On the Expressiveness of Polyadic and Synchronous Communication in Higher-Order Process Calculi

Ivan Lanese   Jorge A. Pérez   Davide Sangiorgi   Alan Schmitt

ICALP 2010, Bordeaux.

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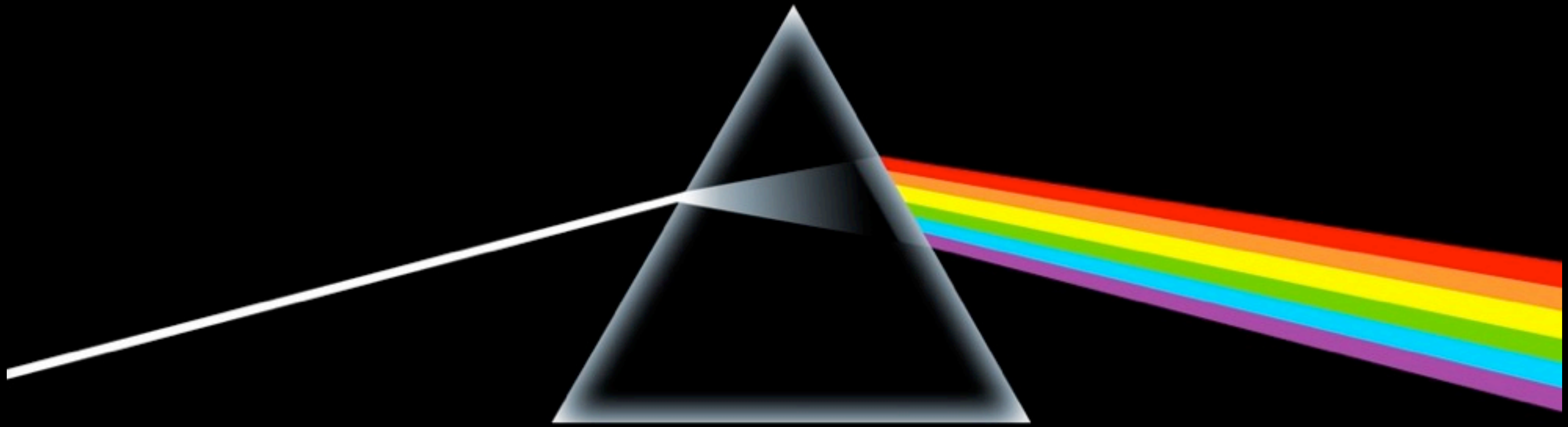
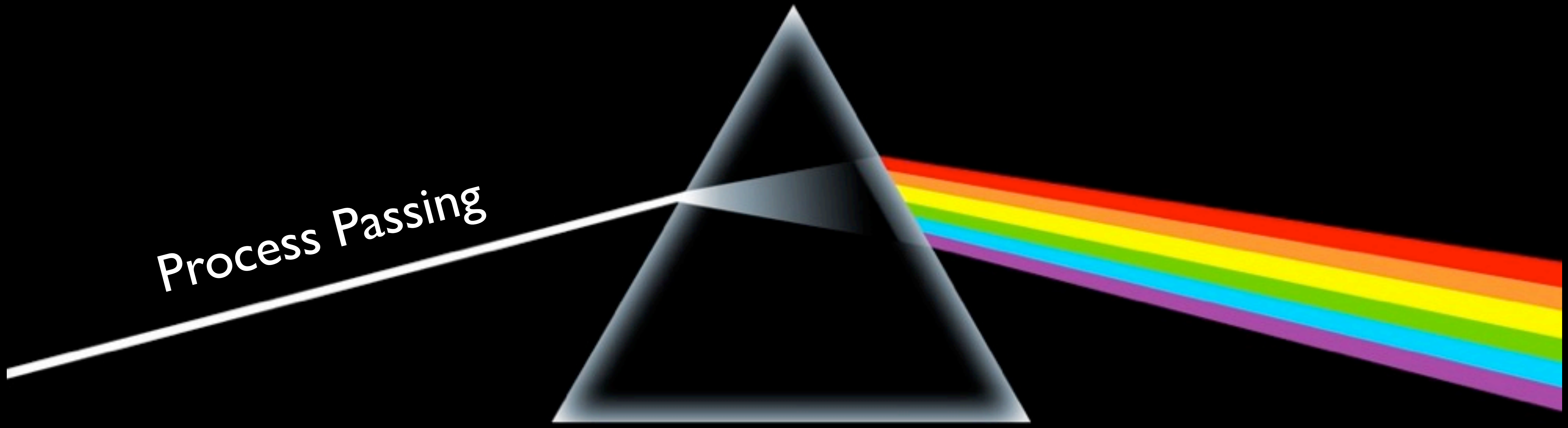
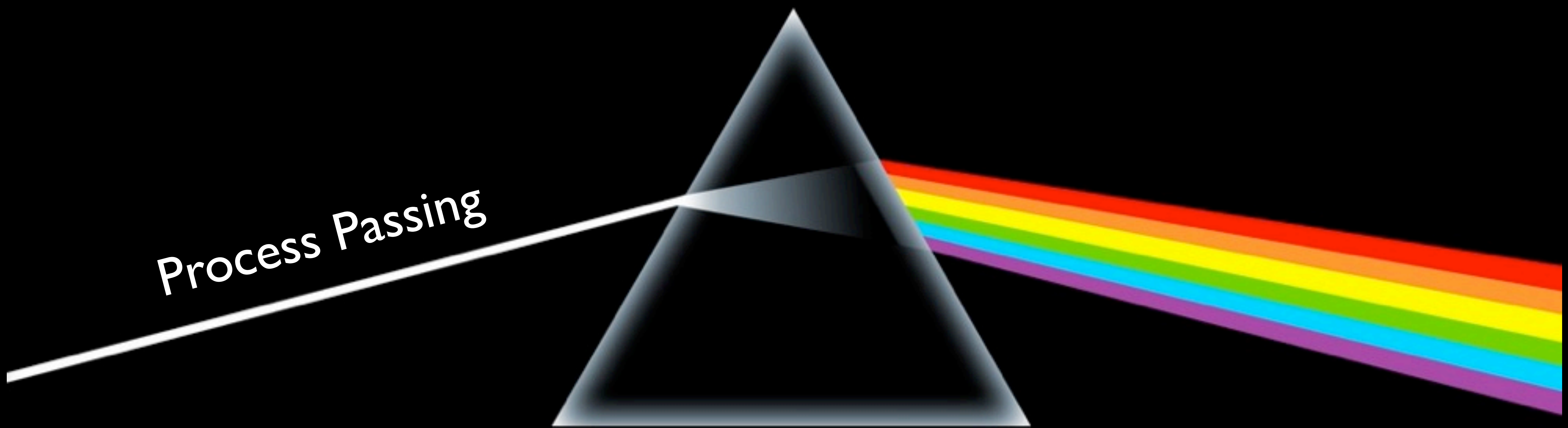
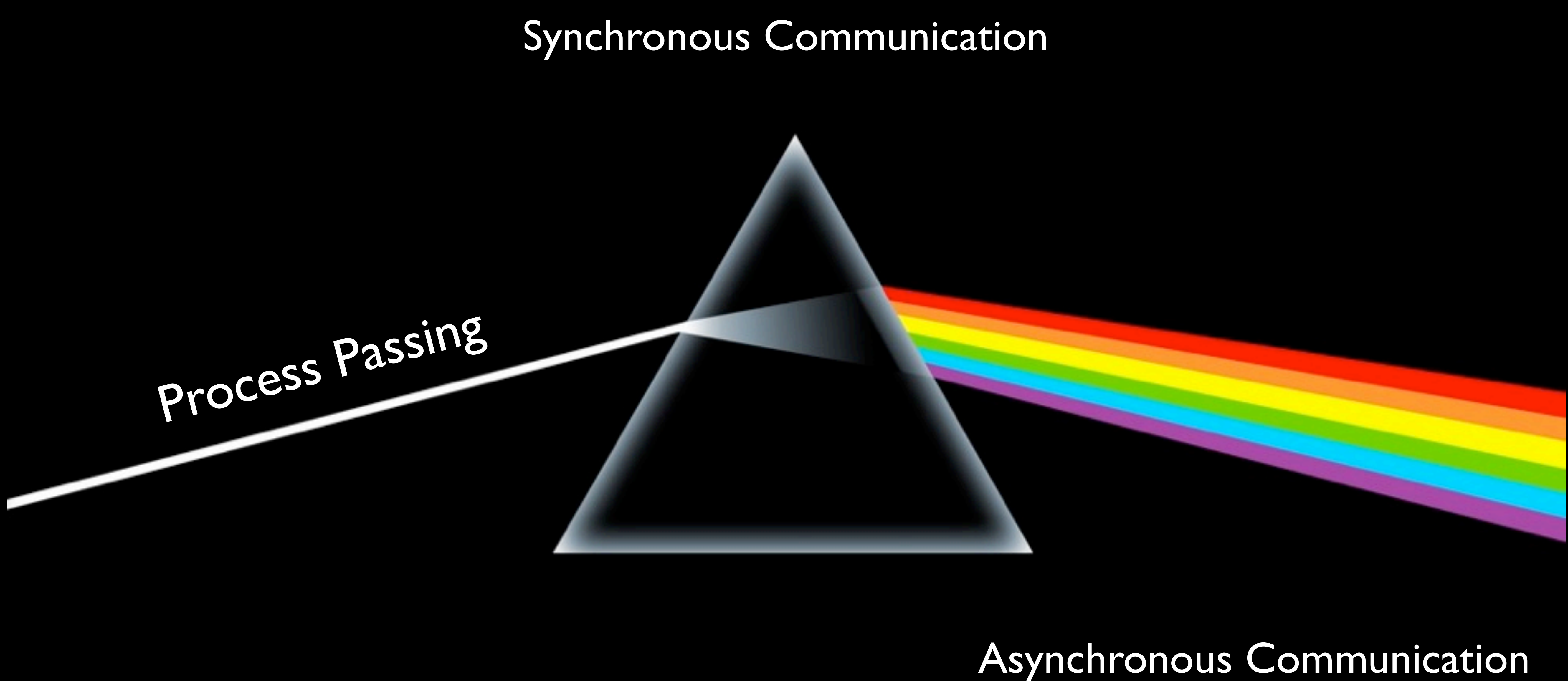



