A Reversible Semantics for Erlang

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Motivation

We have recently introduced a reversible term rewriting principle.

We want to consider a programming language: **Erlang**

Why Erlang?

- Introduces concurrency, a challenge for reversibility
- Used in large-scale distributed systems
- Reversible Computation can contribute to its reliability (debugging, transactions, etc.)

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Erlang

Erlang's features

Main features of Erlang:

- integration of functional and concurrent features
- concurrency model based on asynchronous message-passing
- dynamic typing
- hot code loading

These features make it appropiate for distributed, fault-tolerant applications (Facebook, WhatsApp)

Erlang syntax

We consider a subset of Erlang with this syntax:

$$
\begin{array}{rcl}
\mathit{Module} & ::= & \text{module } \mathit{Atom} = \mathit{fun}_1, \ldots, \mathit{fun}_n \\
\mathit{fun} & ::= & \mathit{fname} = \text{fun } (X_1, \ldots, X_n) \rightarrow \mathit{expr} \\
\mathit{fname} & ::= & \mathit{Atom}/\mathit{Integer} \\
\mathit{lit} & ::= & \mathit{Atom} \mid \mathit{Integer} \mid \mathit{Float} \mid [\,] \\
\mathit{expr} & ::= & \mathit{Var} \mid \mathit{lit} \mid \mathit{fname} \mid [\mathit{expr}_1 | \mathit{expr}_2] \mid \{\mathit{expr}_1, \ldots, \mathit{expr}_n\} \\
 & \mid & \text{call } \mathit{expr}\ (\mathit{expr}_1, \ldots, \mathit{expr}_n) \mid \text{apply } \mathit{expr}\ (\mathit{expr}_1, \ldots, \mathit{expr}_n) \\
 & \mid & \text{case } \mathit{expr} \text{ of } \mathit{clause}_1; \ldots; \mathit{clause}_m \text{ end} \\
 & \mid & \text{let } \mathit{Var} = \mathit{expr}_1 \text{ in } \mathit{expr}_2 \mid \text{receive } \mathit{clause}_1; \ldots; \mathit{clause}_n \text{ end} \\
 & \mid & \text{spawn}(\mathit{expr}, [\mathit{expr}_1, \ldots, \mathit{expr}_n]) \mid \mathit{expr}_1 \,!\, \mathit{expr}_2 \mid \text{self}() \\
\mathit{clause} & ::= & \mathit{pat} \text{ when } \mathit{expr}_1 \rightarrow \mathit{expr}_2 \\
\mathit{pat} & ::= & \mathit{Var} \mid \mathit{lit} \mid [\mathit{pat}_1 | \mathit{pat}_2] \mid \{\mathit{pat}_1, \ldots, \mathit{pat}_n\}
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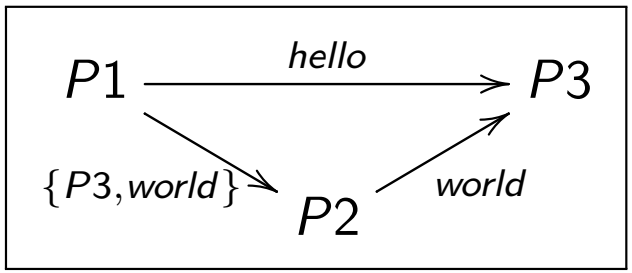

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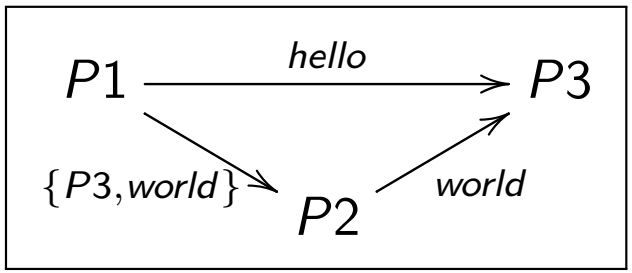

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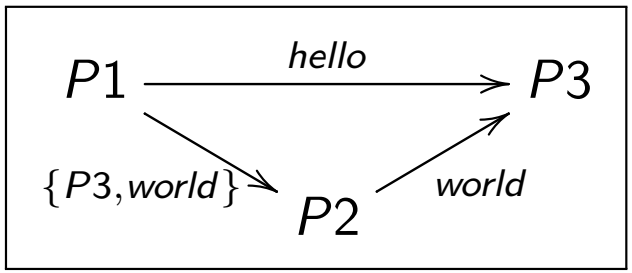

