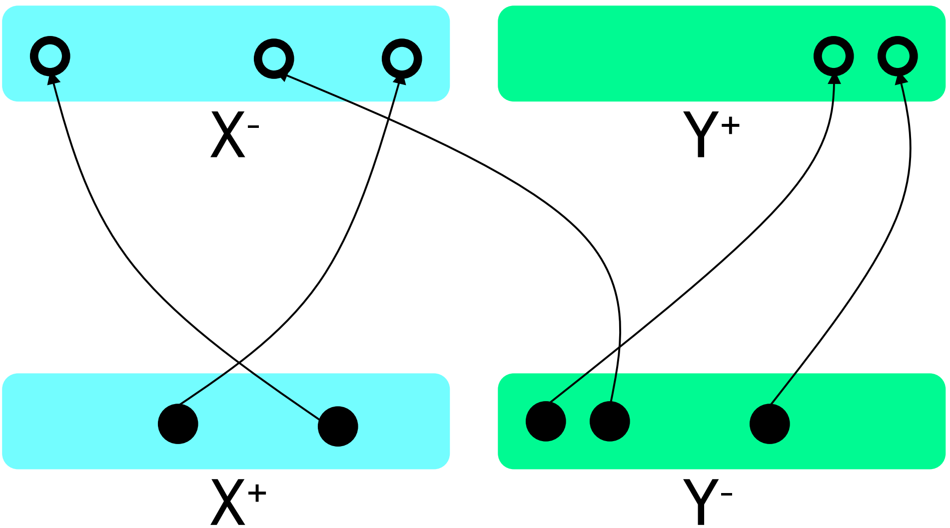
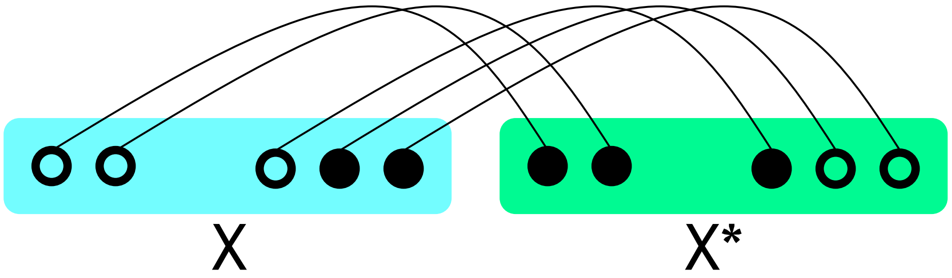


Examples

(i)

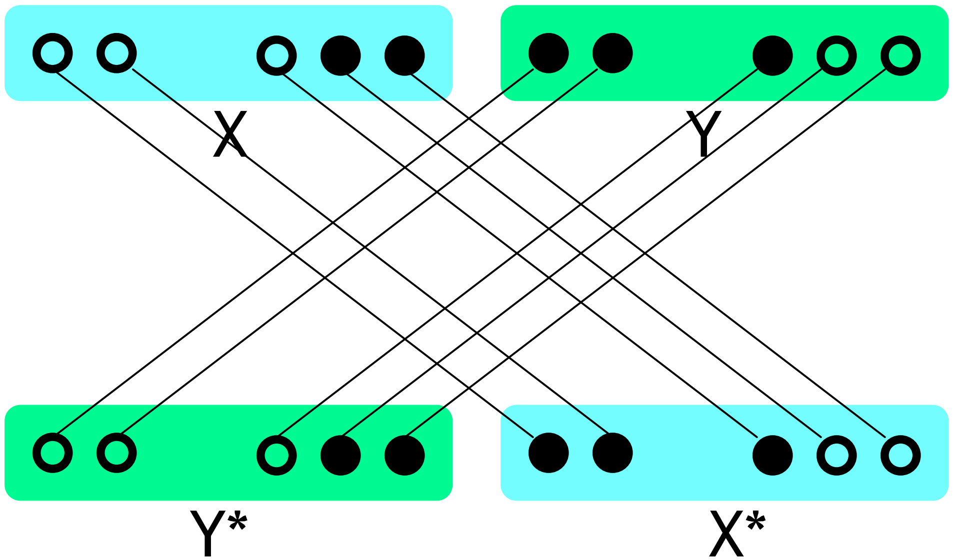


$= \varepsilon_X : X^* + X \rightarrow 0$

$= \eta_X : 0 \rightarrow X + X^*$

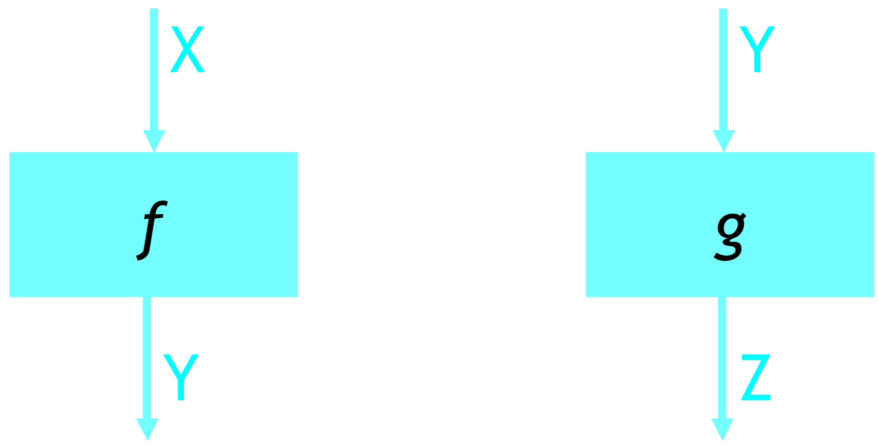
$= 1_X : X \rightarrow X$

(ii)