Process Passing

Synchronous Communication




Synchronous Communication
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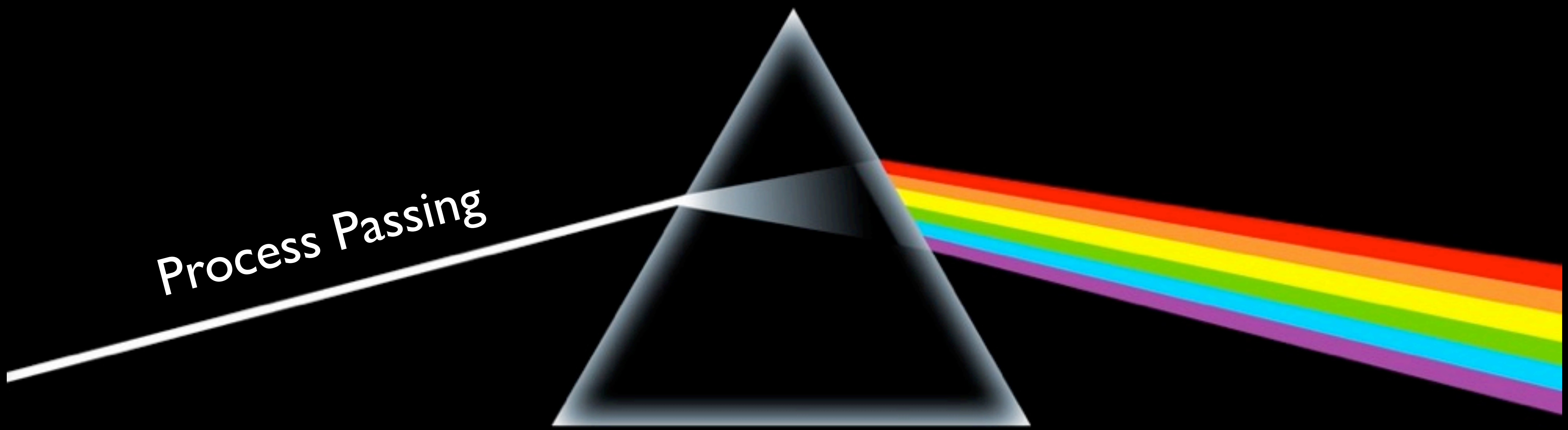
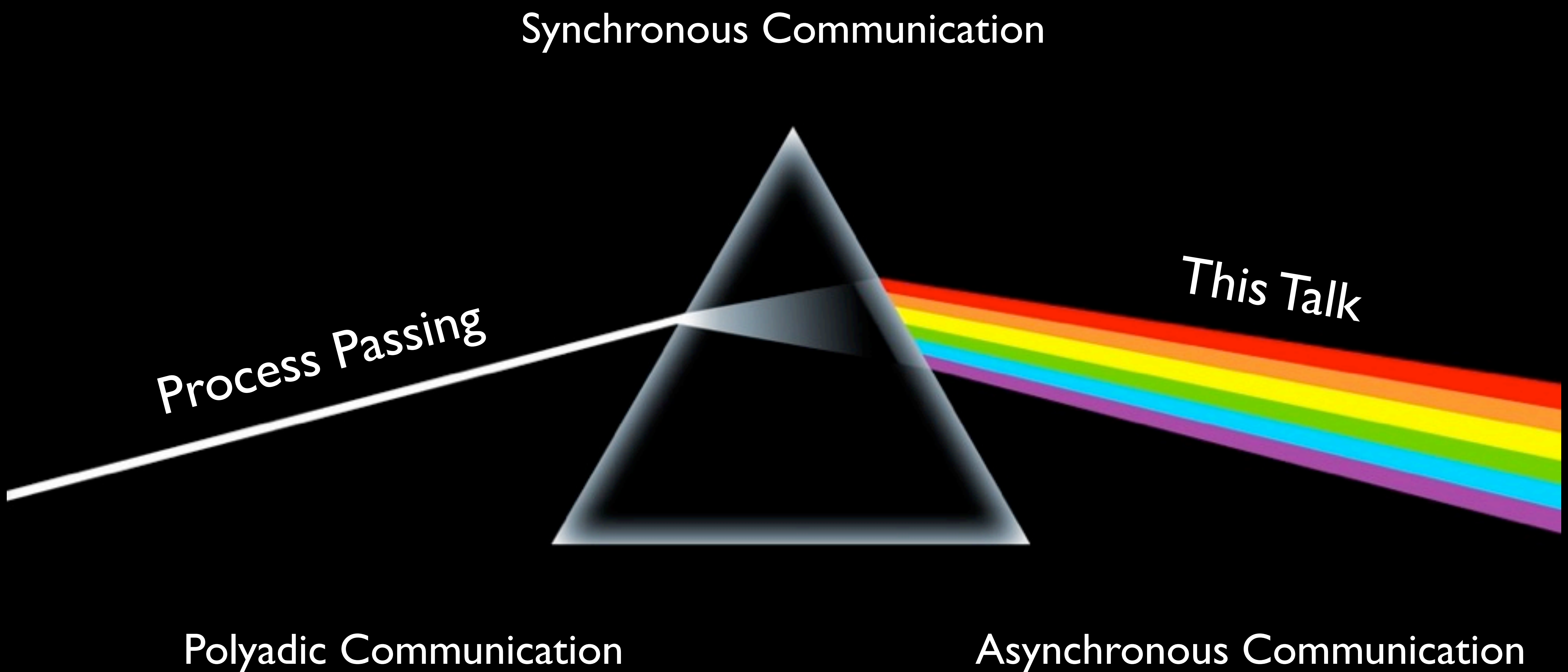