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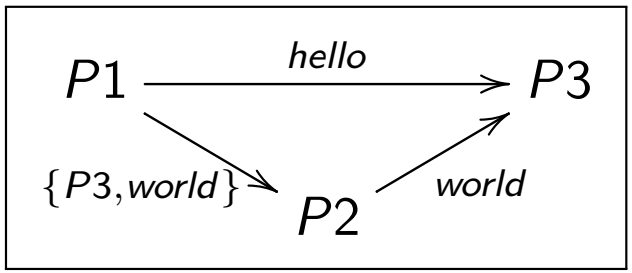

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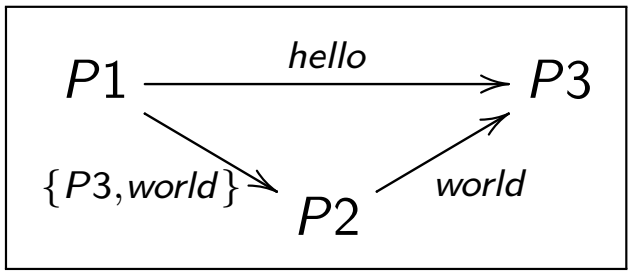

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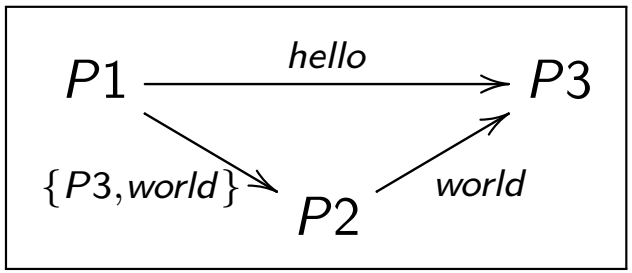

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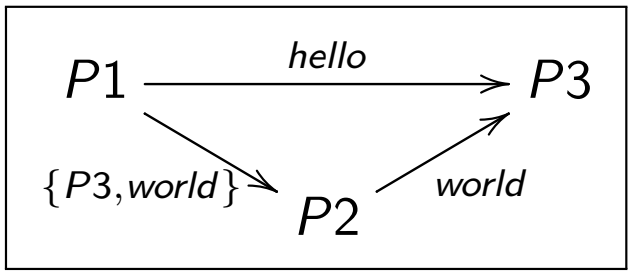

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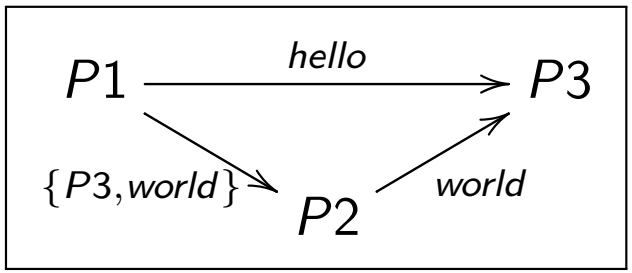

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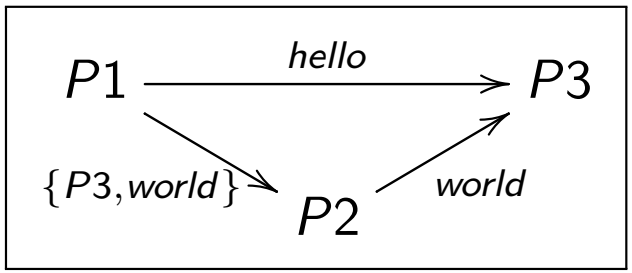

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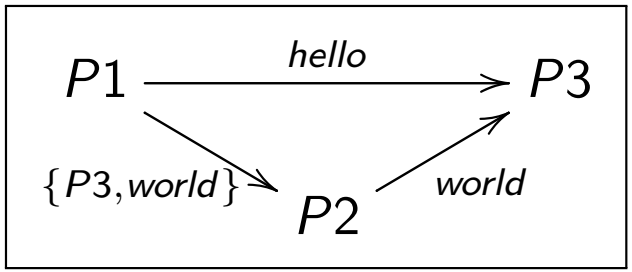

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Hello, World!