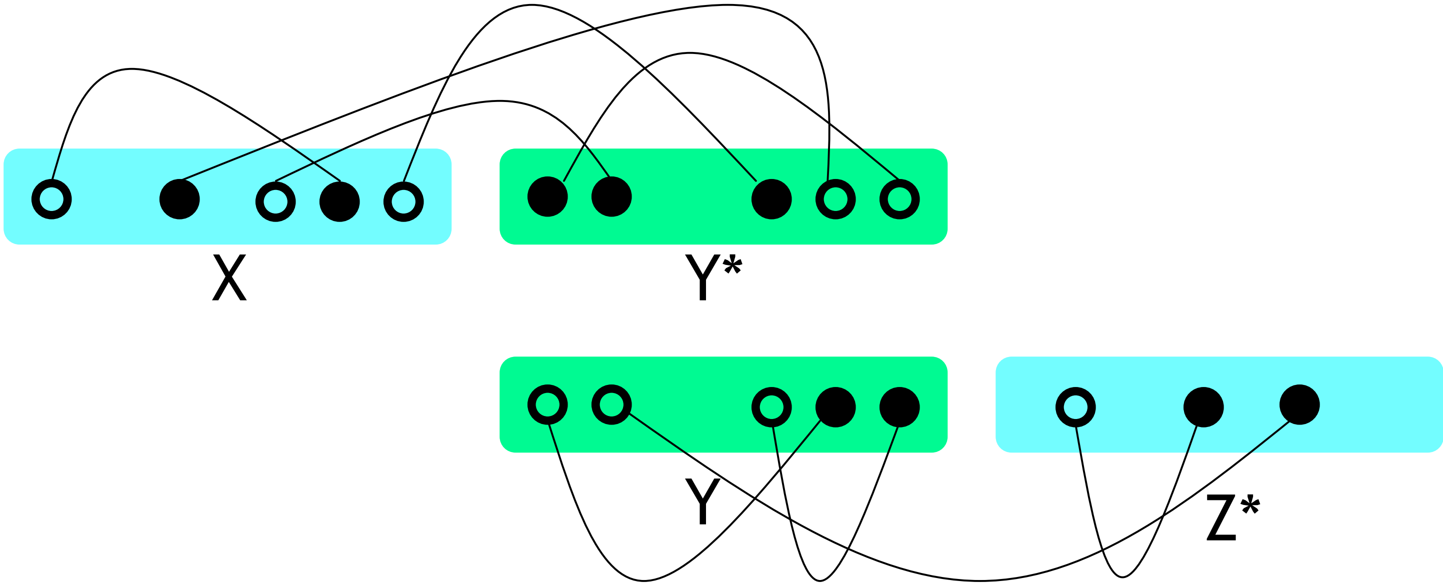


$= \sigma_{XY} : X + Y \rightarrow Y + X$

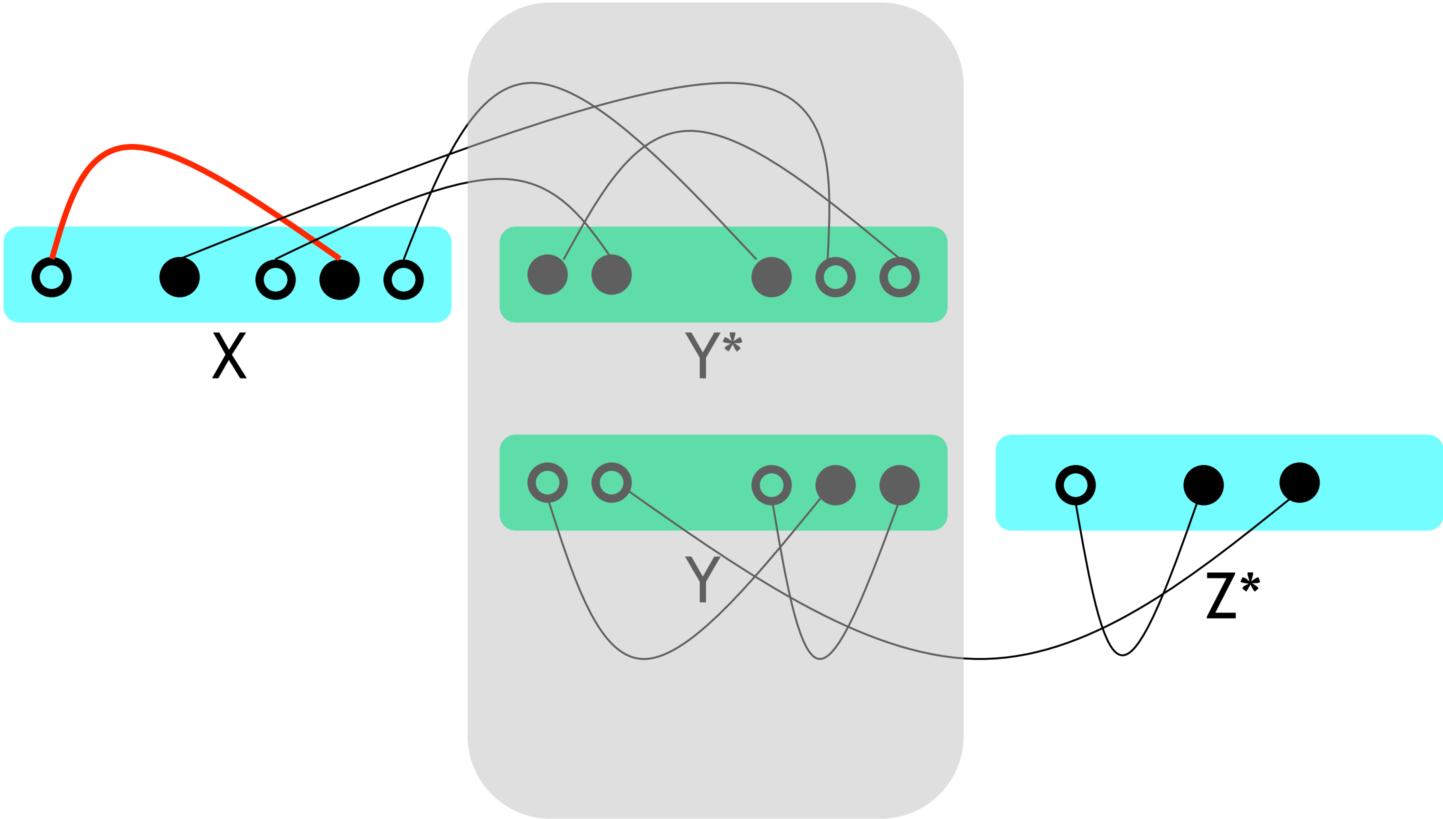
Composition



Example



Some paths



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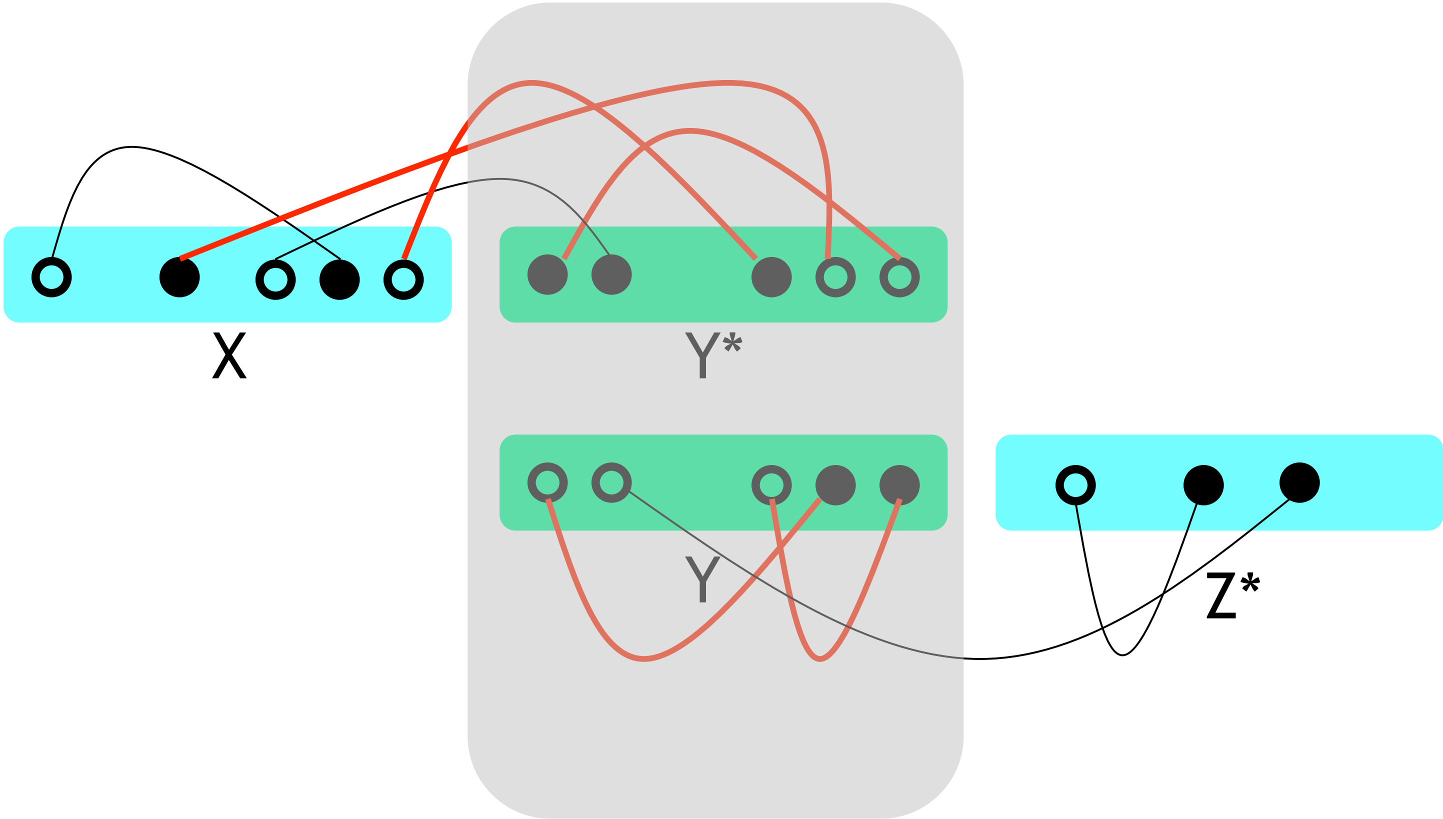

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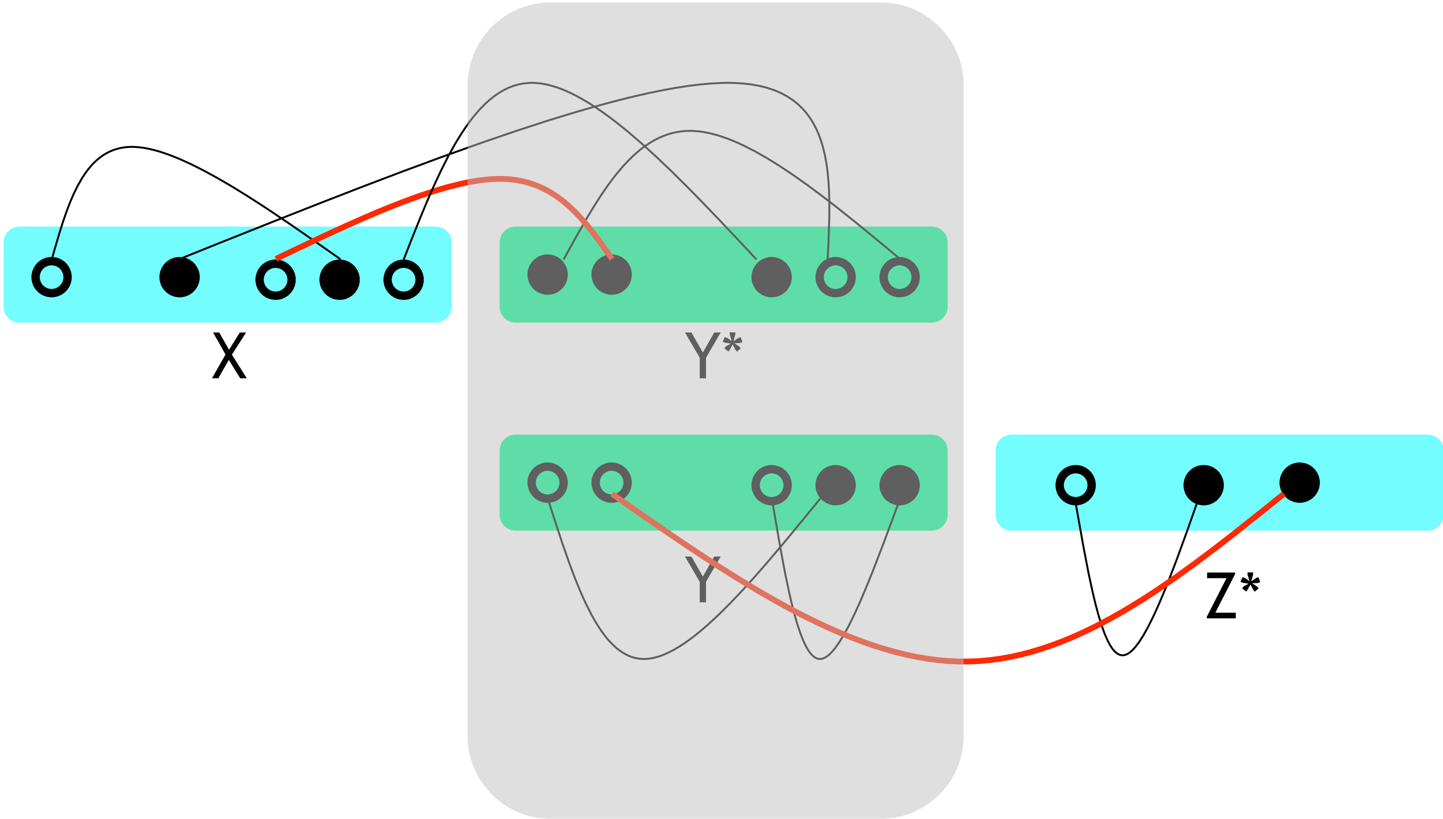
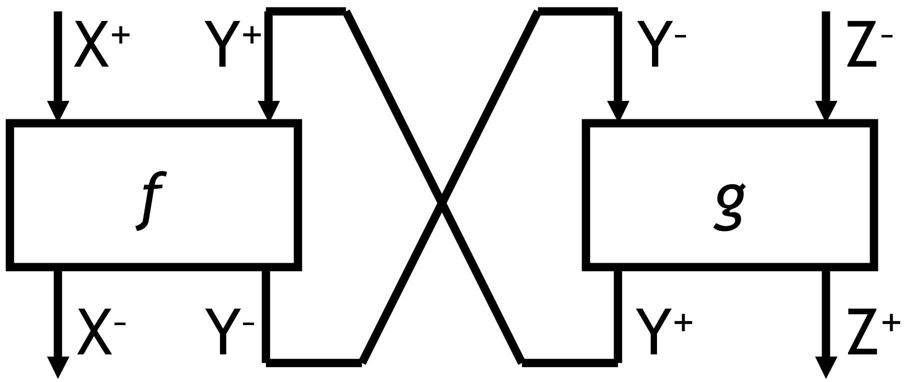


Equivalently: composition via symmetric feedback



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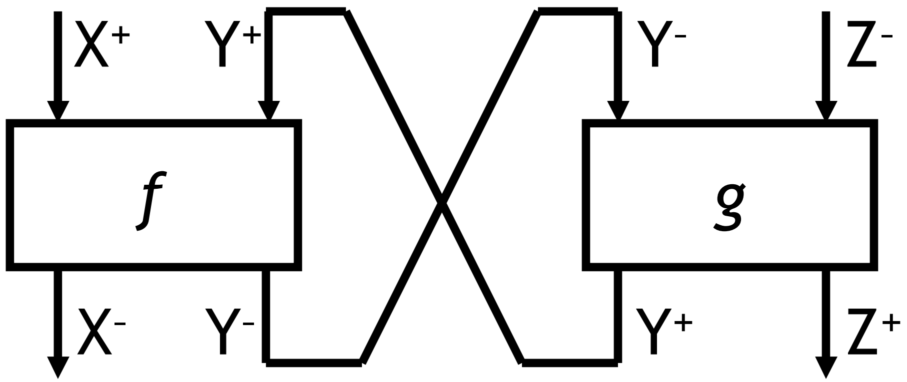


geometry of accounting = geometry of interaction

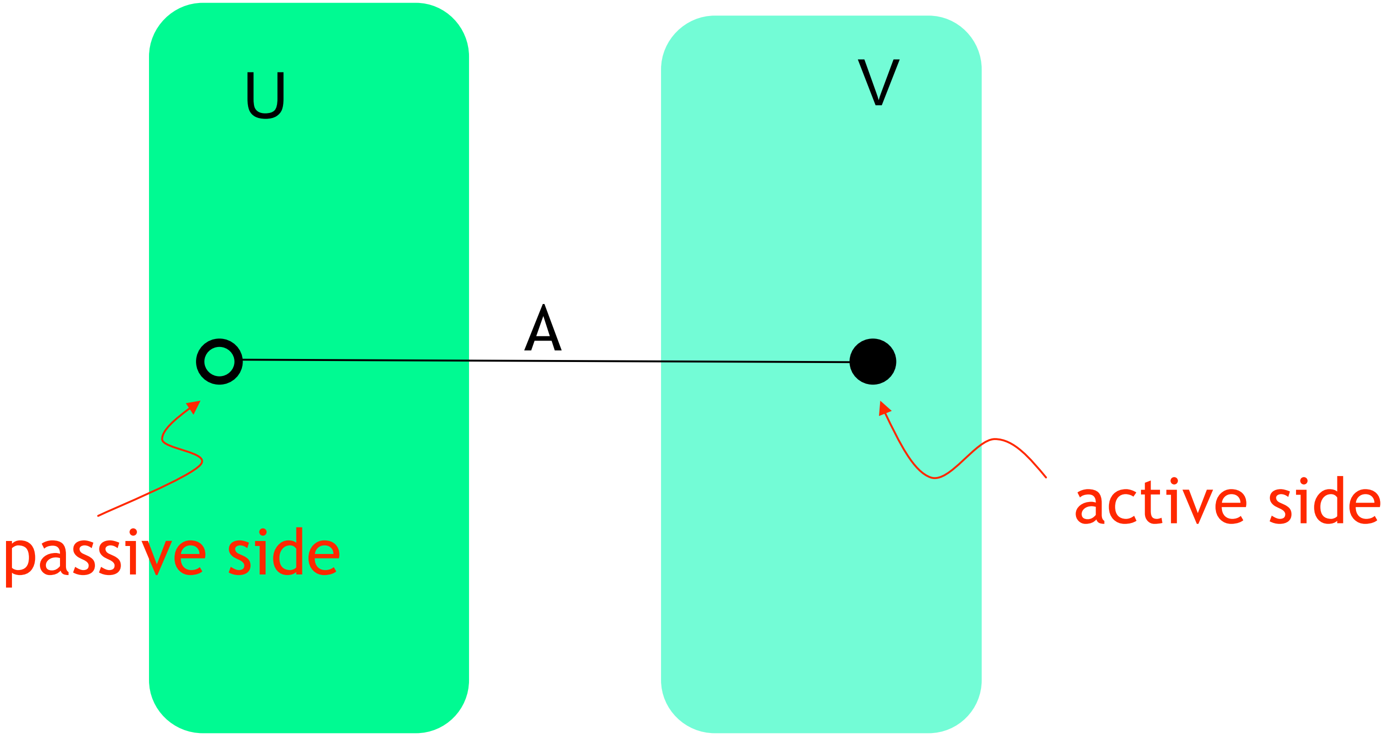
IV. Towards a logic of commitment

Whither now?