Polyadic Communication

Asynchronous Communication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This Talk
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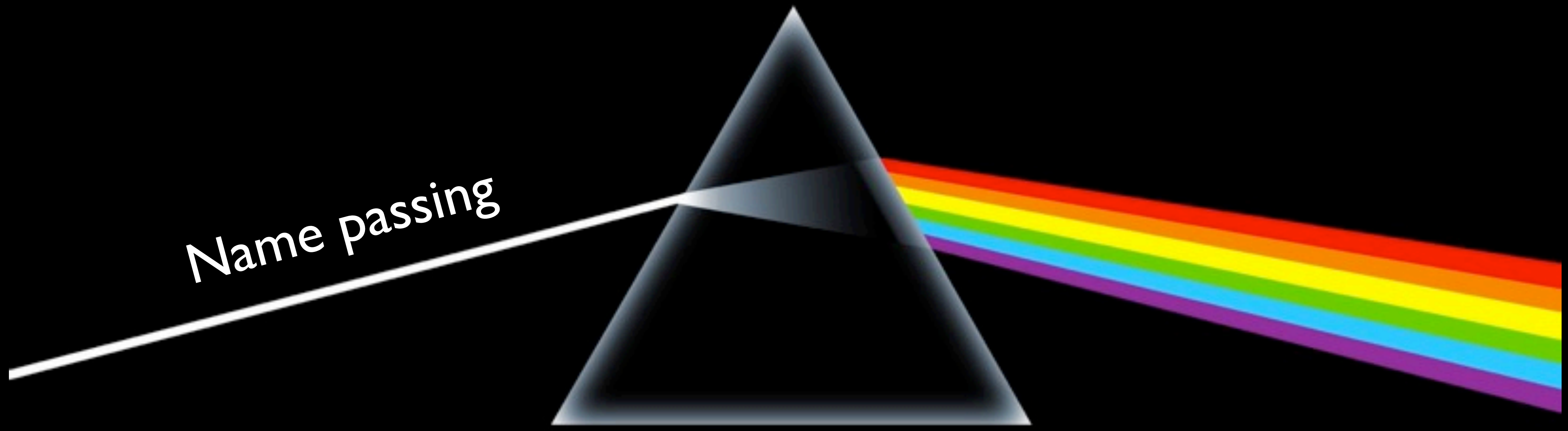
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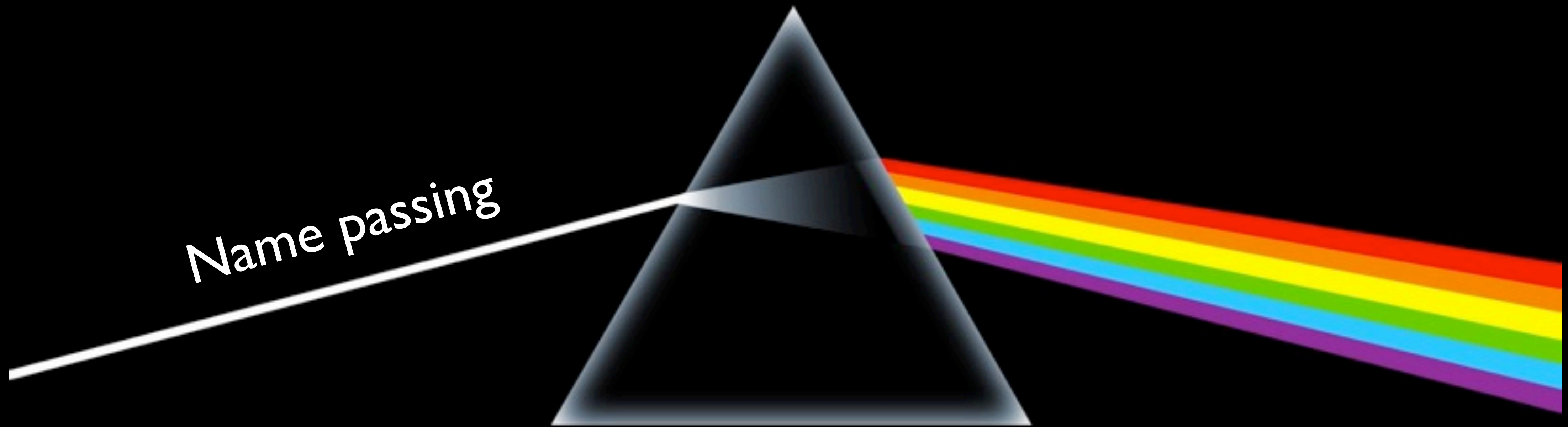

✦ Encodings of synchronous π-calculus *without choice* into asynchronous π [Honda-Tokoro91, Boudol92]

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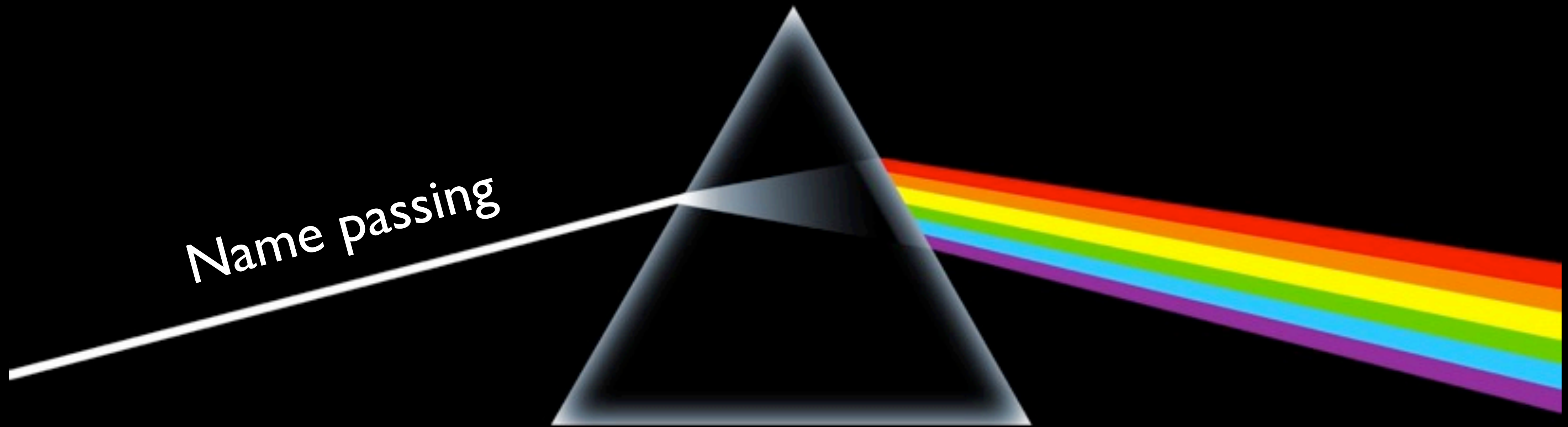

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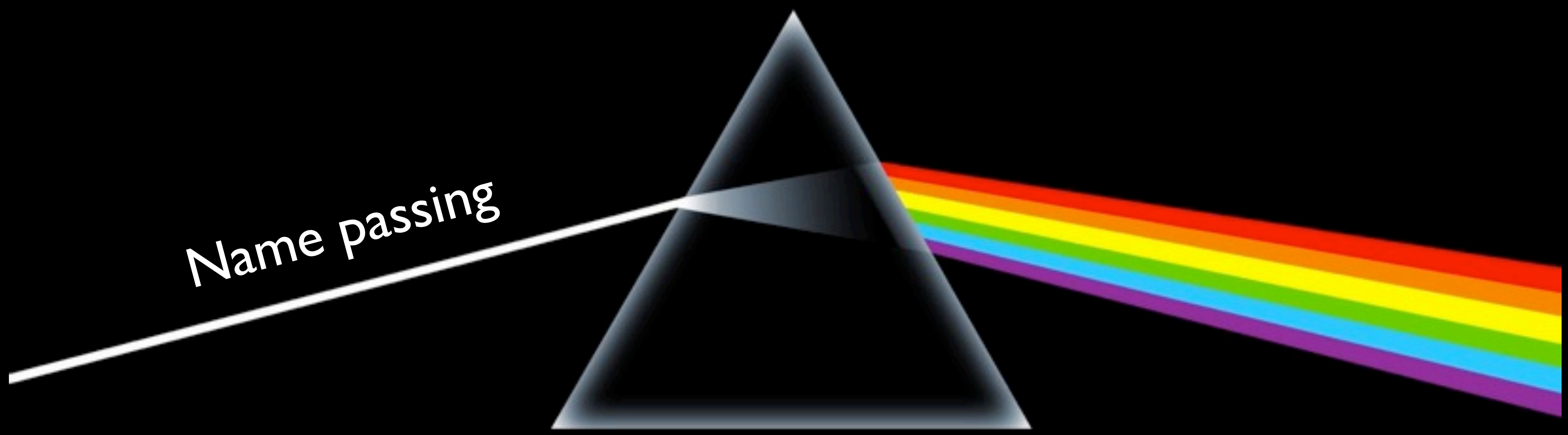