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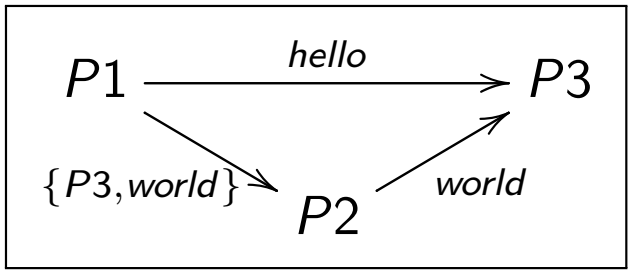

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{hello,world},{world,hello}

Semantics

Previous definitions

Definition (process)

A process is a triple $\langle p, (\theta, e), q \rangle$ where

- p is the pid of the process
- (θ, e) is the control of the state
- q is the process' local mailbox

Definition (system)

A system is denoted by $\Gamma; \Pi$, where

- Γ is the global mailbox of the system
- Π is a pool of processes

We often use $\Gamma; \langle p, (\theta, e), q \rangle \& \Pi$

Expression semantics: sequential actions

$$(Var)\ \frac{}{\theta, X \xrightarrow{\tau} \theta, \theta(X)} \qquad (Tuple)\ \frac{\theta, e_i \xrightarrow{\ell} \theta', e_i'}{\theta, \{\overline{v_{1,i-1}}, e_i, \overline{e_{i+1,n}}\} \xrightarrow{\ell} \theta', \{\overline{v_{1,i-1}}, e_i', \overline{e_{i+1,n}}\}}$$

$$(List1)\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, [e_1|e_2] \xrightarrow{\ell} \theta', [e_1'|e_2]} \qquad (List2)\ \frac{\theta, e_2 \xrightarrow{\ell} \theta', e_2'}{\theta, [v_1|e_2] \xrightarrow{\ell} \theta', [v_1|e_2']}$$

$$(Let1)\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, \text{let } X = e_1 \text{ in } e_2 \xrightarrow{\ell} \theta', \text{let } X = e_1' \text{ in } e_2} \qquad (Let2)\ \frac{}{\theta, \text{let } X = v \text{ in } e \xrightarrow{\tau} \theta[X \mapsto v], e}$$

$$(Case1)\ \frac{\theta, e \xrightarrow{\ell} \theta', e'}{\theta, \text{case } e \text{ of } cl_1; \ldots; cl_n \text{ end} \xrightarrow{\ell} \theta', \text{case } e' \text{ of } cl_1; \ldots; cl_n \text{ end}} \qquad (Case2)\ \frac{\text{match}(v, cl_1, \ldots, cl_n) = \langle \theta_i, e_i \rangle}{\theta, \text{case } v \text{ of } cl_1; \ldots; cl_n \text{ end} \xrightarrow{\tau} \theta\theta_i, e_i}$$

$$(Apply1)\ \frac{\theta, e_i \xrightarrow{\ell} \theta', e_i' \quad i \in \{1, \ldots, n\}}{\theta, \text{apply } a/n\ (\overline{v_{1,i-1}}, e_i, \overline{e_{i+1,n}}) \xrightarrow{\ell} \theta', \text{apply } a/n\ (\overline{v_{1,i-1}}, e_i', \overline{e_{i+1,n}})}$$

$$(Apply2)\ \frac{\mu(a/n) = \text{fun } (X_1, \ldots, X_n) \to e}{\theta, \text{apply } a/n\ (v_1, \ldots, v_n) \xrightarrow{\tau} \{X_1 \mapsto v_1, \ldots, X_n \mapsto v_n\}, e}$$

Expression semantics: concurrent problems

In concurrent actions, we face the following problems:

1. we do not know the result of the actions (fresh variables)
2. we must perform side effects (labels)

Labels

- At expression level, transitions for concurrent actions are labelled
- At system level, labels are used to perform the associated actions

