An Axiomatic Approach to Reversible Computation

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Outline

Reversible computation

Axiomatic approach

Case studies

Reversible computation

Reversible computation allows computation to proceed not only in the standard, forward direction, but also backwards, recovering past states.

Applications in different areas:

- low-power computing (Landauer 1961)
- optimistic parallel simulation (Carothers et al 1999)
- error recovery in robot assembly operations (Laursen et al 2015)
- debugging (GDB since 2009, WinDbg)
- …

Reversible calculi and languages

In many of these areas, concurrent systems are of interest.

Reversible extensions of concurrent formalisms and languages have been proposed

Seminal one, RCCS (Danos & Krivine 2004) is a reversible form of CCS (Milner 1980)

Reversible extensions of π-calculus, Petri Nets, Erlang and others exist

Main idea: add memories so that computations can be reversed

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Our research question

Observation: in all the settings, similar properties are proved.
Techniques are similar but ad hoc.

Can we develop a general theory and then instantiate it on different formalisms?

Advantages:

- prove results once and for all
  - encourages automatic proof checking
- highlight similarities and differences among approaches

Abstraction

We abstract away the syntax of the formalisms

We just consider their (reversible) labelled transition system (LTS):

Forward transition: $P \xrightarrow{a} Q$

Backward transition: $Q \xrightarrow{\underline{a}} P$

Forward or backward transition: $t : P \xrightarrow{\alpha} Q$

Inverse of t always exists (Loop Lemma): $\underline{t} : Q \xrightarrow{\underline{\alpha}} P$

Reversibility and concurrency

In a sequential setting actions are undone in reverse order:

$$P \xrightarrow{a} Q \xrightarrow{b} R \qquad\qquad R \xrightarrow{b} Q \xrightarrow{a} P$$

In concurrent systems, the total order of actions is not relevant and may not even exist.

Causal-consistent reversibility (Danos & Krivine 2004)

An action can be reversed iff all its consequences (if any) have been already reversed.

If $P \xrightarrow{a} Q$ causes $Q \xrightarrow{b} R$ then cannot reverse a before b.

But if $P \xrightarrow{a} Q$ and $Q \xrightarrow{b} R$ are independent (concurrent) we can have

$$P \xrightarrow{a} Q \xrightarrow{b} R \qquad\qquad R \xrightarrow{a} Q' \xrightarrow{b} P$$

Here Q' was not visited going forwards, but could have been:

$$P \xrightarrow{b} Q' \xrightarrow{a} R$$

Causal equivalence

We need some preliminary notions.

Paths r, s are sequences of transitions $t_1 t_2 \ldots t_n$.

Causal equivalence on paths: $r \approx s$ iff s can be obtained from r by

1. swapping adjacent independent transitions
2. adding/removing pairs of do/undo or undo/redo: $t\underline{t} = \underline{t}t = \epsilon$

To do this, one has to fix a notion of independence.

Causal Consistency

Causal-consistent reversibility has been mostly charaterised in terms of the following property

Causal Consistency (CC - Danos & Krivine 2004)

If r and s are coinitial and cofinal paths then $r \approx s$.

It captures the fact that the information in memory is compatible with the notion of causal equivalence.

- non causal equivalent computations produce different memories
- causal equivalent computations produce the same memory

Proofs are quite lengthy but mostly take a similar approach.

The relation with the intuitive definition seems not clear, and has not been much discussed in the literature

Our approach

Our idea

We want to show that properties such as CC follow from a small set of axioms. Proving the axioms should be easier than proving the properties directly.

We use abstract labelled transition systems with independence (LTSIs).

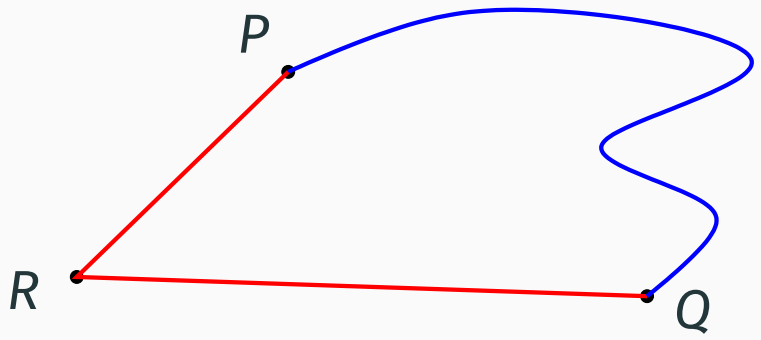
Related to the occurrence LTSIs of Sassone et al (1996).

- We treat reverse transitions as first-class citizens
- We adopt a minimal set of axioms and add more as needed

Classical proof of CC

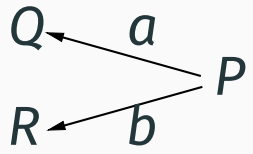
Usual proof of CC goes through the Parabolic Lemma (PL)

PL: every path is causal equivalent to a backward path followed by a forward path



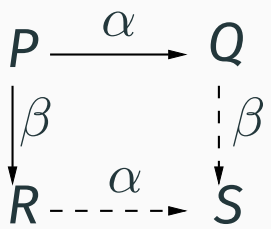
Basic axioms

1. Coinitial backward transitions are independent (BTI):



(generalizes backward determinism from sequential reversibility)

2. Square property (SP): If transitions are coinitial and independent then we can close the diamond:



3. Well-foundedness (WF): no infinite reverse path

$$\ldots \xrightarrow{a_{n+1}} P_n \xrightarrow{a_n} \ldots \xrightarrow{a_2} P_1 \xrightarrow{a_1} P_0$$

Cannot reverse to before starting point.