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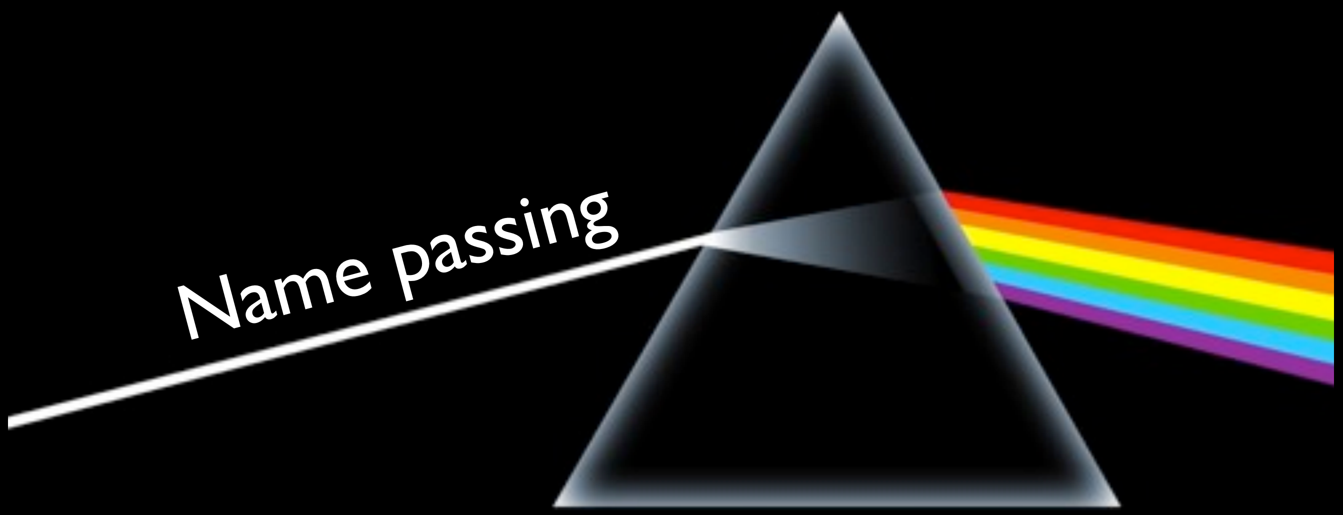
- The encoding of higher-order π-calculus (*name* AND *process* passing) into the π-calculus [Sangiorgi93]



Relies on *private links*
by combining *restriction*
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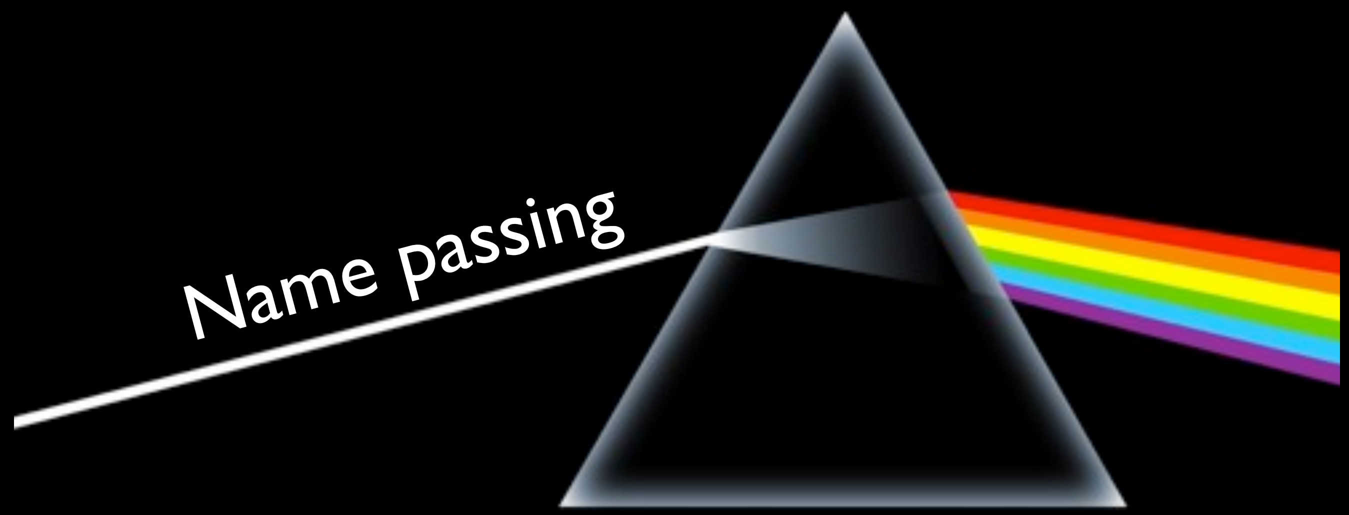

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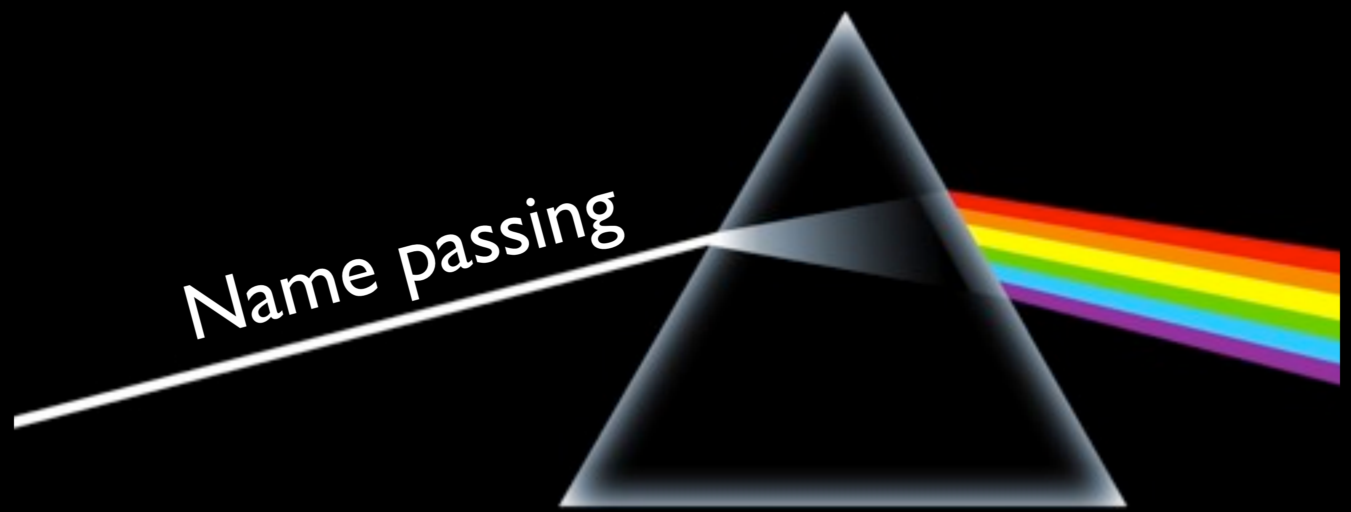
Example: *biadic into monadic* name passing



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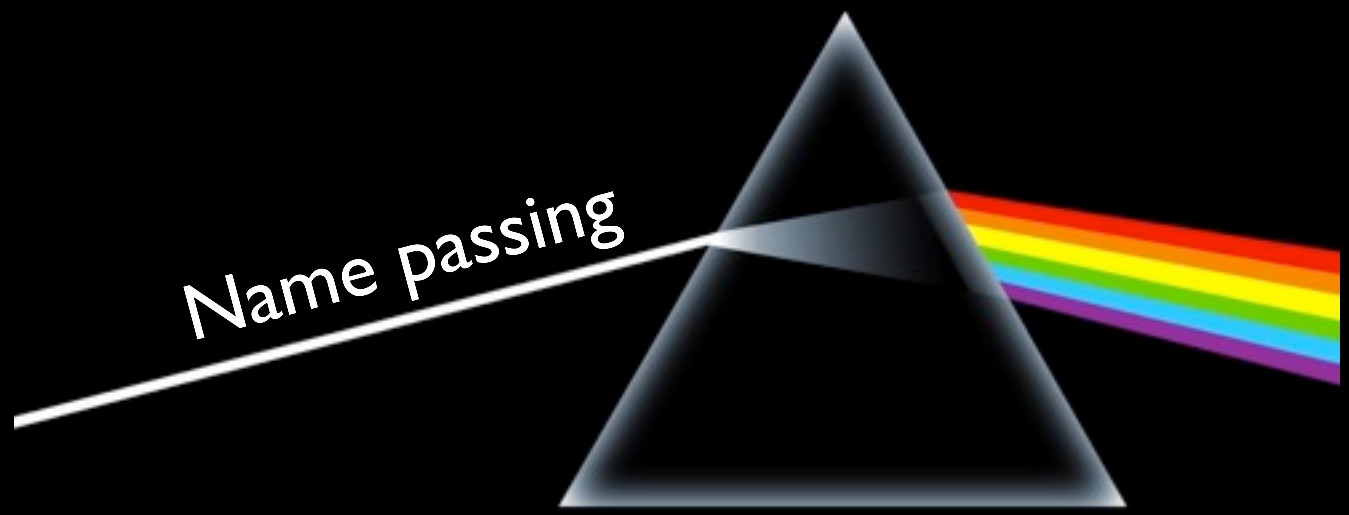