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Expression semantics: rule Send

$$(\mathit{Send1})\ \frac{\theta, e_1 \xrightarrow{\ell} \theta', e_1'}{\theta, e_1 \mathbin{!} e_2 \xrightarrow{\ell} \theta', e_1' \mathbin{!} e_2} \qquad \frac{\theta, e_2 \xrightarrow{\ell} \theta', e_2'}{\theta, v_1 \mathbin{!} e_2 \xrightarrow{\ell} \theta', v_1 \mathbin{!} e_2'}$$

$$(\mathit{Send2})\ \frac{}{\theta, v_1 \mathbin{!} v_2 \xrightarrow{\mathsf{send}(v_1, v_2)} \theta, v_2}$$

Expression semantics: rule Self

$$(Self)\ \frac{}{\theta, \mathsf{self}() \xrightarrow{\mathsf{self}(\kappa)} \theta, \kappa}$$

Expression semantics: rules Spawn and Receive

$$(\mathit{Spawn})\ \frac{}{\theta, \mathsf{spawn}(a/n, [e_1, \ldots, e_n]) \xrightarrow{\mathsf{spawn}(\kappa, a/n, [\overline{e_n}])} \theta, \kappa}$$

$$(\mathit{Receive})\ \frac{}{\theta, \mathsf{receive}\ cl_1; \ldots; cl_n\ \mathsf{end} \xrightarrow{\mathsf{rec}(\kappa, \overline{cl_n})} \theta, \kappa}$$

Semantics

System-level rules

System semantics: rule Seq

$$(Seq)\quad \frac{\theta, e \xrightarrow{\tau} \theta', e'}{\Gamma; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma; \langle p, (\theta', e'), q\rangle \& \Pi}$$

System semantics: rule Self

$$(Self)\quad \frac{\theta, e \xrightarrow{\mathsf{self}(\kappa)} \theta', e'}{\Gamma; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma; \langle p, (\theta', e'\{\kappa \mapsto p\}), q\rangle \& \Pi}$$

System semantics: rule Send

$$(Send)\quad \frac{\theta, e \xrightarrow{\mathsf{send}(p'',v)} \theta', e'}{\Gamma; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma \cup (p'', v); \langle p, (\theta', e'), q\rangle \& \Pi}$$

System semantics: rule Receive

$$(\mathit{Receive})\quad \frac{\theta, e \xrightarrow{\mathsf{rec}(\kappa,\overline{cl_n})} \theta', e' \quad \mathsf{matchrec}(\overline{cl_n}, q) = (\theta_i, e_i, v)}{\Gamma; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma; \langle p, (\theta'\theta_i, e'\{\kappa \mapsto e_i\}), q \backslash\backslash v\rangle \& \Pi}$$

System semantics: rule Spawn

$$(Spawn)\quad \frac{\theta, e \xrightarrow{\text{spawn}(\kappa, a/n, [\overline{e_n}])} \theta', e' \qquad p' \text{ is a fresh pid}}{\Gamma; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma; \quad \begin{array}{l}\langle p, (\theta', e'\{\kappa \mapsto p'\}), q\rangle \& \\ \langle p', (\theta', \text{apply } a/n\ (\overline{e_n})), [\,]\rangle \& \Pi\end{array}}$$

System semantics: rule Sched

$$(Sched)\quad \frac{}{\Gamma \cup \{(p, v)\}; \langle p, (\theta, e), q\rangle \& \Pi \hookrightarrow \Gamma; \langle p, (\theta, e), v : q\rangle \& \Pi}$$

Reversible Semantics

Reversible semantics

Our reversible semantics is composed of

- the forward semantics (we record steps given in h)
- the backward semantics (we use the recorded information)

Uncontrolled semantics

Rules (both forward and backward) are fired in a non-deterministic fashion

Forward (reversible) semantics

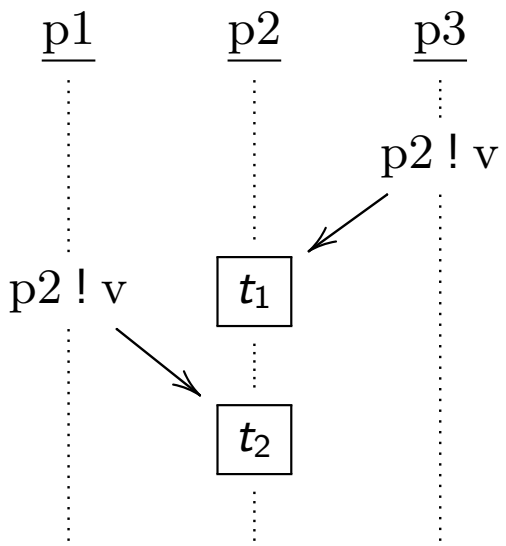
$$(Seq)\qquad \frac{\theta, e \xrightarrow{\tau} \theta', e'}{\Gamma; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup \Gamma; \langle \tau(\theta, e) : h, p, (\theta', e'), q\rangle \& \Pi}$$