Whither now? Back to dialogues!

Whither now? Back to dialogues!

Look at the basic form of a commitment as a contract



Whither now? Back to dialogues!

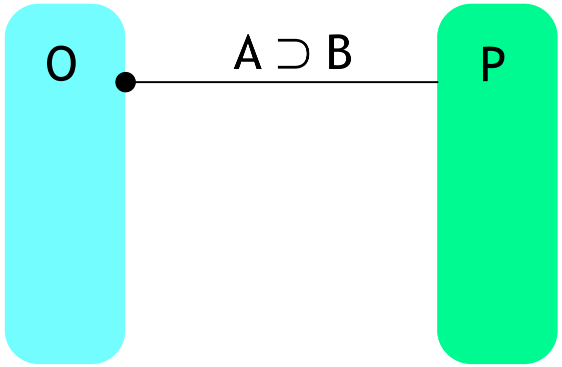
Think of $A \supset B$ as a contract between a Proponent (passive) and an Opponent (active).

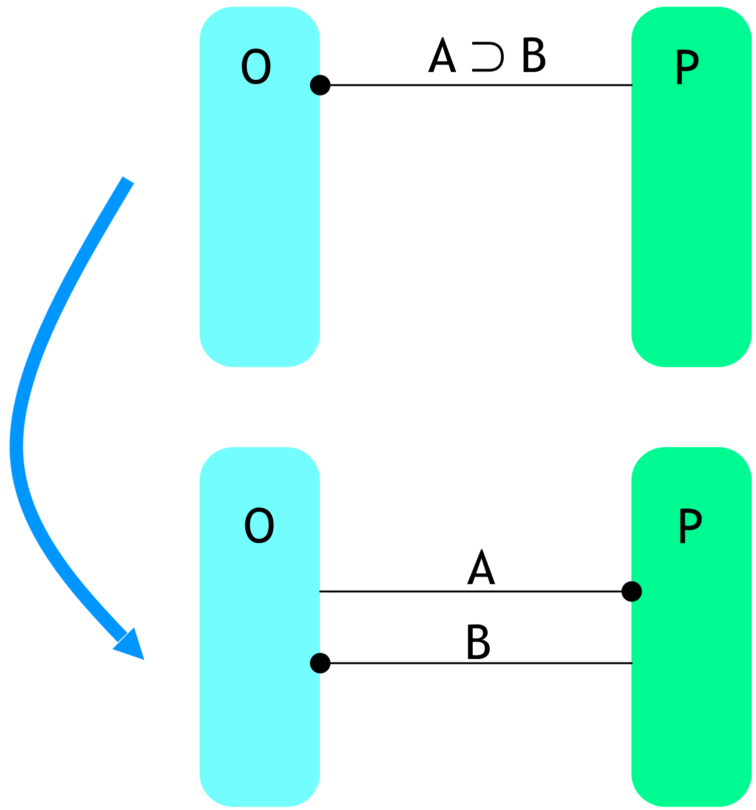
The execution of this contract is started by the active party, replacing

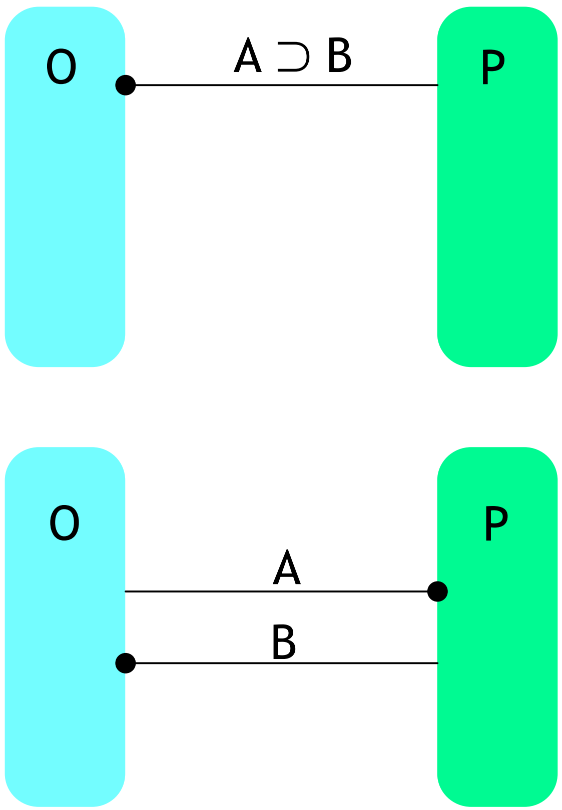
$$A \supset B$$

by a couple of contracts:

A where P is active, and
B where O is active

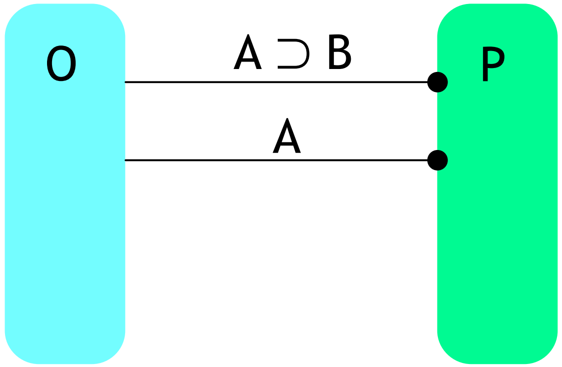

O
A ⊃ B
P


O
A ⊃ B
P
O
P
A
B



$A \supset B$ valid if — after performance — P is a 0-account

Validity of *modus ponens*



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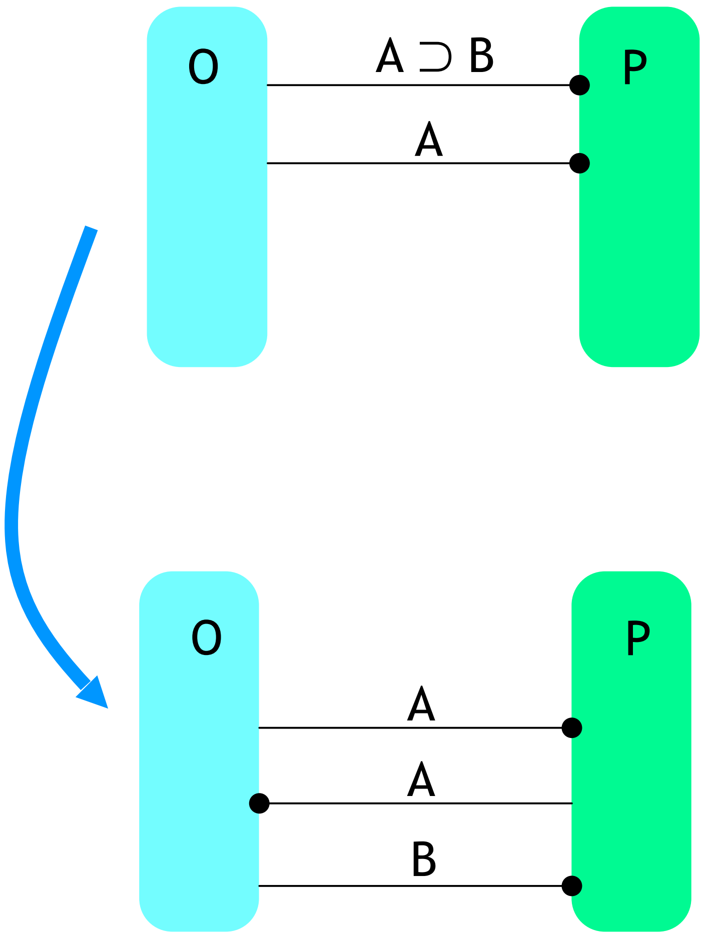

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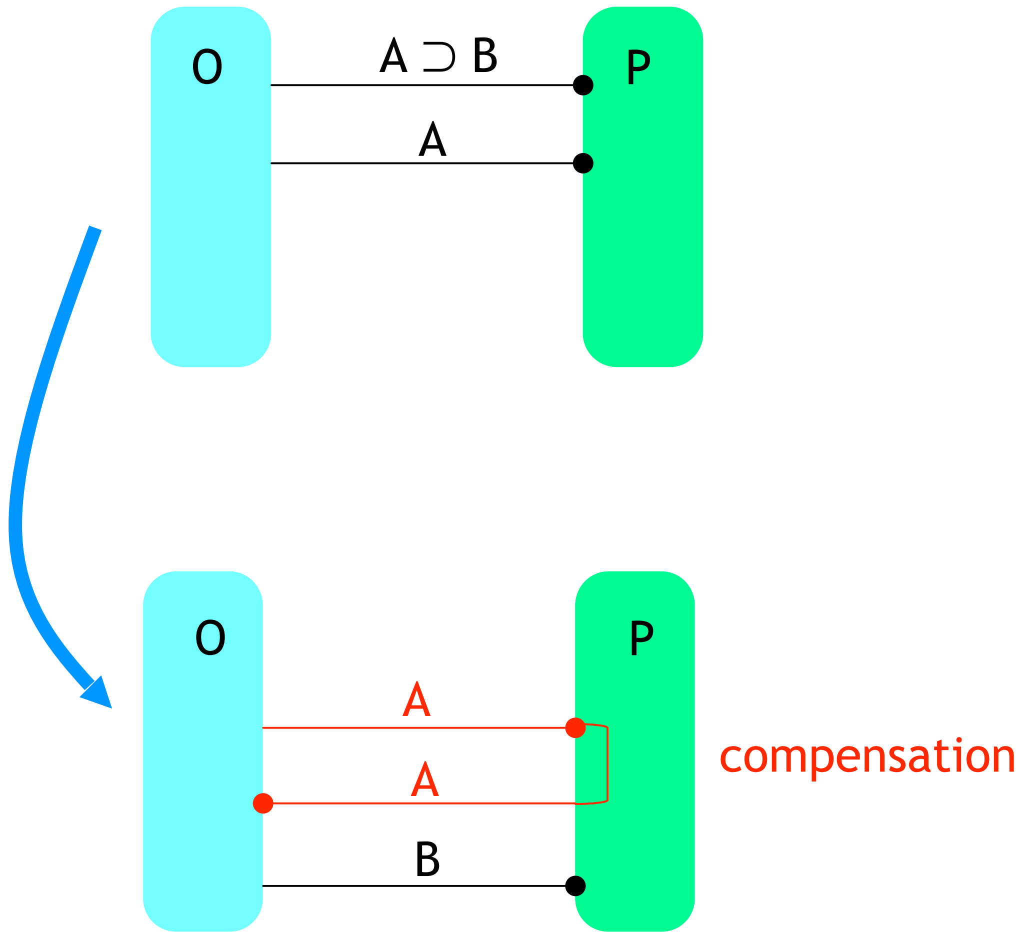

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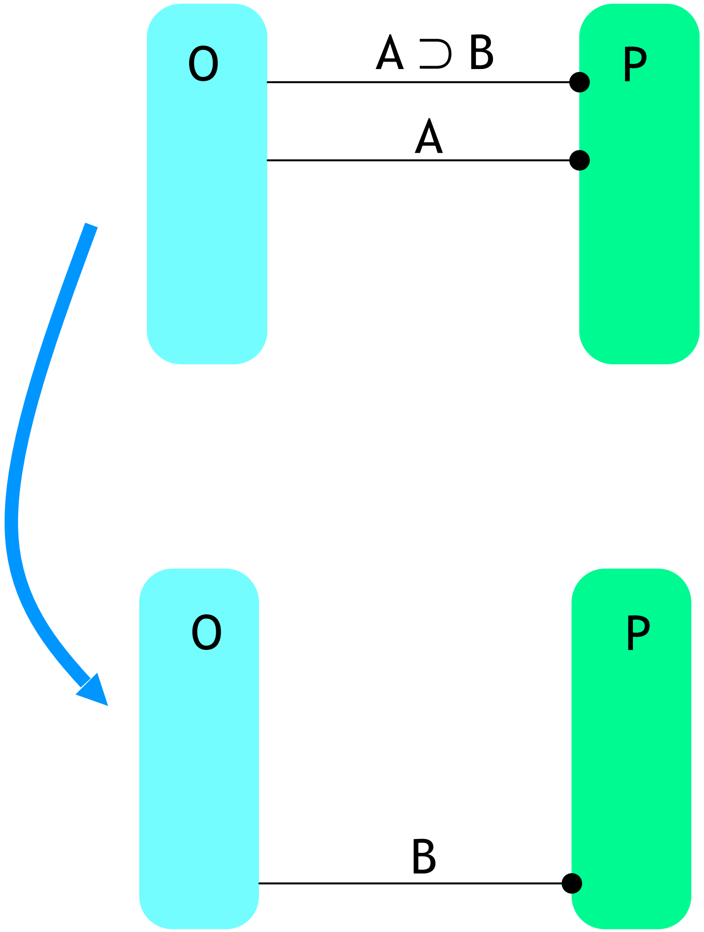