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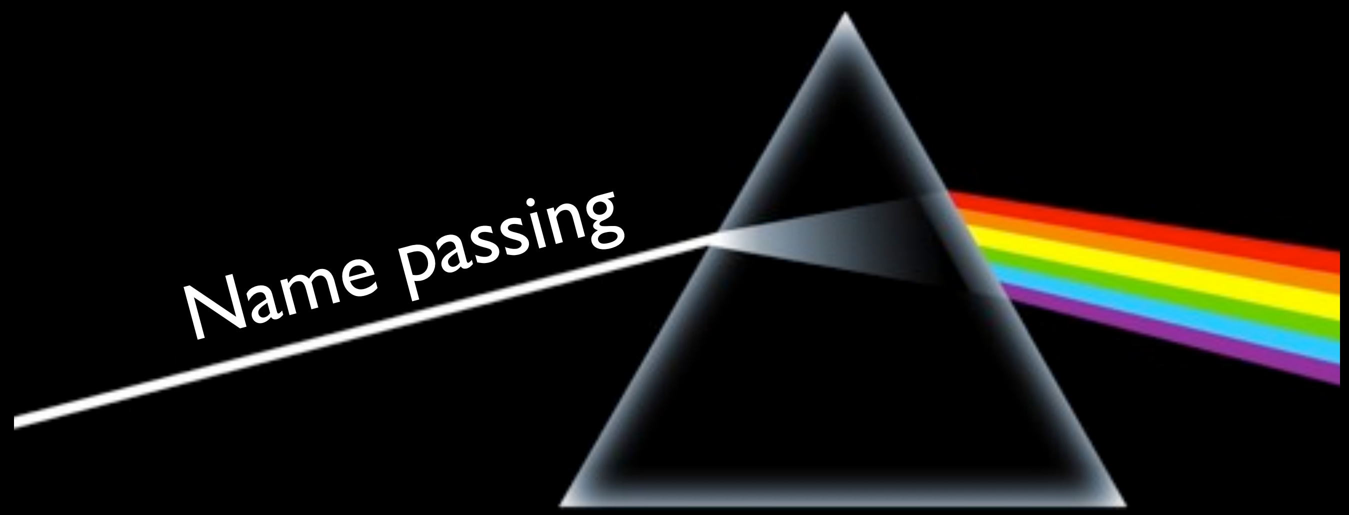

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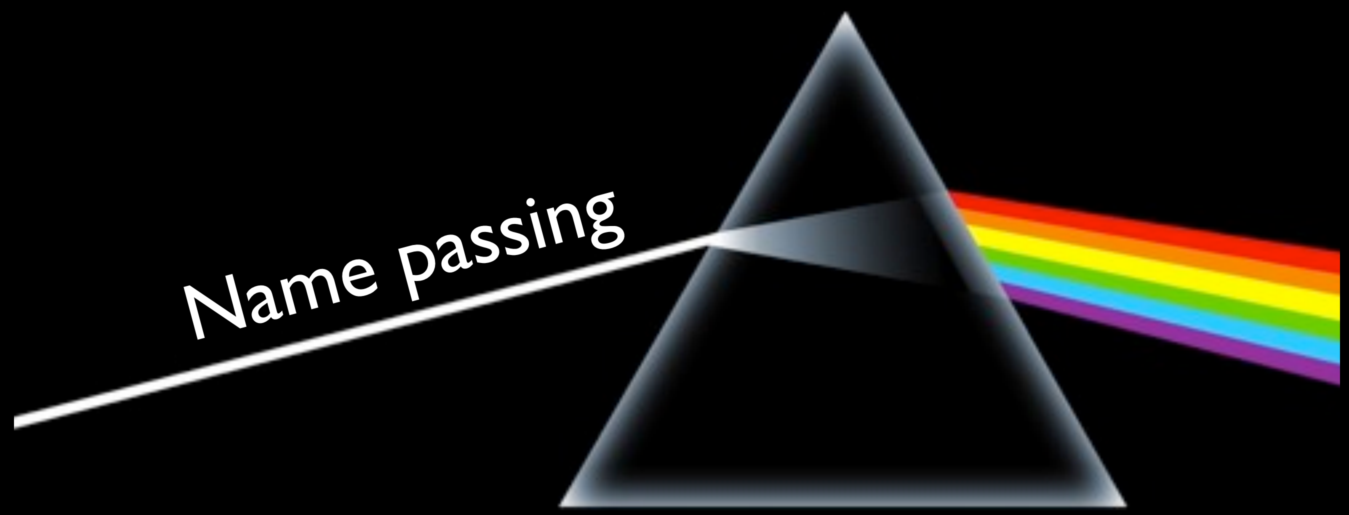

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Encodings are compact and robust wrt interferences

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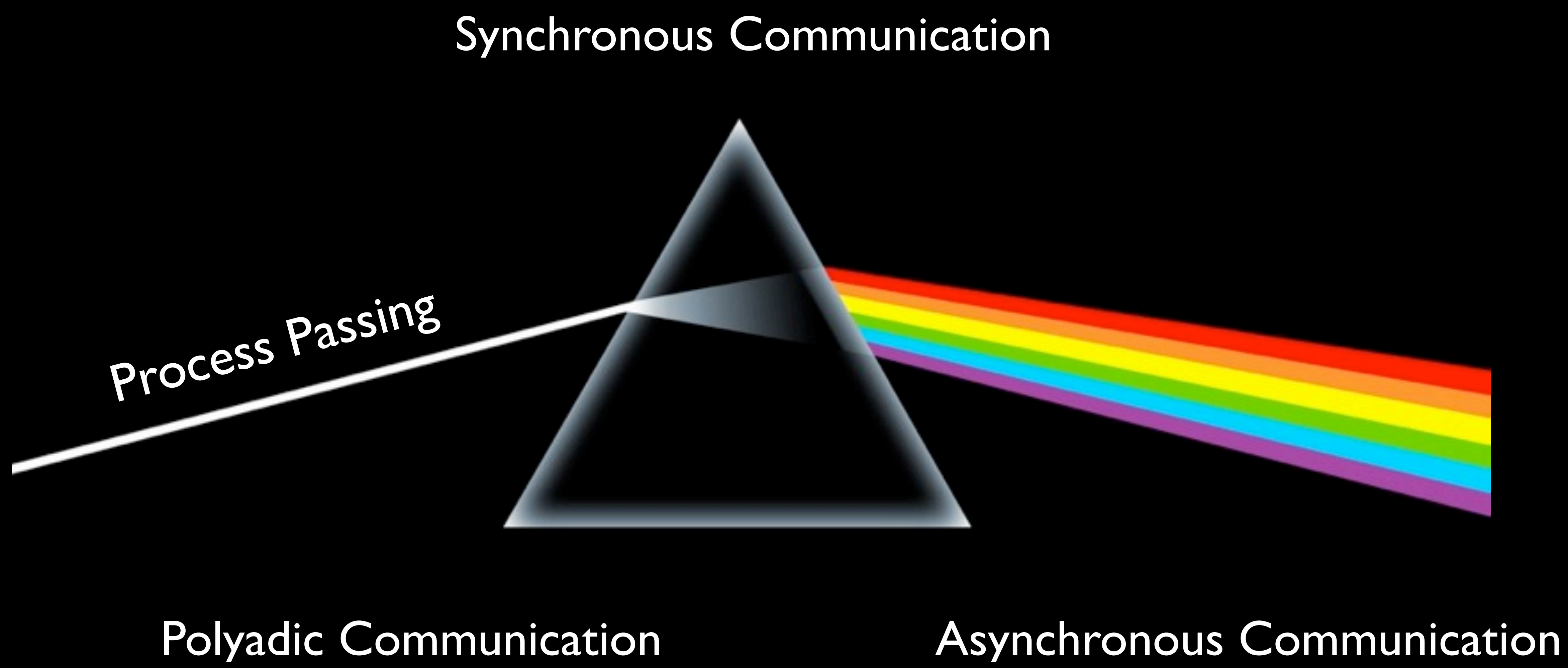