$$(Self)\qquad \frac{\theta, e \overset{\mathsf{self}(y)}{\rightarrow} \theta', e'}{\Gamma; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup \Gamma; \langle \mathsf{self}(\theta, e) : h, p, (\theta', e'\{y \mapsto p\}), q\rangle \& \Pi}$$

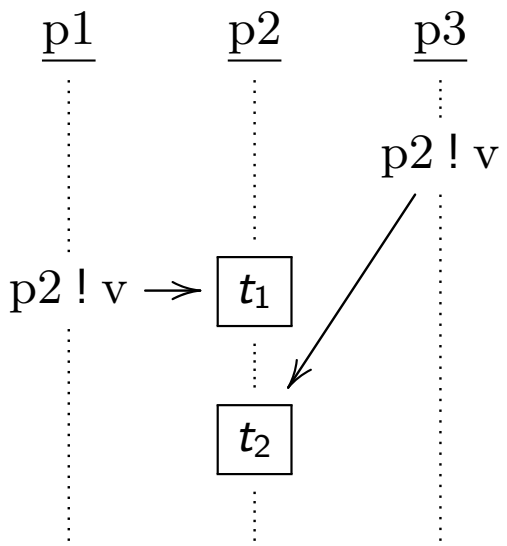
$$(Spawn)\qquad \frac{\theta, e \overset{\mathsf{spawn}(y, a/n, [e_1, \ldots, e_n])}{\rightarrow} \theta', e' \quad p' \text{ is a fresh pid}}{\begin{array}{ll}\Gamma; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup & \Gamma; \langle \mathsf{spawn}(\theta, e, p') : h, p, (\theta', e'\{y \mapsto p'\}), q\rangle \\ & \& \langle [\,], p', (\theta, \mathsf{apply}\ a/n\ (e_1, \ldots, e_n)), [\,]\rangle \& \Pi\end{array}}$$

Communication with different processes

Messsages could be distinguished by attaching the pid of the processes



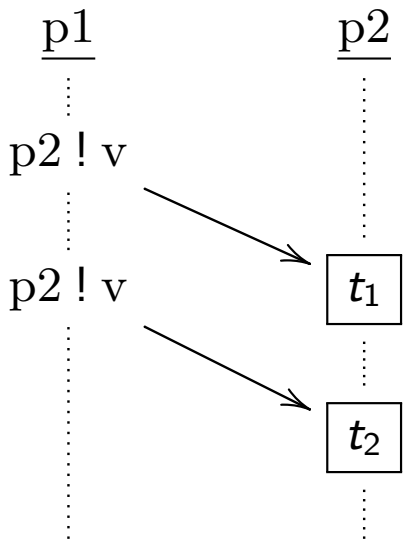
(a)



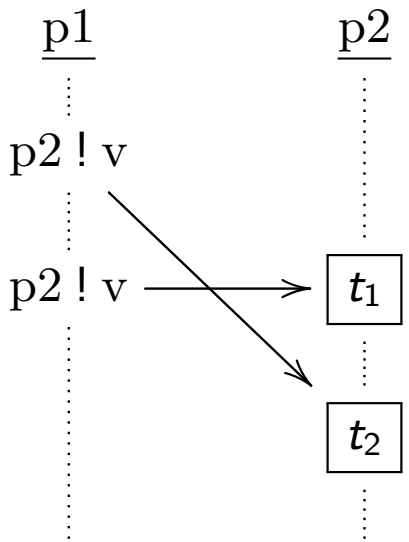
(b)

Communication with a single process

The pid of the process is not enough, we need timestamps!