Example

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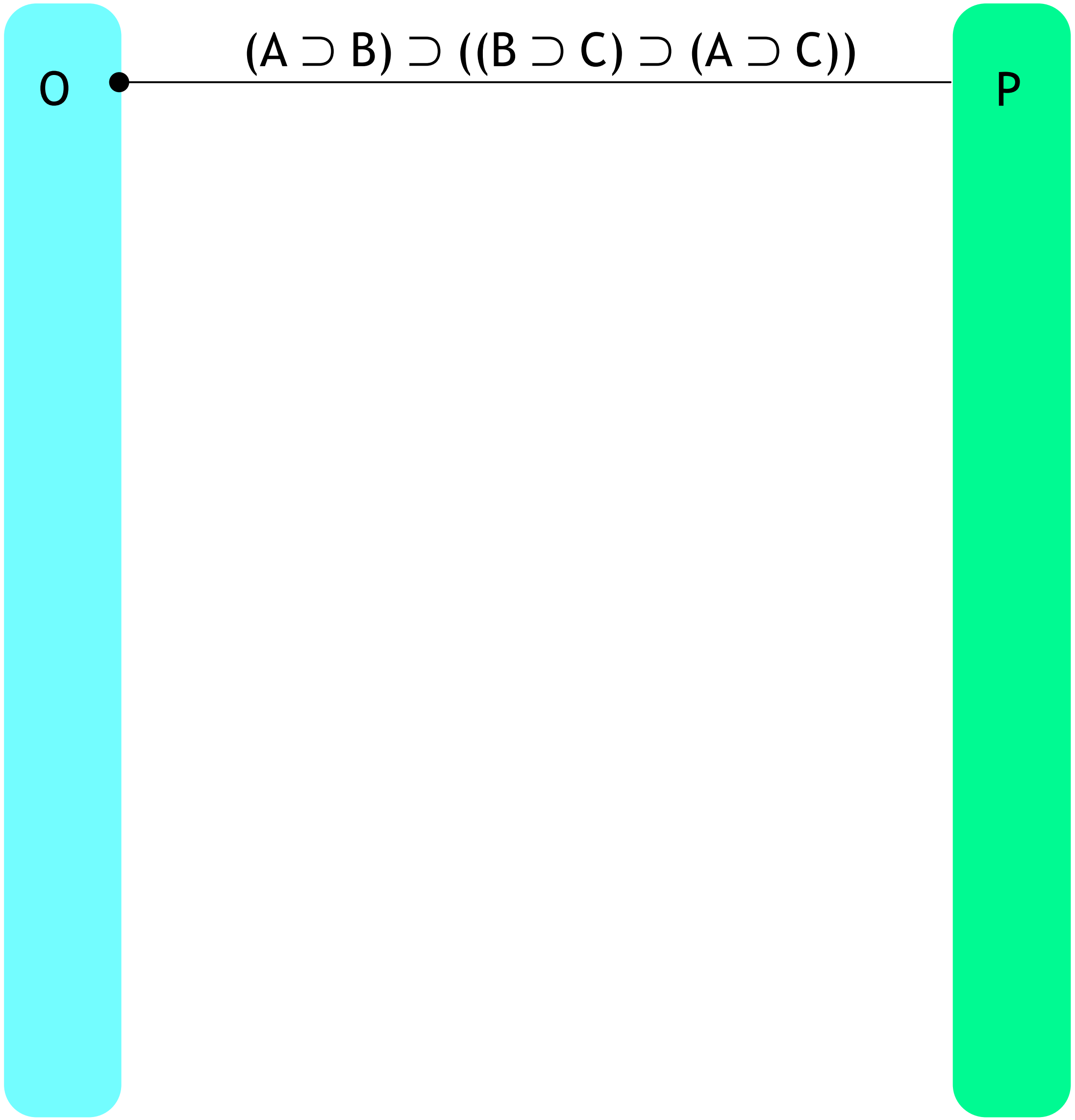
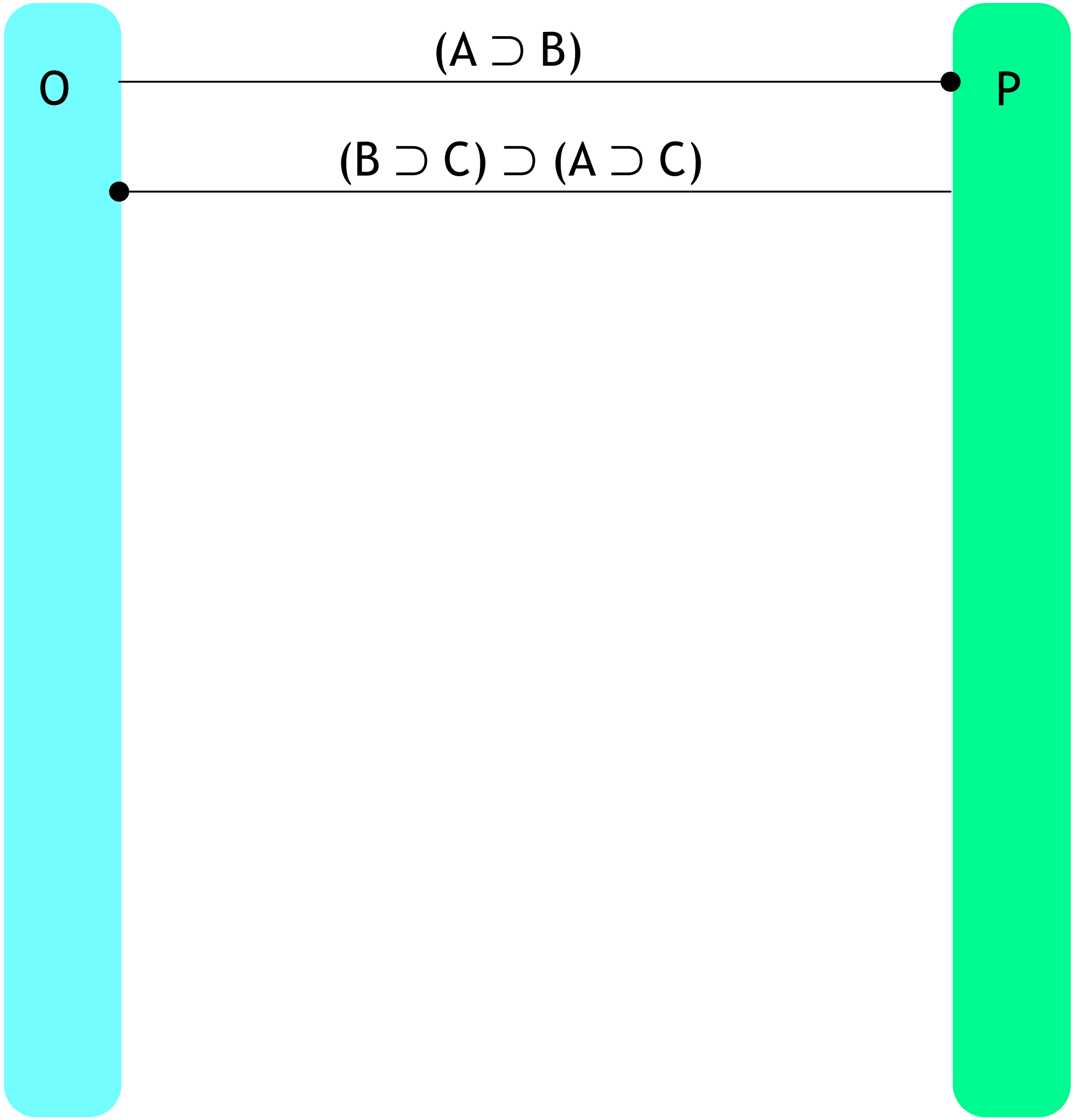
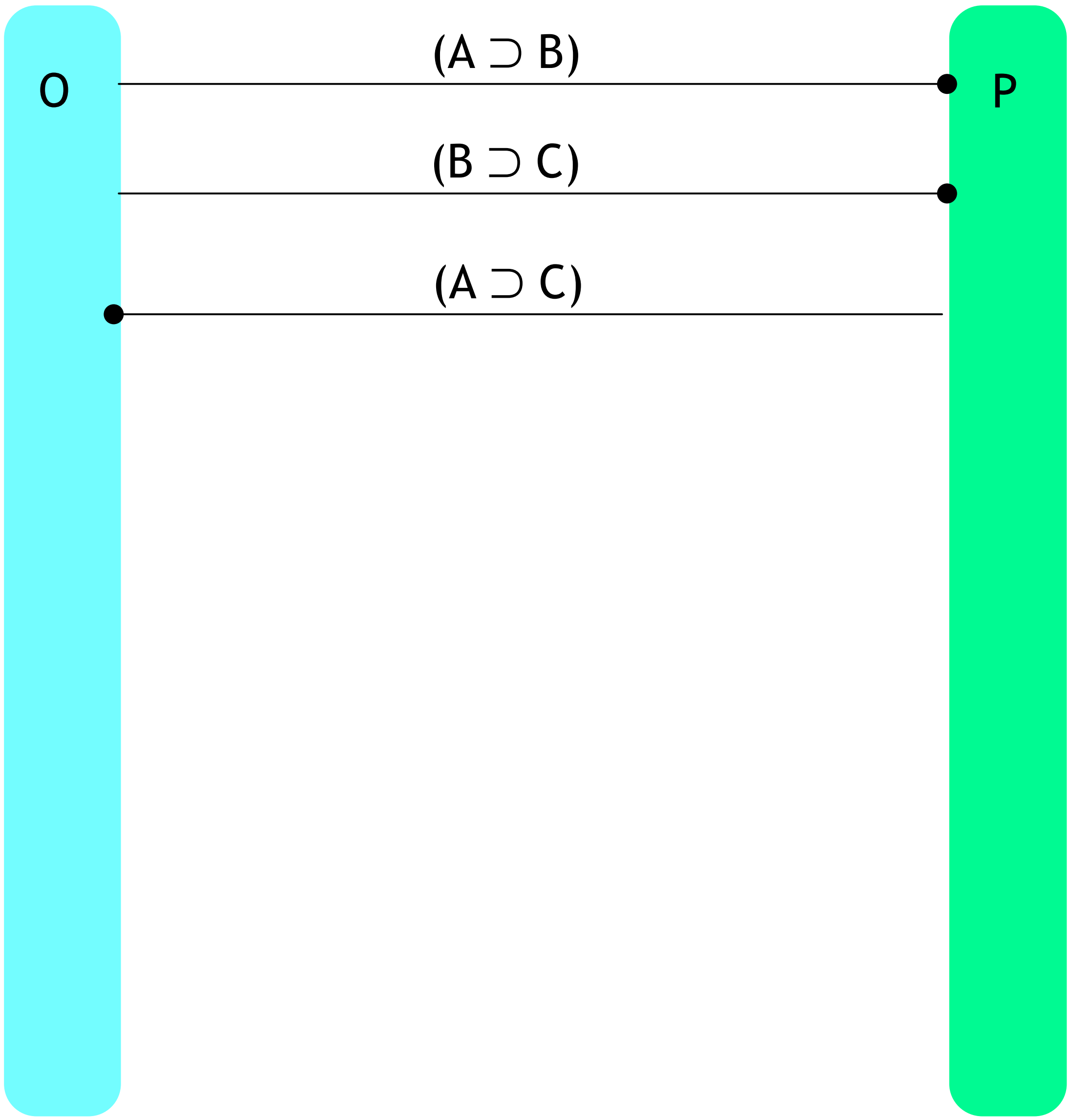



O
P
(A ⊃ B)
(B ⊃ C) ⊃ (A ⊃ C)