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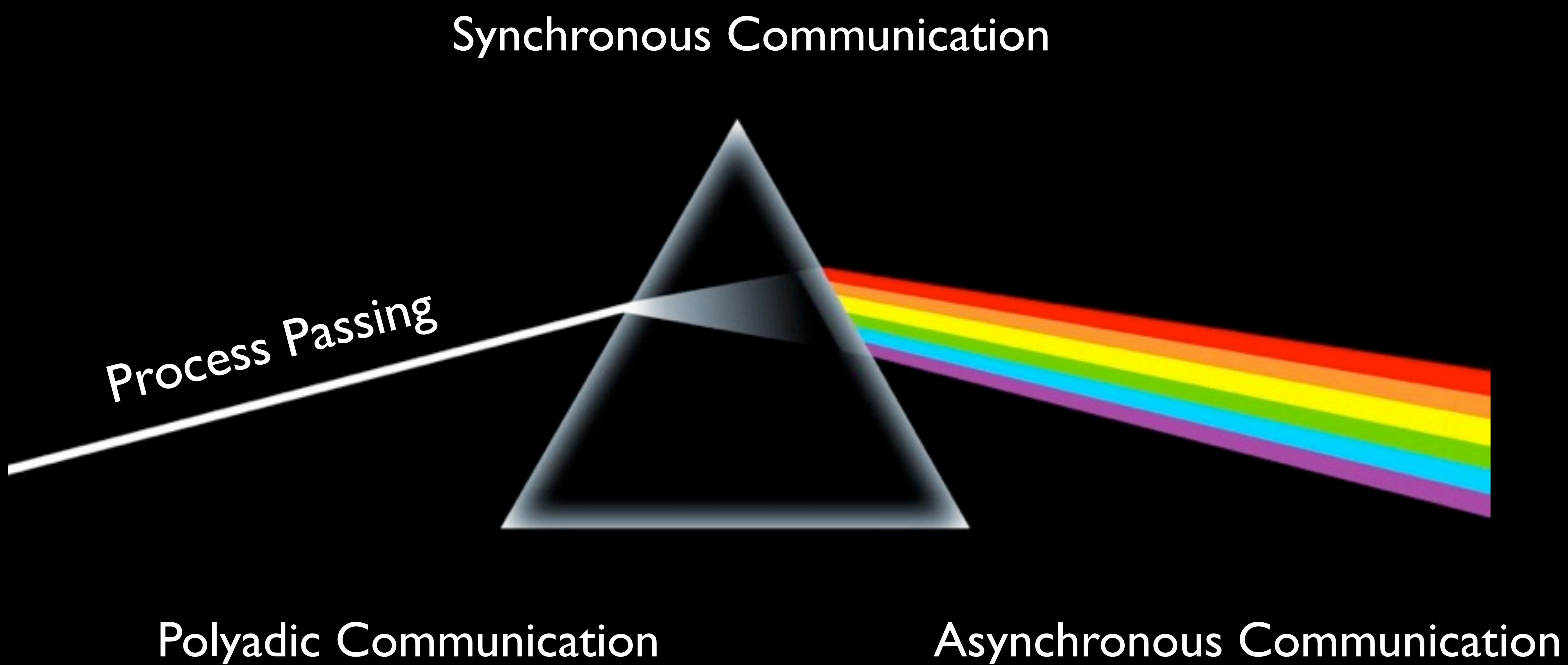

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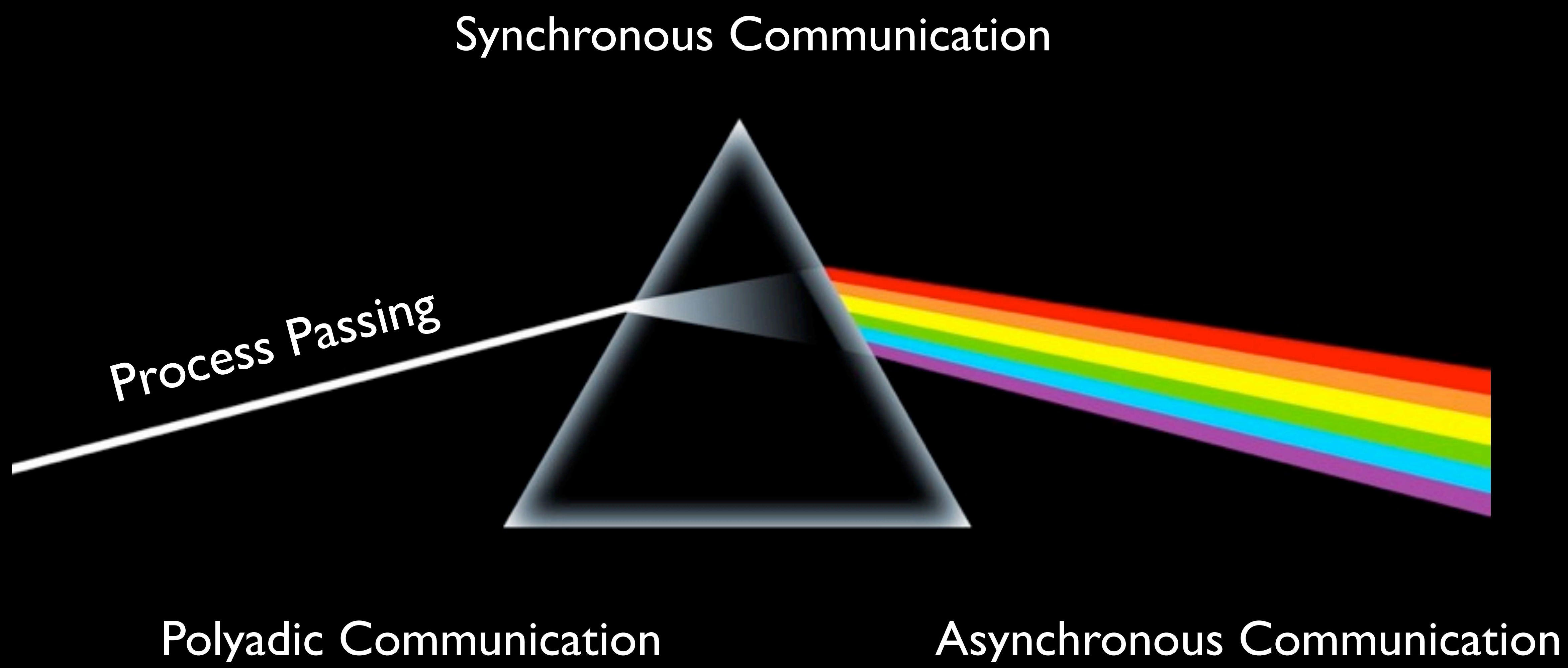

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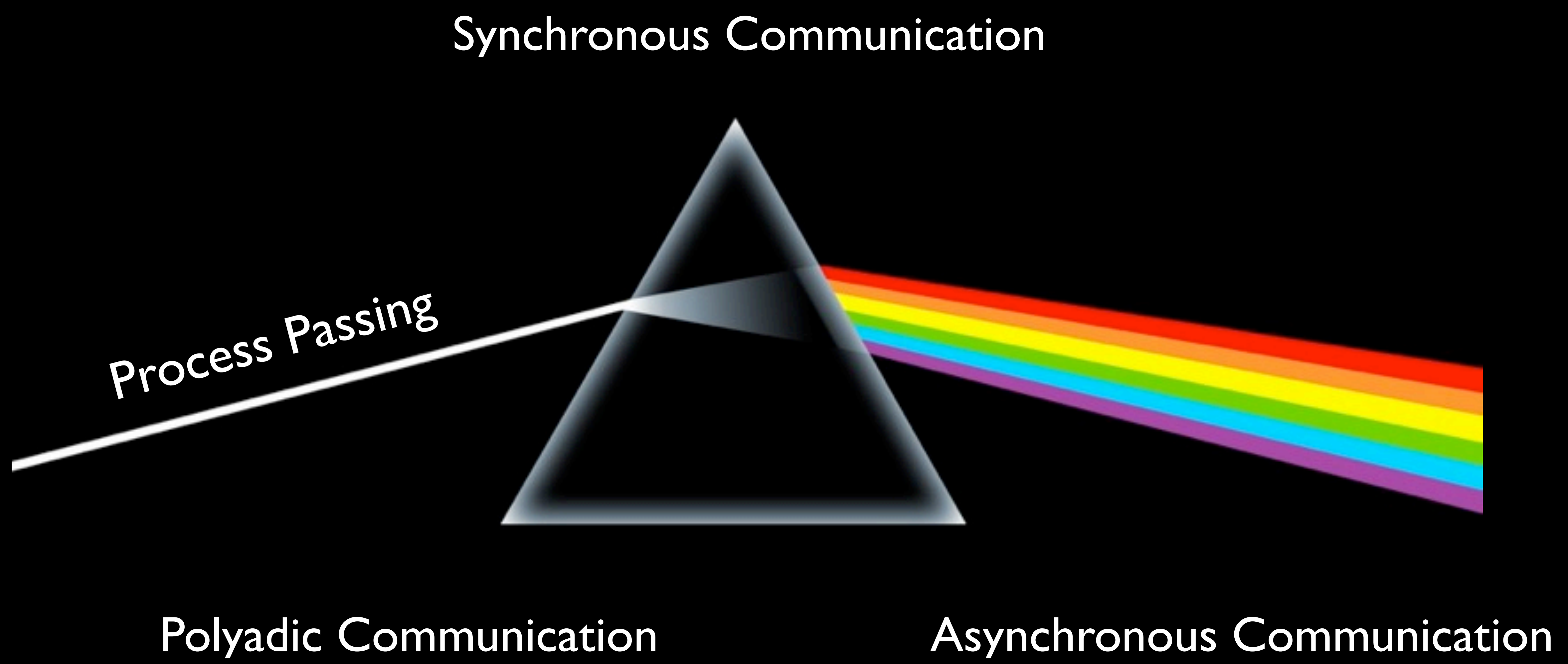