First results

Theorem

If BTI and SP then PL.

Theorem

If WF and PL then CC.

- Proof much shorter than existing proofs
- Success for the axiomatic approach
- Shows that CC is not much stronger than PL

Beyond CC

If CC is weaker than thought, how should we characterise causal-consistent reversibility?

Split its informal definition into:

- Causal Safety: if we can reverse t, then all events after t are independent of t (consequences have been undone)
- Causal Liveness: if all events after t are independent of t, then we can reverse t

Formalising CS and CL

We give three definitions of CS/CL:

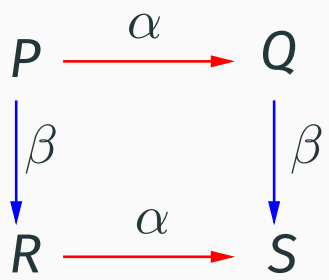
- via independence of transitions ($P \xrightarrow{a} Q \iota Q_1 \xrightarrow{c} Q_2$)
- via independence of events ($[P \xrightarrow{a} Q]$ ci $[Q_1 \xrightarrow{c} Q_2]$)
- via ordering of events ($[P \xrightarrow{a} Q] \not< [Q_1 \xrightarrow{c} Q_2]$)

With minimal axioms these are all different, but with our full set of axioms they become equivalent.

But what are events?

Events are equivalence classes of transitions.

Equate transitions representing the same action executed at different points in the computation.



If coinitial transitions in the square are independent then we let

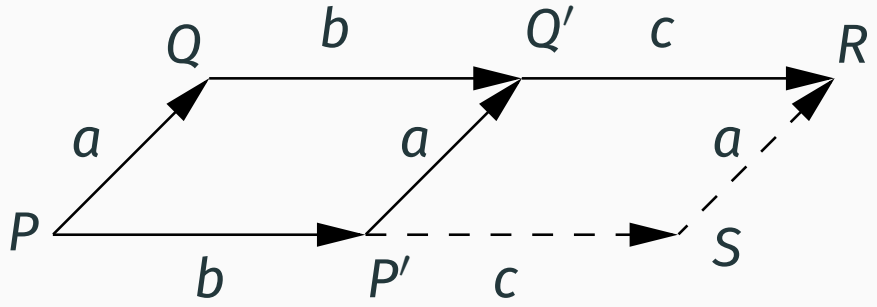
$$P \xrightarrow{\alpha} Q \sim R \xrightarrow{\alpha} S \qquad P \xrightarrow{\beta} R \sim Q \xrightarrow{\beta} S$$

Get two events $[P \xrightarrow{\alpha} Q]$ and $[P \xrightarrow{\beta} R]$ as equivalence classes.

Lift independence to events: $[t_1]$ ci $[t_2]$ if have representatives t_1' and t_2' which are coinitial and independent.

CC does not imply CS or CL

Satisfies all axioms so far, hence PL+CC, both with and without dashed dransitions:



Independence: by BTI and CPI.

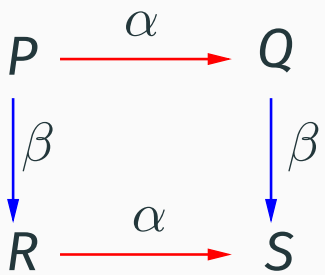
CS fails on $abc\underline{a}$, with dashed transitions.

CL fails on abc, without dashed transitions, and adding $\underline{P \xrightarrow{a} Q} \iota Q' \xrightarrow{c} R$.

We provide further axioms from which CS and CL can be deduced.

Axioms

1. SP: square property
2. BTI: backward transitions are independent
3. WF: well-founded
4. CPI: coinitial propagation of independence (around a square)



5. IRE: independence respects events (if $t \sim t' \iota u$ then $t \iota u$)
6. IEC: independence of events is coinitial (if $t \iota u$ then $[t]$ ci $[u]$)

You can find other properties that can be proved from such axioms in the paper.

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Independence coincides with concurrency.

All the axioms are satisfied. Mostly proved in the original paper or trivial. CPI and IRE easy since independence is defined on labels.

We get for free PL, CC, CS and CL (and other minor results).

$\rho\pi$ and Erlang

Similar to RCCS, the main difference is that it has a reduction semantics. However, richer labels have been defined using the memories involved in the transition.

We get for free PL, CC, CS and CL (and other minor results).

Summary

- We presented basic axioms which are satisfied by RCCS and other reversible formalisms.
- Verifying these axioms is easier than verifying the properties directly.
- Causal Consistency provides limited information, and should be supplemented by Causal Safety and Causal Liveness.
- Our abstract proofs should be relatively easy to formalise in a proof assistant.