O
P
(A ⊃ B)
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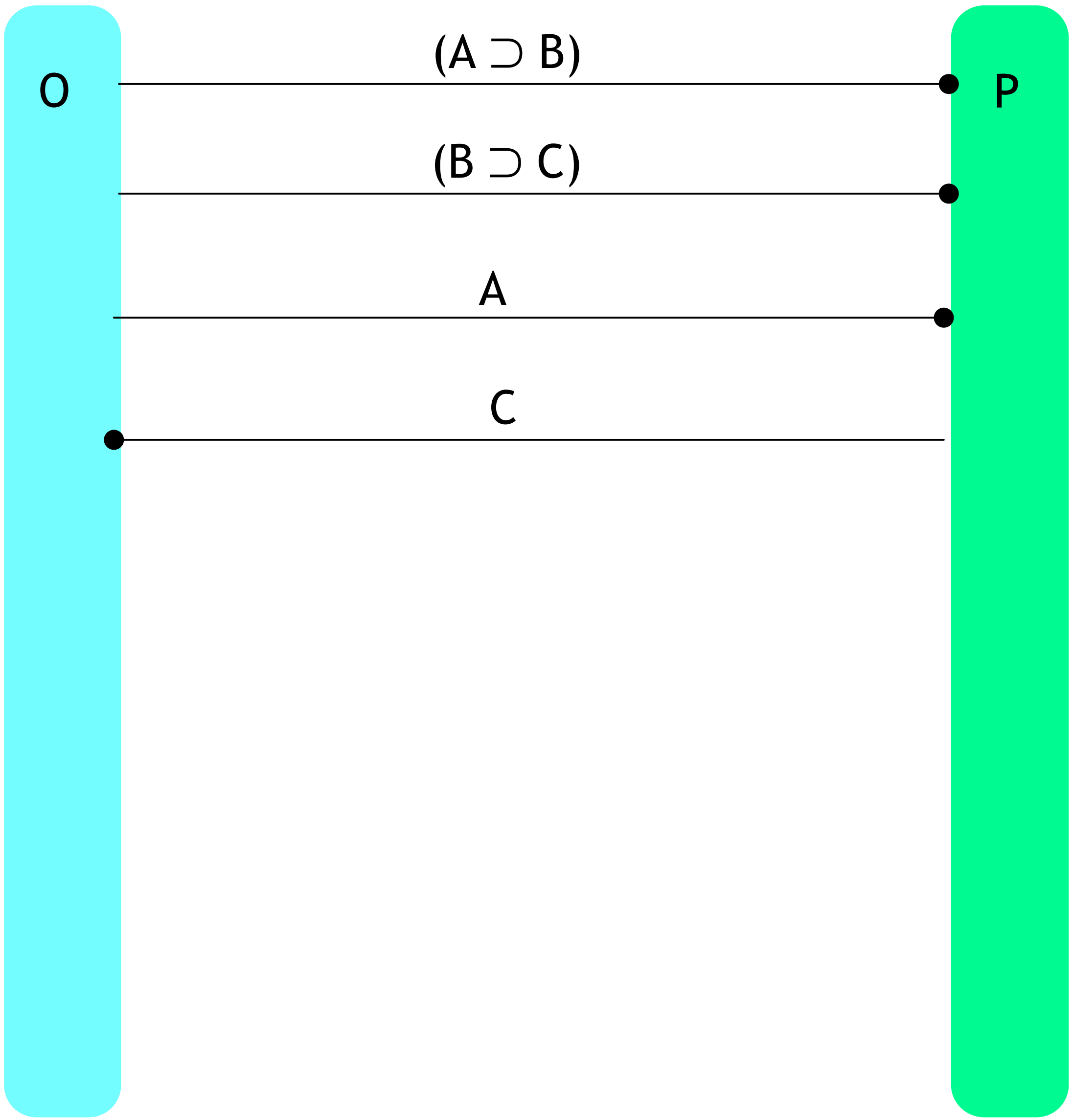
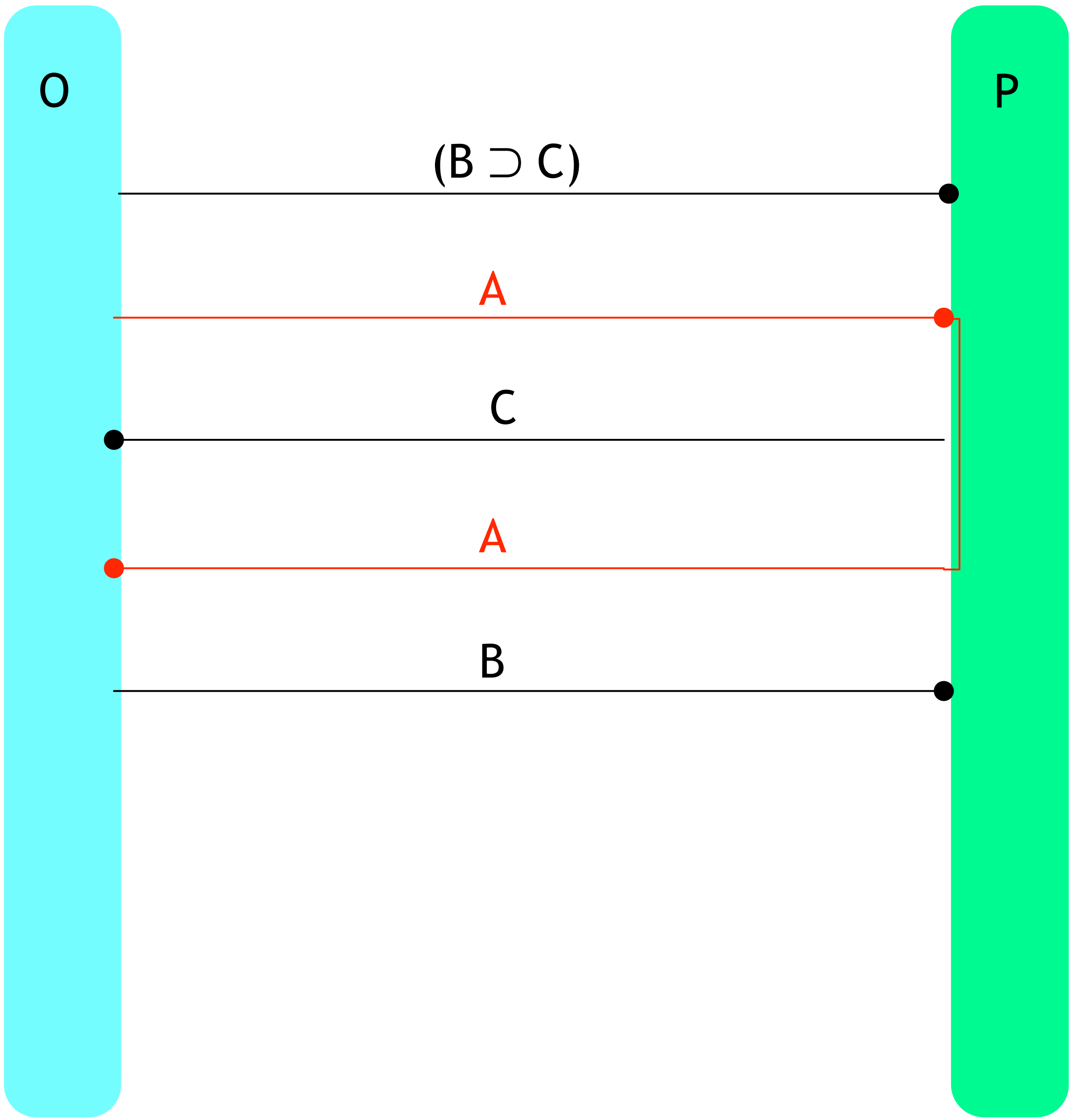
(A ⊃ C)