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Here: *pure* process passing

Processes as black boxes

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  But a receiver cannot "dig into" the structure of a process. So it *cannot actually use* such names.
- "Hollow" scope extrusions: the scope expands but their effect is limited

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$$\nu n\, (\overline{a}\langle P\rangle.S') \parallel a(x).R'$$

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In *R'*, name *n* <u>*can only be*</u> used as defined in *P and S'*

Our Results

1. Synchronous communication *can be encoded* into asynchronous communication
2. Polyadic communication of arity *n* *cannot be encoded* into communication of arity *n-1*
3. *Abstraction passing* *cannot be encoded* into polyadic communication

The Languages

Synchronous pure process passing of arity *n* (SHO^n)

$$P, Q ::= a(\tilde{x}).P \;\mid\; \overline{a}\langle\tilde{Q}\rangle.P \;\mid\; P_1 \parallel P_2 \;\mid\; \nu r\, P \;\mid\; x \;\mid\; \mathbf{0}$$

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$$P, Q ::= \cdots \;\;|\;\; (x)P \;\;|\; P_1\lfloor P_2 \rfloor$$

Semantics

- A Labeled Transition System (LTS) that enforces a closer look into synchronizations
- Two kinds of Internal behavior:
  - *internal* synchronizations τ
  - *public* synchronizations $a\tau$

The LTS for SHO^n

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$$\textsc{Inp}\ \frac{}{a(\widetilde{x}).\,P \xrightarrow{a(\widetilde{x})} P} \qquad \textsc{Out}\ \frac{}{\overline{a}\langle\widetilde{Q}\rangle.\,P \xrightarrow{\overline{a}\langle\widetilde{Q}\rangle} P} \qquad \textsc{Act1}\ \frac{P_1 \xrightarrow{\alpha} P_1' \quad \mathsf{cond}(\alpha, P_2)}{P_1 \parallel P_2 \xrightarrow{\alpha} P_1' \parallel P_2}$$

$$\textsc{Open}\ \frac{P \xrightarrow{(\nu\widetilde{s})\overline{a}\langle\widetilde{P}''\rangle} P' \quad r \neq a,\ r \in \mathsf{fn}(\widetilde{P}'') - \widetilde{s}}{\nu r\, P \xrightarrow{(\nu r\widetilde{s})\overline{a}\langle\widetilde{P}''\rangle} P'} \qquad \textsc{Res}\ \frac{P \xrightarrow{\alpha} P' \quad r \notin \mathsf{n}(\alpha)}{\nu r\, P \xrightarrow{\alpha} \nu r\, P'}$$

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Rule for the variant with abstraction passing:

$$\textsc{App}\ \frac{}{(x)P\lfloor Q\rfloor \xrightarrow{\tau} P\{Q/x\}}$$

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Encodings are *composable*: the composition of two encodings is an encoding


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- Challenge: to encode SHO^n into AHO^n

  Our solution: Send the first n-1 objects as they are, and use the n-th object to send BOTH the last object AND the continuation

Encoding Synchronous into Asynchronous

The basic case: SHO^1 into AHO^1

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The encoding is a homomorphism for the other operators.
Guarded choice is a derived construct in SHO^n

- Object and continuation together in a guarded sum
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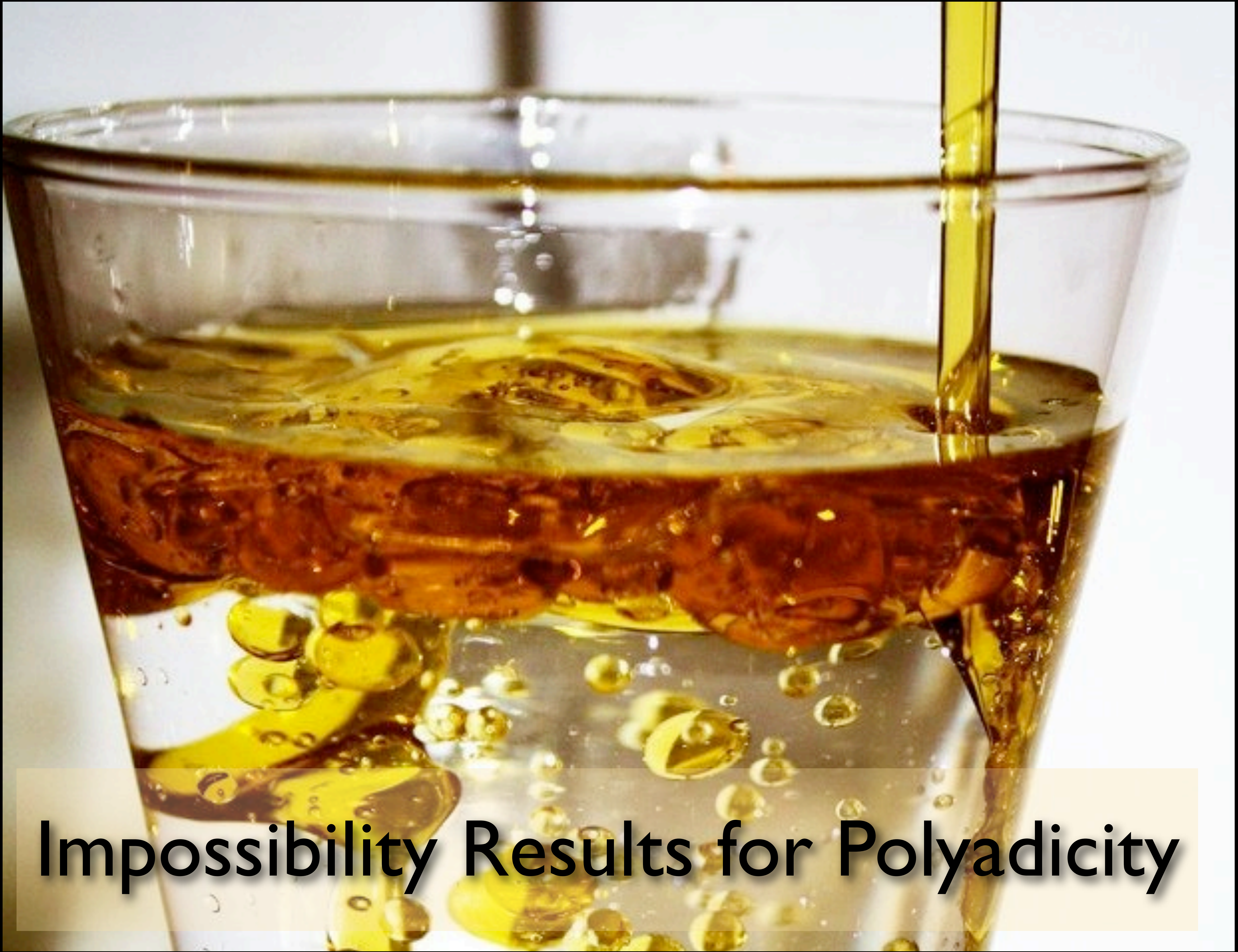
- The generalization to the *n*-adic case is immediate



Impossibility Results for Polyadicity

Restricted names are like oil and water

- They do not really “mix” after communications --- “hollow” extrusions
- This separation prevents private link establishment

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Our approach to separation:

- *Disjoint form*: our way of formalizing separation of restricted names *after* a public synchronization
- *Stability conditions*: when/how processes remain in disjoint form along computations

Disjoint Forms

Two biadic processes that do not share private names
They can communicate through a public name:

$$\nu\tilde{n}\,(\overline{a}\langle R_1, R_2\rangle.P) \parallel a(x_1, x_2).Q$$

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The scope expands but this is a hollow extrusion

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The private names of C and those of P, R_1, R_2 are *disjoint*

Disjoint Forms

Definition 10 (Disjoint Form) *Let* $T \equiv \nu\widetilde{n}(P \parallel C[\widetilde{R}])$ *be a* SHO^m *process where*

1. $\widetilde{n}$ *is a set of names such that* $\widetilde{n} \subseteq \mathsf{fn}(P, \widetilde{R})$ *and* $\widetilde{n} \cap \mathsf{fn}(C) = \emptyset$*;*
2. C *is a k-ary (guarded, multihole) context;*
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We then say that T *is in* k-adic disjoint form with respect to $\widetilde{n}$, $\widetilde{R}$, and P.

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Stability Conditions:

Disjoint forms are preserved by internal synchronizations and certain output actions

The impossibility result

Theorem. *There is no encoding of SHO^2 into SHO^1*

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3. Show that the encoding of *P* mimics such communication and gets into *monadic disjoint form (MDF)*
4. Show that the MDF is preserved along relevant computations
5. Using a causality analysis, show that the (limited) structure of the MDF causes the encoding of *P* to exhibit behavior that *P* doesn't have: contradiction.