(c)



(d)

Forward (reversible) semantics (2)

$$(Send)\quad \frac{\theta, e \overset{\mathsf{send}(p'', v)}{\rightarrow} \theta', e' \quad \text{and} \quad \lambda \text{ is a fresh identifier}}{\Gamma; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup \Gamma \cup (p'', \{v, \lambda\}); \langle \mathsf{send}(\theta, e, \{v, \lambda\}) : h, p, (\theta', e'), q\rangle \& \Pi}$$

$$(Receive)\quad \frac{\theta, e \overset{\mathsf{rec}(y, \overline{cl_n})}{\rightarrow} \theta', e' \quad \mathsf{matchrec}(\overline{cl_n}, q) = (\theta_i, e_i, \{v, \lambda\})}{\Gamma; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup \Gamma; \langle \begin{array}{l} \mathsf{rec}(\theta, e, \{v, \lambda\}, q, q \backslash\backslash \{v, k\}) : h, \\ p, (\theta' \theta_i, e' \{y \mapsto e_i\}), q \backslash\backslash \{v, k\} \end{array} \rangle \& \Pi}$$

$$(Sched)\quad \frac{}{\Gamma \cup \{(p, \{v, \lambda\})\}; \langle h, p, (\theta, e), q\rangle \& \Pi \rightharpoonup \Gamma; \langle h, p, (\theta, e), \{v, \lambda\} : q\rangle \& \Pi}$$

Backward semantics

$$(\overline{Seq}) \qquad \Gamma; \langle \tau(\theta, e) : h, p, (\theta', e'), q \rangle \ \& \ \Pi \leftharpoondown \Gamma; \langle h, p, (\theta, e), q \rangle \ \& \ \Pi$$

$$(\overline{Self}) \qquad \Gamma; \langle \mathsf{self}(\theta, e) : h, p, (\theta', e'), q \rangle \ \& \ \Pi \leftharpoondown \Gamma; \langle h, p, (\theta, e), q \rangle \ \& \ \Pi$$

$$(\overline{Spawn}) \qquad \begin{array}{l} \Gamma; \langle \mathsf{spawn}(\theta, e, p'') : h, p, (\theta, e), q \rangle \ \& \ \langle [\,], p'', (\theta'', e''), q'' \rangle \ \& \ \Pi \\ \leftharpoondown \Gamma; \langle h, p, (\theta, e), q \rangle \ \& \ \Pi \end{array}$$

Backward semantics (2)

$$(\overline{Send}) \quad \Gamma \cup \{(p', \{v, \lambda\})\}; \langle \mathsf{send}(\theta, e, \{v, \lambda\}) : h, p, (\theta', e'), q\rangle \ \&\ \Pi \leftharpoonup \Gamma; \langle h, p, (\theta, e), q\rangle \ \&\ \Pi$$

$$(\overline{Receive}) \quad \Gamma; \langle \mathsf{rec}(\theta, e, \{v, \lambda\}, q, q') : h, p, (\theta', e'), q'\rangle \ \&\ \Pi \leftharpoonup \Gamma; \langle h, p, (\theta, e), q\rangle \ \&\ \Pi$$

$$(\overline{Sched}) \quad \Gamma; \langle h, p, (\theta, e), \{v, \lambda\} : q\rangle \ \&\ \Pi \leftharpoonup \Gamma \cup (p, \{v, \lambda\}); \langle h, p, (\theta, e), q\rangle \ \&\ \Pi$$
if the topmost rec(...) item in h (if any) is different from $\mathsf{rec}(\theta', e', \{v, \lambda\}, q'', \{v, \lambda\} : q)$

Results

Theorem (conservative)

Let $\mathcal{P}$ be a program without occurrences of "check". Let s_1 be a system of the standard semantics and rs_1 a system of the reversible semantics with $\overline{rs_1} = s_1$. Then, $s_1 \longmapsto^* s_2$ iff $rs_1 \rightharpoonup^* rs_2$ and $\overline{rs_2} = s_2$.

Lemma (loop lemma)

For every pair of systems, s_1 and s_2, we have $s_1 \rightharpoonup_{p,r,k} s_2$ iff $s_2 \leftharpoondown_{p,\overline{r},k} s_1$.