O
P
(A ⊃ B)
(B ⊃ C)
A
C


O
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A
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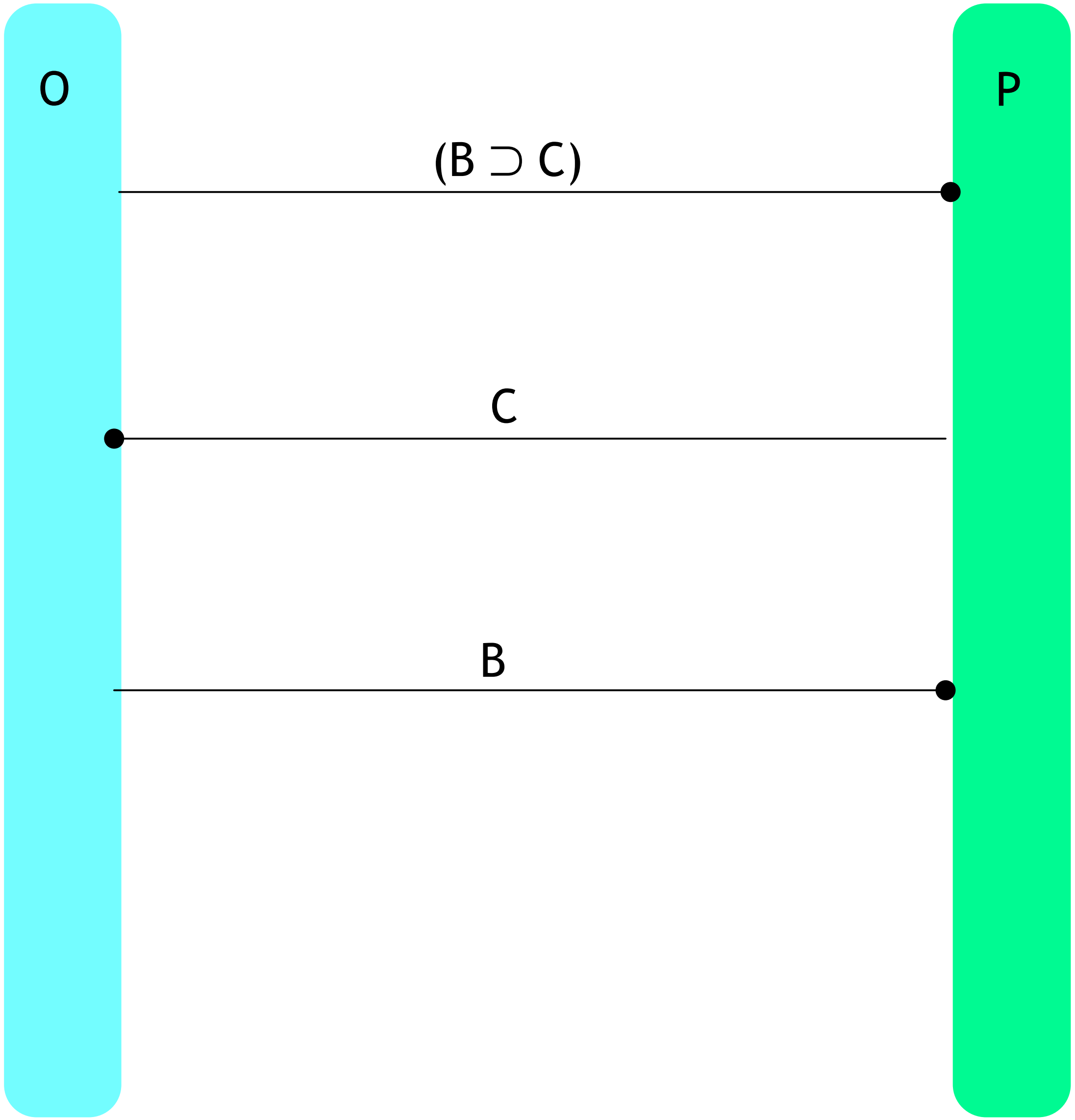
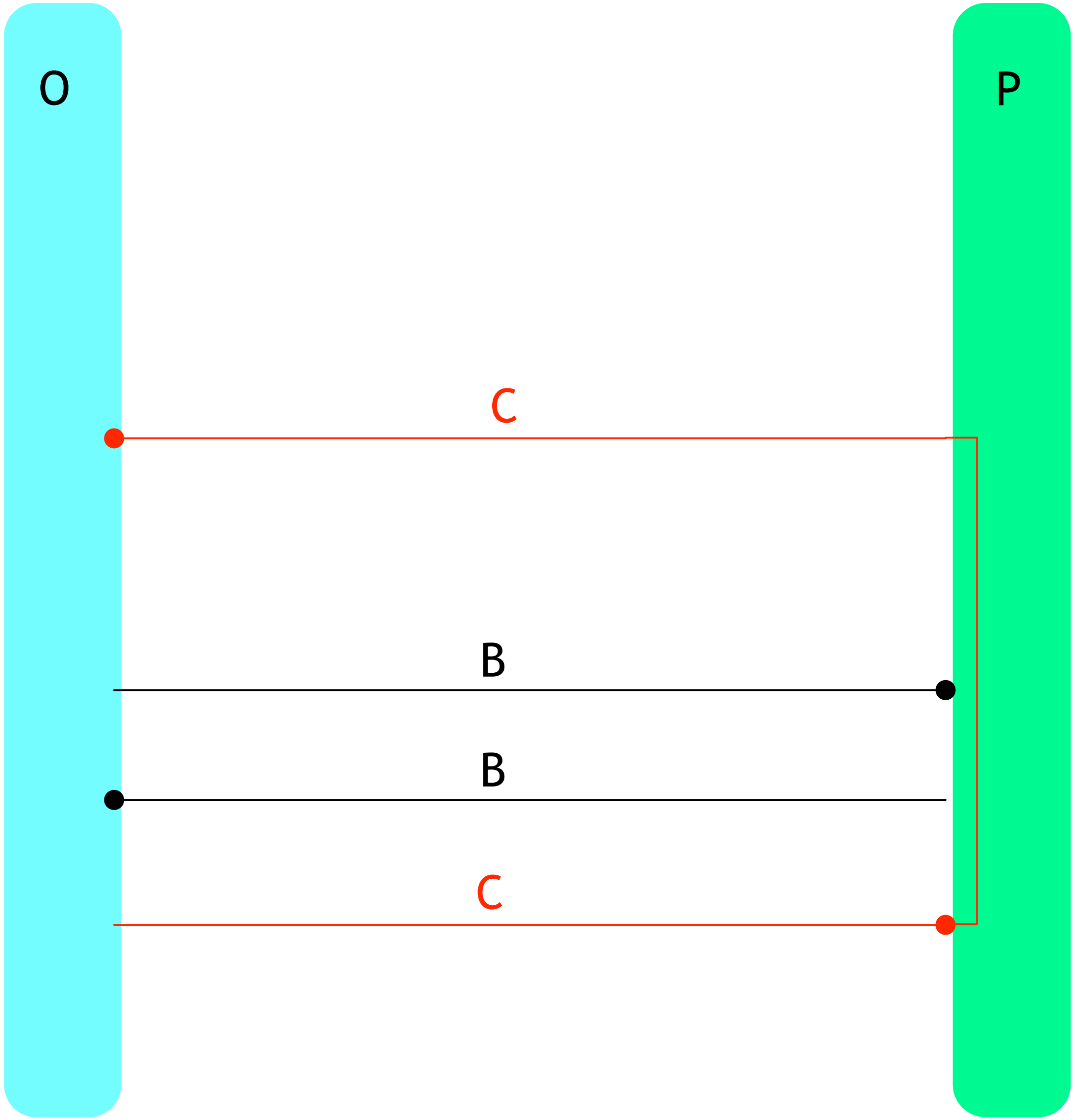
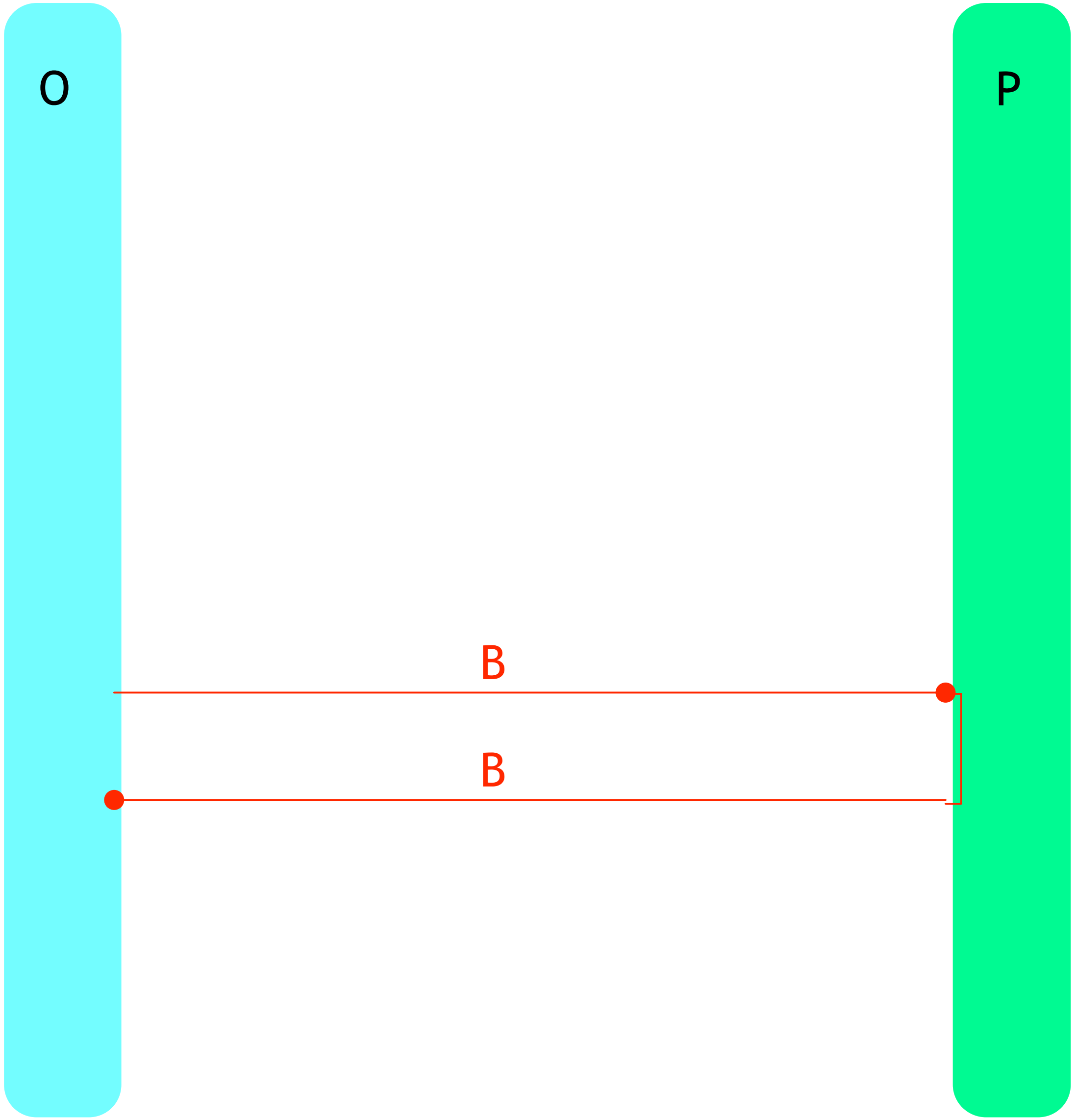
A
B


O
P
(B ⊃ C)
C
B


O
P
C
B
B
C

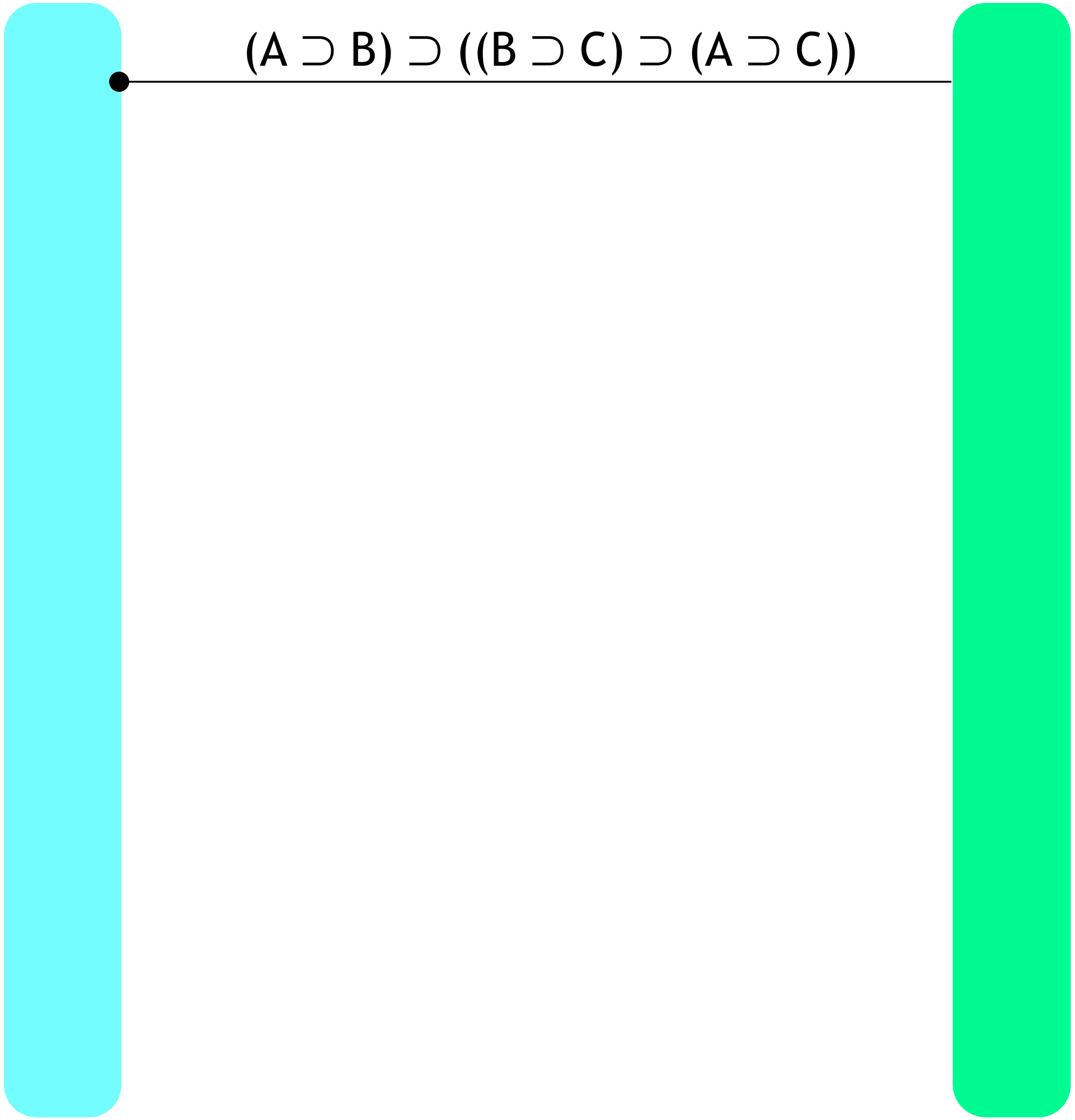

O
P
B
B

O

P

Decorating with λ-terms

Decorating with λ-terms



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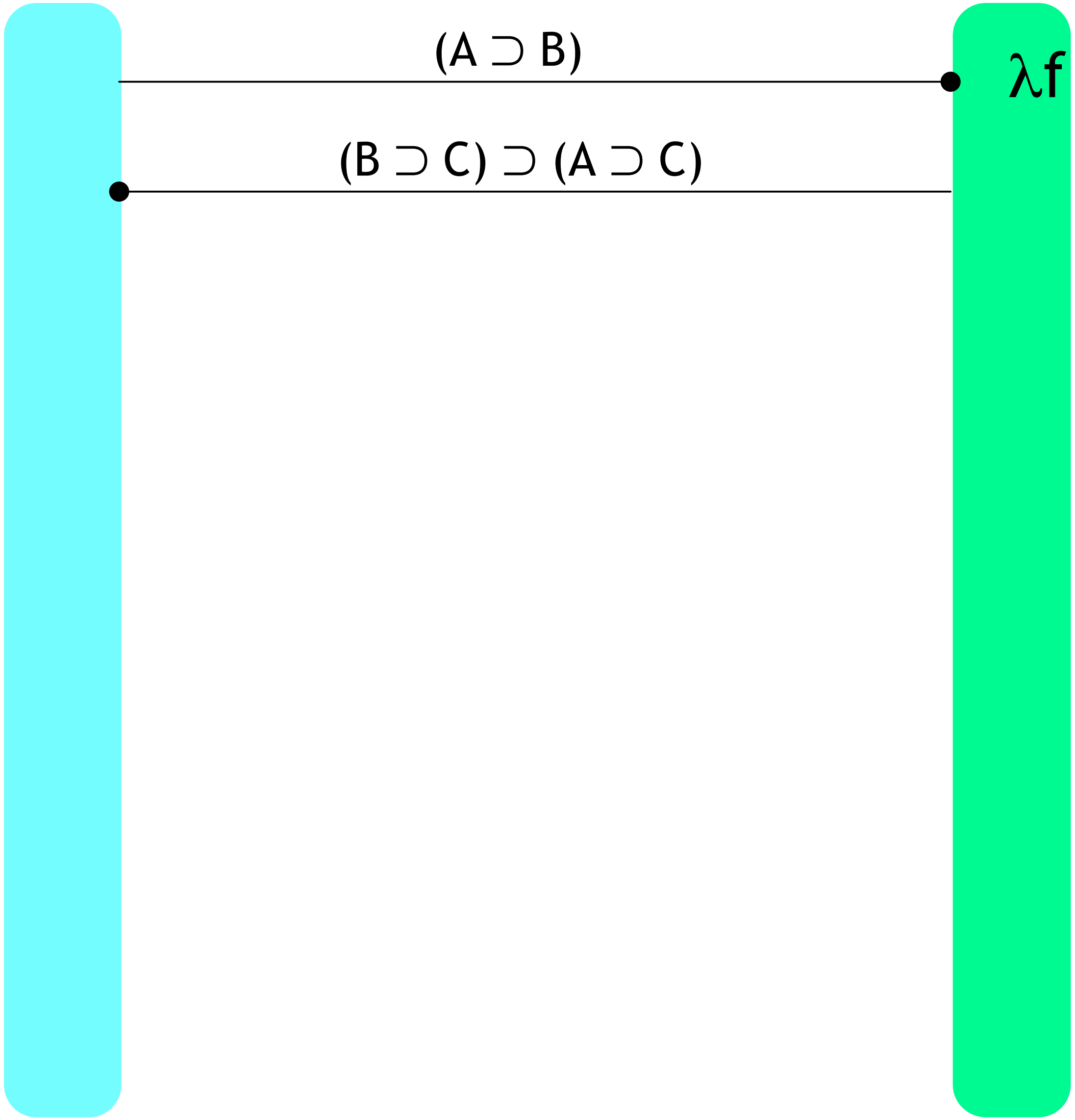