The hierarchy

Theorem. *There is no encoding of* SHO^n *into* SHO^{n-1}, *for every* $n>1$

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- Proofs follow by an extension of all notions and auxiliary results
- The hierarchy holds also for asynchronous calculi


The power of abstraction passing

Abstraction passing

- Sending functions as in the λ-calculus
- It is specific to the higher-order setting --- not present in the name passing
- We only consider abstractions of order 1: functions from processes to processes

Private links with abstraction passing

Encoding SHO^2 into SHO^1 with abstraction passing (SHO^1_a):

$$\begin{aligned}
[\![\overline{a}\langle P_1, P_2\rangle.\, R]\!] &= a(z).\,([\![R]\!] \parallel \nu m\, n\, c\,(\overline{n} \parallel z\lfloor n.\,(\overline{c} \parallel \overline{m}) + m.\,([\![P_1]\!] \parallel \overline{m})\rfloor \\
&\qquad \parallel c.\, z\lfloor [\![P_2]\!] \rfloor)) \\
[\![a(x_1, x_2).\, Q]\!] &= \nu b\,(\overline{a}\langle (y)\overline{b}\langle y\rangle\rangle \parallel b(x_1).\,(x_1 \parallel b(x_2).\,[\![Q]\!]))
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- The receiver takes the initiative and sends an abstraction with a restricted name *b*

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Abstraction passing goes beyond process passing

Theorem. *For every $m, n > 1$, there is no encoding of $SHO^n{}_a$ into SHO^m*

Abstraction passing goes beyond process passing

Theorem. *For every $m, n > 1$, there is no encoding of SHO^n_a into SHO^m*

Proof:

- Suppose there is an encoding A[[-]]: $SHO^n_a \rightarrow SHO^m$
- We know there is an encoding B[[-]]: $SHO^{m+1} \rightarrow SHO^n_a$
- By composability of encodings, we have the encoding
  A◦B[[-]]: $SHO^{m+1} \rightarrow SHO^m$
- However, such an encoding doesn't exist: contradiction

Asynchronous Communication

Abstraction Passing

Polyadic Communication

Asynchronous Communication

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On the Expressiveness of Polyadic and Synchronous Communication in Higher-Order Process Calculi

Ivan Lanese   Jorge A. Pérez   Davide Sangiorgi   Alan Schmitt

ICALP 2010, Bordeaux.

The notion of encoding (Formally)

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Definition 5 (Syntactic Conditions) *Let* $[\![\cdot]\!] : \mathcal{P}_{\mathsf{s}} \to \mathcal{P}_{\mathsf{t}}$ *be a translation of* $\mathcal{L}_{\mathsf{s}}$ *into* $\mathcal{L}_{\mathsf{t}}$. *We say that* $[\![\cdot]\!]$ *is:*

1. compositional *if for every k-ary operator* op *of* $\mathcal{L}_{\mathsf{s}}$ *and for all* $S_1, \ldots, S_k$ *with* $\mathsf{fn}(S_1, \ldots, S_k) = N$, *there exists a k-ary context* $C^N_{\mathsf{op}} \in \mathcal{P}_{\mathsf{t}}$ *that depends on* N *and* op *such that* $[\![\mathsf{op}(S_1, \ldots, S_k)]\!] = C^N_{\mathsf{op}}[[\![S_1]\!], \ldots, [\![S_k]\!]]$;
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Definition 6 (Semantic Conditions) *Let* $[\![\cdot]\!] : \mathcal{P}_\mathsf{s} \to \mathcal{P}_\mathsf{t}$ *be a translation of* $\mathcal{L}_\mathsf{s}$ *into* $\mathcal{L}_\mathsf{t}$. *We say that* $[\![\cdot]\!]$ *is:*

1. complete *if for every* $S, S' \in \mathcal{P}_\mathsf{s}$ *and* $\alpha \in \mathcal{A}_\mathsf{s}$ *such that* $S \stackrel{\alpha}{\Longrightarrow}_\mathsf{s} S'$, *it holds that* $[\![S]\!] \stackrel{\beta}{\Longrightarrow}_\mathsf{t} \approx_\mathsf{t} [\![S']\!]$, *where* $\beta \in \mathcal{A}_\mathsf{t}$ *and* $\mathsf{sig}(\alpha) = \mathsf{sig}(\beta)$;
2. sound *if for every* $S \in \mathcal{P}_\mathsf{s}$, $T \in \mathcal{P}_\mathsf{t}$, $\beta \in \mathcal{A}_\mathsf{t}$ *such that* $[\![S]\!] \stackrel{\beta}{\Longrightarrow}_\mathsf{t} T$ *there exists an* $S' \in \mathcal{P}_\mathsf{s}$ *and an* $\alpha \in \mathcal{A}_\mathsf{s}$ *such that* $S \stackrel{\alpha}{\Longrightarrow}_\mathsf{s} S'$, $T \Rightarrow\approx_\mathsf{t} [\![S']\!]$, *and* $\mathsf{sig}(\alpha) = \mathsf{sig}(\beta)$;
3. adequate *if for every* $S, S' \in \mathcal{P}_\mathsf{s}$, *if* $S \approx_\mathsf{s} S'$ *then* $[\![S]\!] \approx_\mathsf{t} [\![S']\!]$;
4. diverge-reflecting *if for every* $S \in \mathcal{P}_\mathsf{s}$, $[\![S]\!]$ *diverges only if* S *diverges.*

Example: Sync into Async

A synchronous "duplicator" process:

$$\overline{a}\langle P\rangle.S \parallel a(x).(x \parallel x) \xrightarrow{a\tau} S \parallel P \parallel P$$

$$
\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
\end{aligned}
$$

$$
\begin{aligned}
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= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \parallel a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \ & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \ \parallel \ a(x).(x \parallel [\![x \parallel x]\!]) \\
= \ & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \ \parallel \ a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \ & \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \ ) \\
\xrightarrow{\tau} \ & \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \ ) \\
\xrightarrow{\tau} \ & \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \ ) \\
\xrightarrow{\tau} \ & \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \; & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \; & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \; & \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \; & [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
\end{aligned}
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\begin{aligned}
& \llbracket \overline{a}\langle P\rangle.S \rrbracket \parallel \llbracket a(x).(x \parallel x) \rrbracket \\
= \;& \nu k\, l\, (\overline{a}\langle k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel \llbracket x \parallel x \rrbracket) \\
= \;& \nu k\, l\, (\overline{a}\langle k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\overline{l} \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k}) \\
& \qquad\quad \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k}) \\
& \qquad\quad \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, (\llbracket S\rrbracket \parallel \overline{k} \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k}) \\
& \qquad\qquad\qquad\qquad \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, l\, (\llbracket S\rrbracket \parallel \llbracket P\rrbracket \parallel \overline{k} \parallel k.(\llbracket P\rrbracket \parallel \overline{k}) + l.(\llbracket S\rrbracket \parallel \overline{k})\;) \\
\xrightarrow{\tau} \;& \nu k\, (\llbracket S\rrbracket \parallel \llbracket P\rrbracket \parallel \llbracket P\rrbracket \parallel \overline{k}) \\
\approx \;& \llbracket S\rrbracket \parallel \llbracket P\rrbracket \parallel \llbracket P\rrbracket
\end{aligned}
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
=\ & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \ \parallel\ a(x).(x \parallel [\![x \parallel x]\!]) \\
=\ & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \ \parallel\ a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau}\ & \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau}\ & \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau}\ & \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau}\ & \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \; & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \; & \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \overline{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \; & \nu k\, l\, (\overline{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad\quad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \; & \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \; & [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
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\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad\qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \;) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
\end{aligned}
$$

$$
\begin{aligned}
& [\![\overline{a}\langle P\rangle.S]\!] \parallel [\![a(x).(x \parallel x)]\!] \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel [\![x \parallel x]\!]) \\
= \;& \nu k\, l\, (\overline{a}\langle k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\rangle \parallel \bar{l}) \;\parallel\; a(x).(x \parallel x \parallel x) \\
\xrightarrow{a\tau} \;& \nu k\, l\, (\bar{l} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k}) \\
& \qquad\qquad \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, l\, ([\![S]\!] \parallel [\![P]\!] \parallel \overline{k} \parallel k.([\![P]\!] \parallel \overline{k}) + l.([\![S]\!] \parallel \overline{k})\ ) \\
\xrightarrow{\tau} \;& \nu k\, ([\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!] \parallel \overline{k}) \\
\approx \;& [\![S]\!] \parallel [\![P]\!] \parallel [\![P]\!]
\end{aligned}
$$