Concurrent transitions

Definition (concurrent transitions)

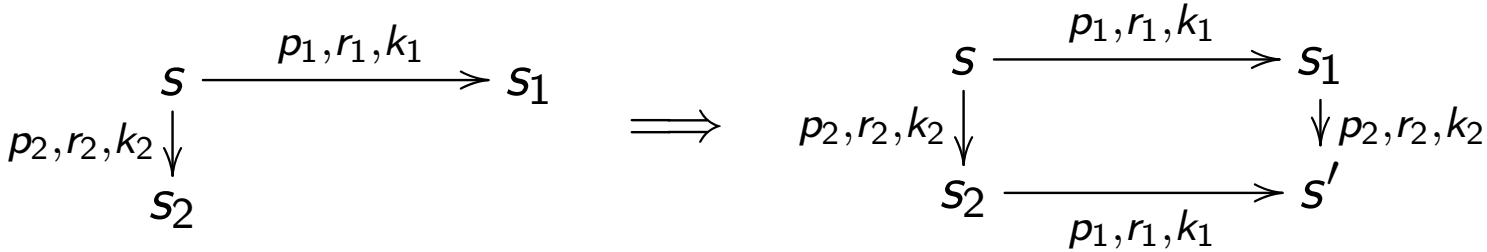
$t_1 = (s \rightleftharpoons_{p_1,r_1,k_1} s_1)$ and $t_2 = (s \rightleftharpoons_{p_2,r_2,k_2} s_2)$ are *in conflict* if:

- **both transitions are forward**, $p_1 = p_2$, and either $r_1 = r_2 = Sched$ or one is *Sched* and the other *Receive*
- one is **forward**, $p_1 = \mathrm{p}$, and the other one is a **backward** transition that undoes the creation of p
- one is a **forward** transition that delivers a message $\{v, \lambda\}$ and the other one is a **backward** transition that undoes the sending $\{v, \lambda\}$
- one is a **forward** transition and the other one is a **backward** transition such that $p_1 = p_2$ and either i) both applied rules are different from *Sched* and $\overline{Sched}$; ii) one rule is *Sched* and the other one is $\overline{Sched}$; iii) one rule is *Sched* and the other one is $\overline{Receive}$; or iv) one rule is $\overline{Sched}$ and the other one is *Receive*

Results

Lemma (square lemma)

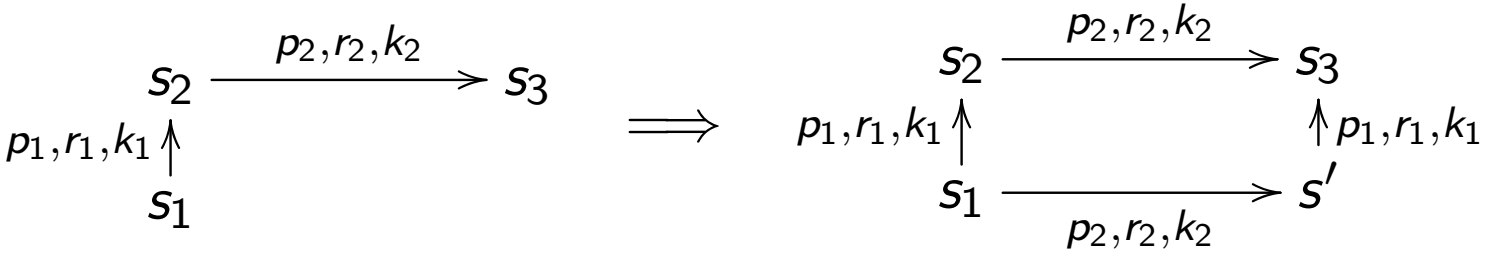
Let $\mathcal{P}$ be a program. Given two coinitial transitions $t_1 = (s \leftrightharpoons_{p_1,r_1,k_1} s_1)$ and $t_2 = (s \leftrightharpoons_{p_2,r_2,k_2} s_2)$, there exists two cofinal transitions $t_2/t_1 = (s_1 \leftrightharpoons_{p_2,r_2,k_2} s')$ and $t_1/t_2 = (s_2 \leftrightharpoons_{p_1,r_1,k_1} s')$.