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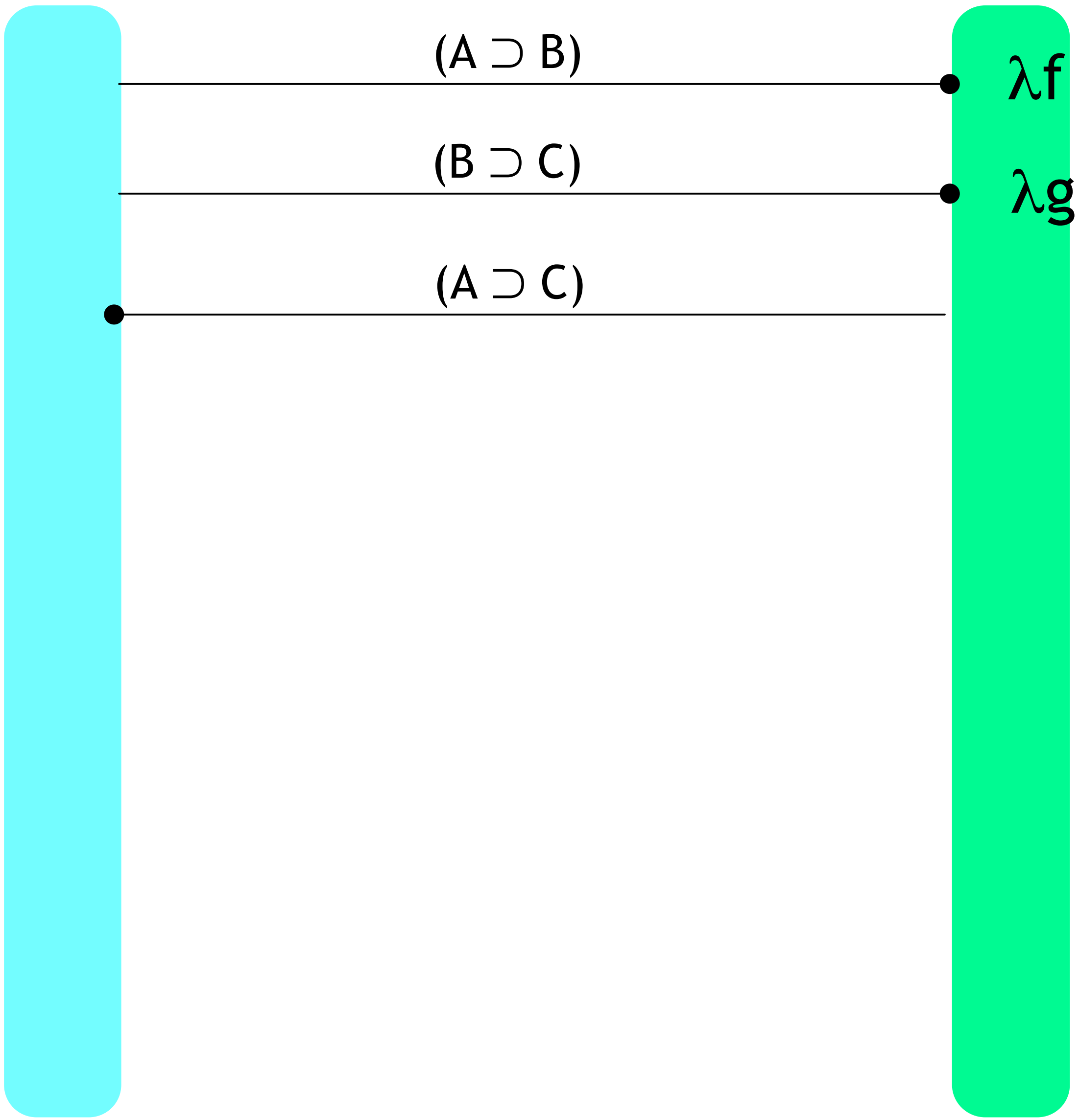
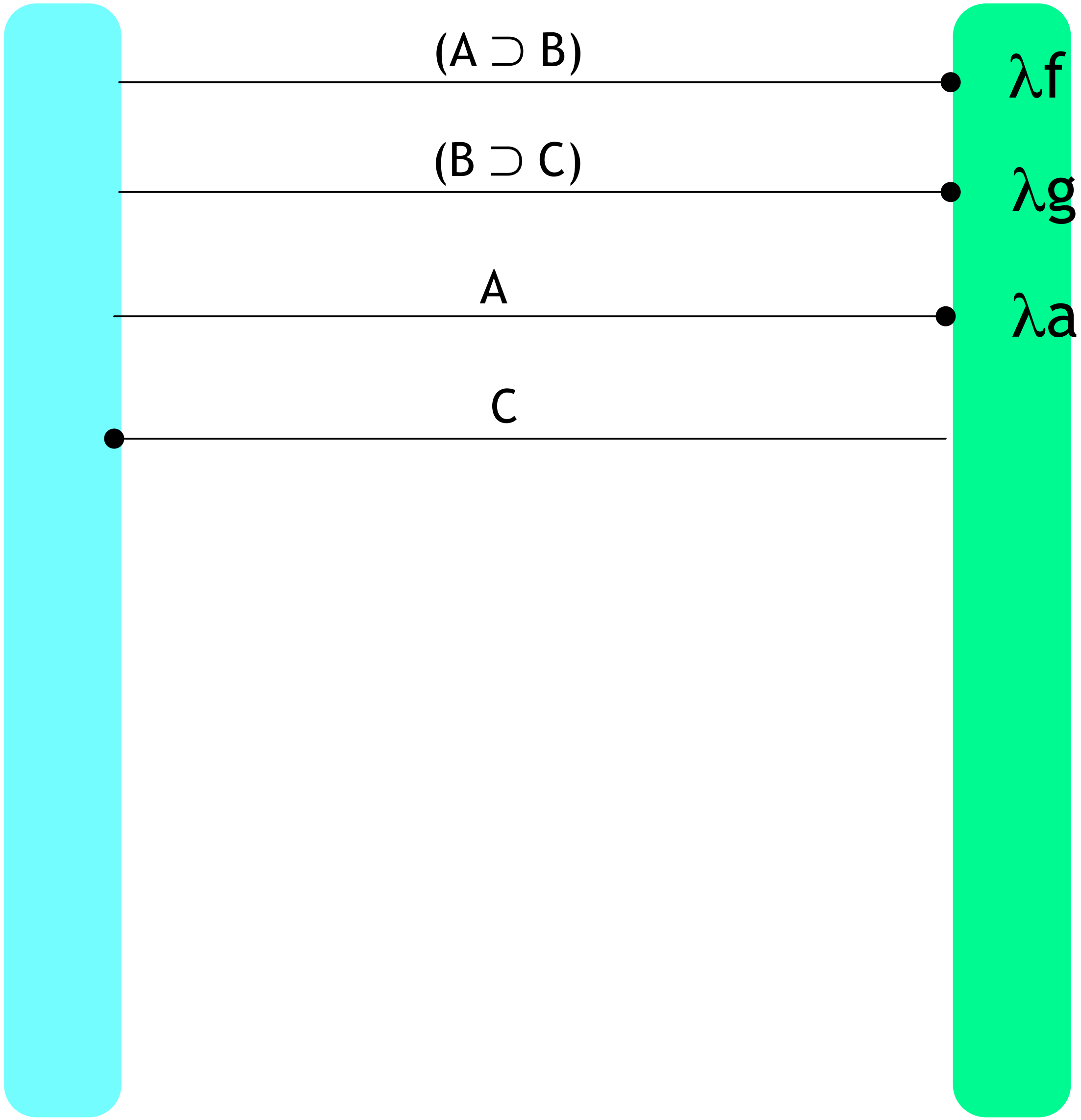


Decorating with λ-terms



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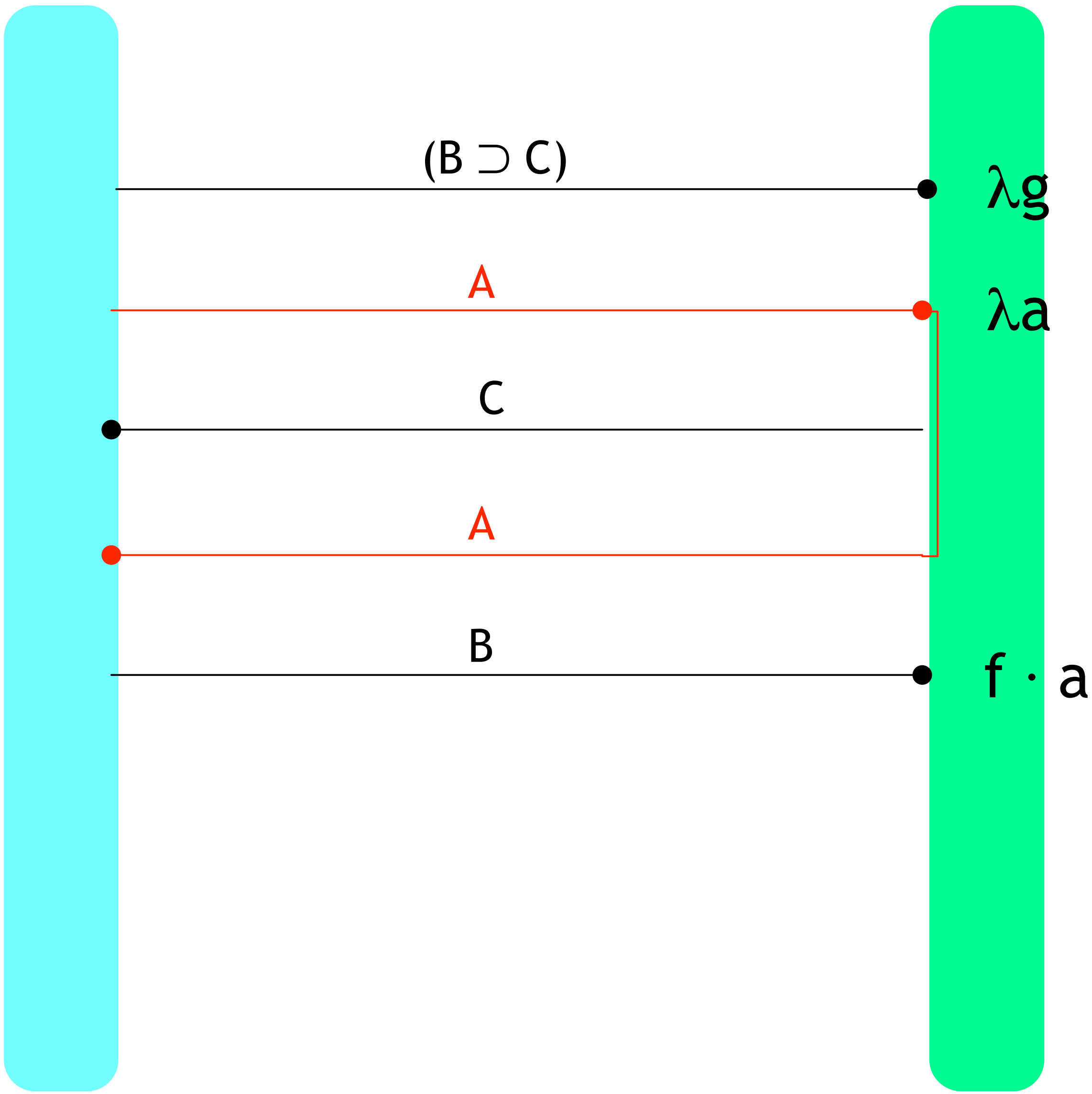

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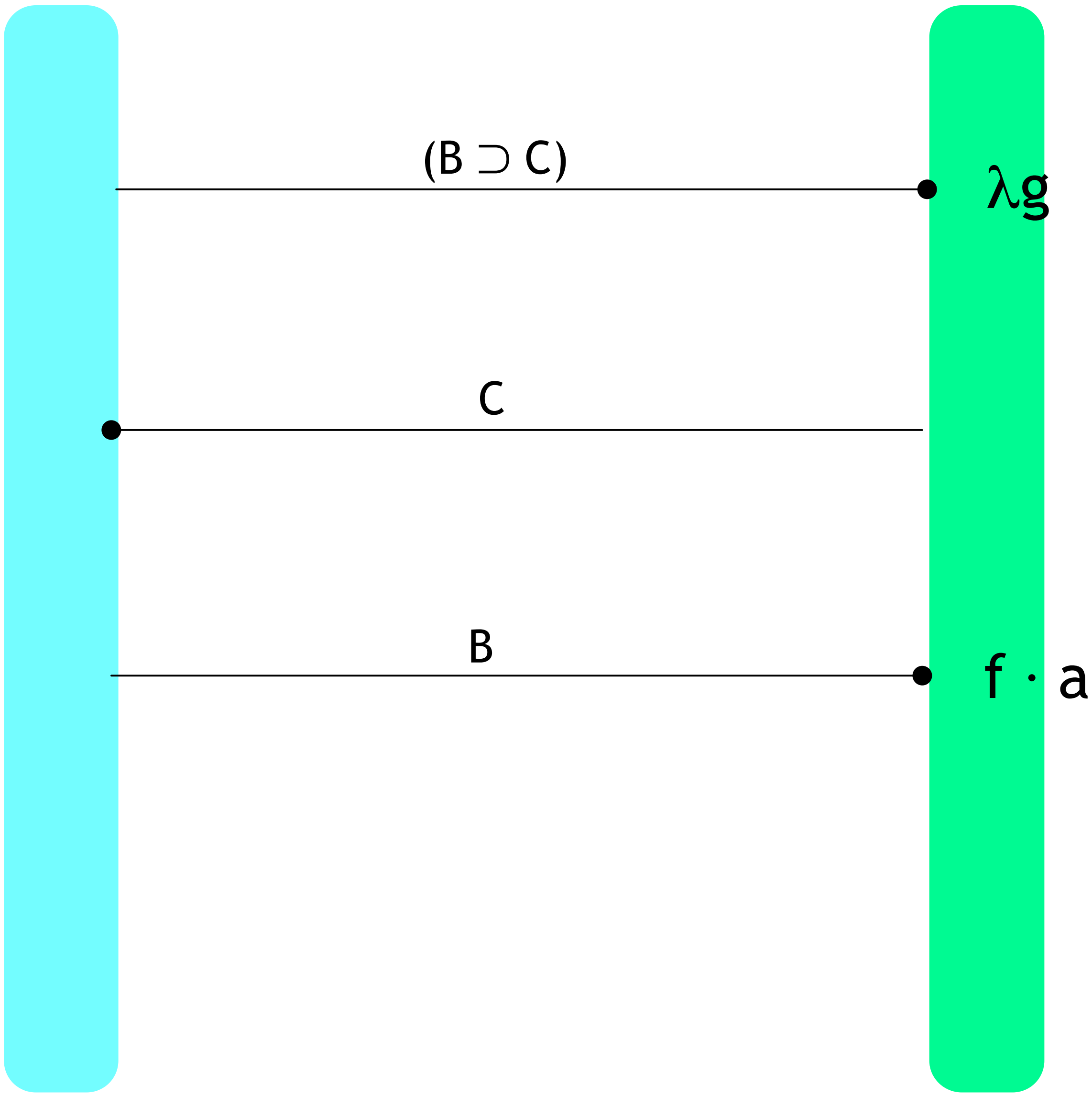

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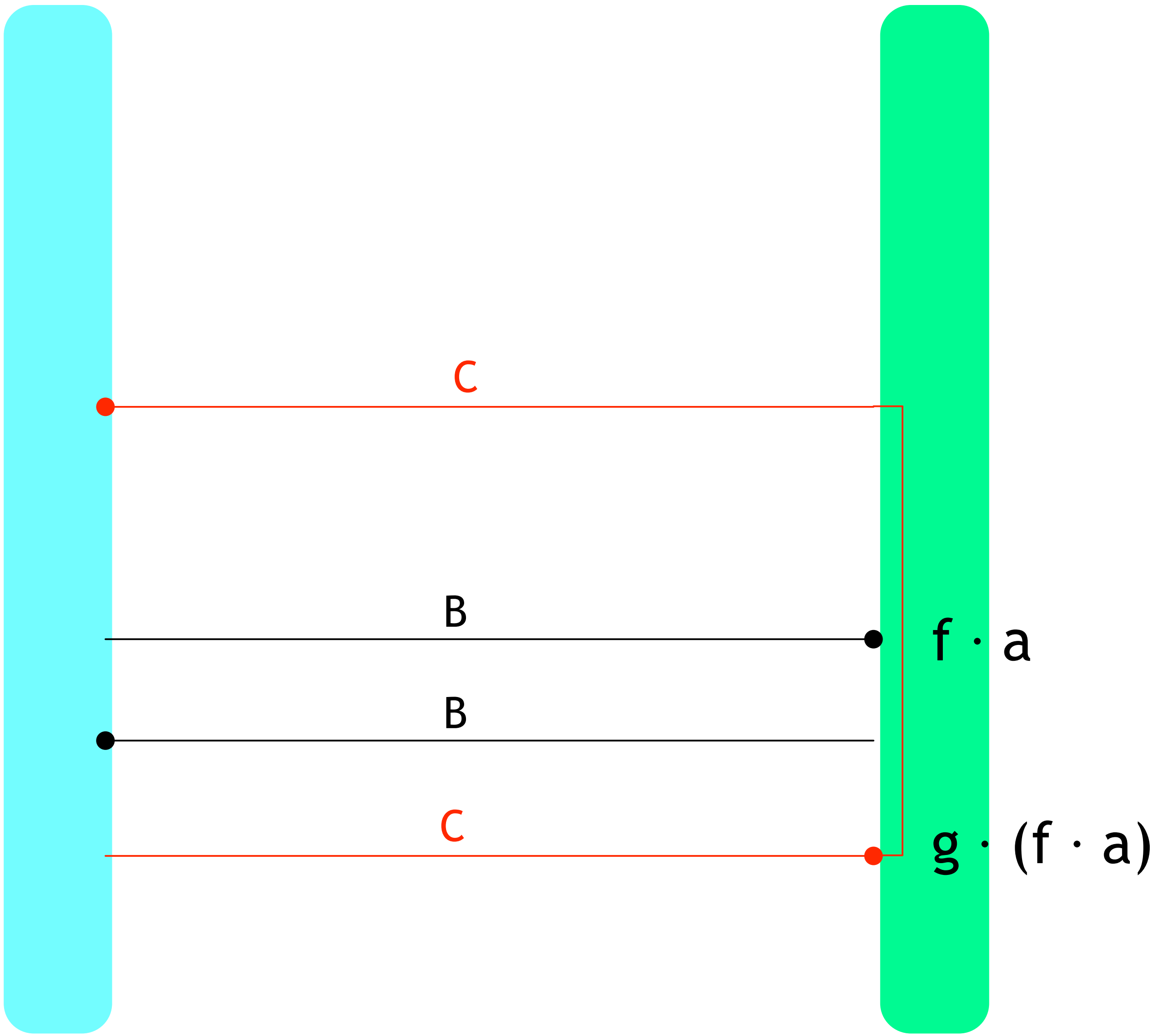

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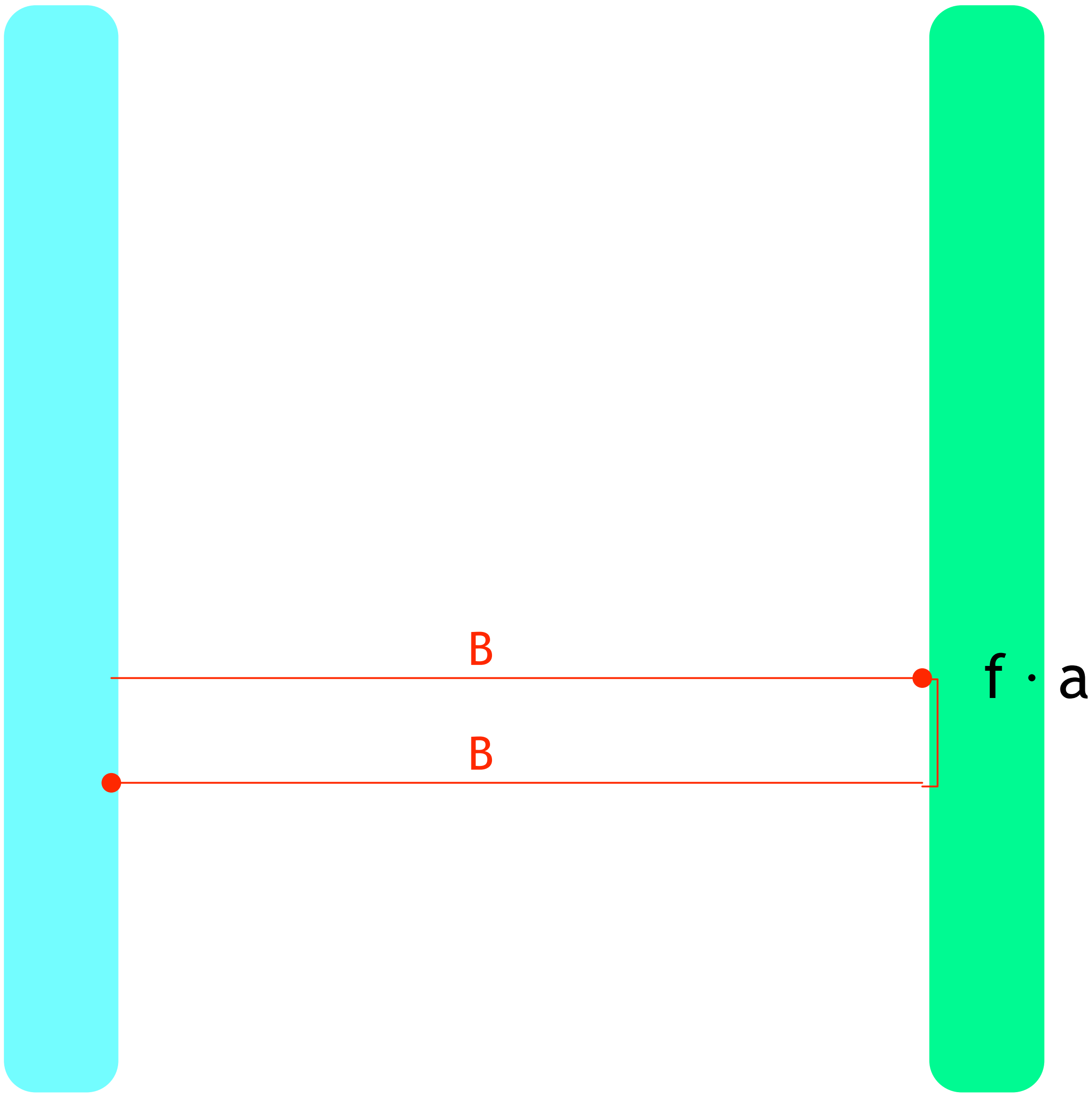

Decorating with λ-terms

Loose ends

Relations of matchings with Kelly-MacLane graphs;

Structure of the category $\mathcal{A}cc$;

Formulae as contracts/proofs as contract performances: what logical structure?

Applications to design and planning (use cases, design by contract, interaction design, design rationale)?

❖

Philosophy: commitment as an item in a new vocabulary for computing (along with, e.g., interaction)?

The end.

The end.

Thank you.