Results

Lemma (switching lemma)

Given two composable transitions of the form $t_1 = (s_1 \leftrightharpoons_{p_1,r_1,k_1} s_2)$ *and* $t_2 = (s_2 \leftrightharpoons_{p_2,r_2,k_2} s_3)$ *such that* t_1^{-1} *and* t_2 *are concurrent, there exist a system* s_4 *and two composable transitions* $t_1' = (s_1 \leftrightharpoons_{p_2,r_2,k_2} s_4)$ *and* $t_2' = (s_4 \leftrightharpoons_{p_1,r_1,k_1} s_3)$.



Causal consistency

Definition (causal equivalence)

$$t_1; t_2/t_1 \approx t_2; t_1/t_2$$

$$t; t^{-1} \approx \epsilon_{init(t)}$$

Theorem (causal consistency)

Let d_1 and d_2 be coinitial derivations. Then, $d_1 \approx d_2$ iff d_1 and d_2 are cofinal.

Controlled semantics

Finally, we add control to this semantics (rollback operator):

$$\lfloor proc \rfloor_{\Psi}$$

There are **three kinds** of rollbacks:

- introduced by the programmer: $\#^{\mathtt{t}}_{\mathsf{ch}}$

$$\text{let } \_ = \text{check}(\mathtt{t}) \text{ in } expr$$

- internal: $\#_{\mathsf{sch}}$ (message dispatching) and $\#_{\mathsf{sp}}$ (spawning a process)

Rollback example

Undo actions from a process that has spawned another process

Rollback example (1)

Start rollback

To start a rollback, we fire the rule *Undo*

$$(\overline{Undo}) \quad \Gamma; \lfloor \langle h, p, (\theta', e'), q \rangle \rfloor_{\Psi} \,\&\, \Pi \leftharpoondown \Gamma; \lfloor \langle h, p, (\theta, e), q \rangle \rfloor_{\Psi \cup \{\#^{t}_{ch}\}} \,\&\, \Pi$$

Rollback example (2)

Rollback propagation

Propagate the rollback to the spawned process

$(\overline{Spawn2})$

$$\Gamma; \lfloor \langle \mathsf{spawn}(\theta, e, p'') : h, p, (\theta, e), q \rangle \rfloor_{\Psi} \;\&\; \lfloor \langle h', p'', (\theta'', e''), q'' \rangle \rfloor_{\Psi'} \;\&\; \Pi$$
$$\leftharpoondown \Gamma; \lfloor \langle \mathsf{spawn}(\theta, e, p'') : h, p, (\theta, e), q \rangle \rfloor_{\Psi} \;\&\; \lfloor \langle h', p'', (\theta'', e''), q'' \rangle \rfloor_{\Psi' \cup \{\#_{\mathsf{sp}}\}} \;\&\; \Pi$$
$$\text{if } h' \neq [\,] \text{ and } \#_{\mathsf{sp}} \notin \Psi$$

Rollback example (3)

Causal consistent undo

Undo action when all dependent actions have been undone

$(\overline{Spawn1})$

$$\Gamma; \lfloor \langle \mathsf{spawn}(\theta, e, p'') : h, p, (\theta, e), q \rangle \rfloor_{\Psi} \;\&\; \lfloor \langle [\,], p'', (\theta'', e''), q'' \rangle \rfloor_{\Psi'} \;\&\; \Pi \leftharpoondown \Gamma; \lfloor \langle h, p, (\theta, e), q \rangle \rfloor_{\Psi} \;\&\; \Pi$$

Rollback example (4)

Rollback removal

The rollback is removed when we reach the corresponding checkpoint

$$(\overline{Check}) \quad \Gamma; \lfloor \langle \mathsf{check}(\theta, e, \mathsf{t}) : h, p, (\theta', e'), q \rangle \rfloor_{\Psi} \;\&\; \Pi \;\leftharpoondown\; \Gamma; \lfloor \langle h, p, (\theta, e), q \rangle \rfloor_{\Psi \setminus \{\#^{\mathsf{t}}_{\mathsf{ch}}\}} \;\&\; \Pi$$

Conclusions

Conclusions

We have:

- proposed a new semantics for Erlang
- defined a **reversible semantics for a subset of Erlang**
- proofs for the main properties
- developed a rollback operator for the reversible semantics

Ongoing work:

- implementation of the reversible semantics
- design concrete applications (debugging, fault-tolerance, etc.)

Thanks for your attention!