Assume-Guarantee verification of Hybrid Systems in ARIADNE

Davide Bresolin and Tiziano Villa

University of Verona

Games 2009
Udine, Italy

Outline

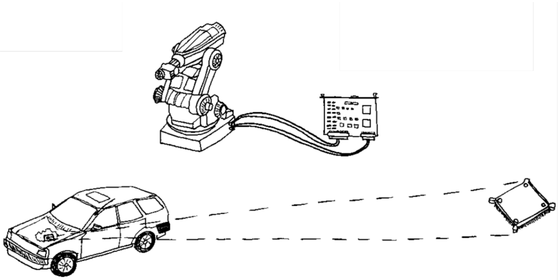
1. Introduction to Hybrid Systems
2. The software package ARIADNE
3. Assume-guarantee reasoning in ARIADNE
4. Conclusions

Outline

Hybrid Systems

Many real systems have a double nature:

- they evolve in a contiuous way;
- they are controlled by a discrete system.



How to model them?

Hybrid Systems/Automata

Hybrid Automata: Definition

Definition (Hybrid Automaton, Alur et al. 1992)

A *hybrid automaton* is a tuple $H = \langle \mathcal{V}, \mathcal{E}, \mathbb{R}^k, Inv, Dyn, Act, Reset \rangle$:

1. $\langle \mathcal{V}, \mathcal{E} \rangle$ is a finite directed graph; the vertexes, $\mathcal{V}$, are called *locations* or *control modes*, and the directed edges, $\mathcal{E}$, are called *control switches*;
2. Each location $v \in \mathcal{V}$ is labeled by the predicate $Inv(v)$ on the set $\mathbb{R}^k$ and the transitive relation $Dyn(v)$ on $\mathbb{R}^k \times \mathbb{R}^k \times \mathbb{R}^{\geq 0}$;
3. Each edge $e \in \mathcal{E}$ is labeled by the predicate $Act(e)$ on $\mathbb{R}^k$ and the relation $Reset(e)$ on $\mathbb{R}^k \times \mathbb{R}^k$.

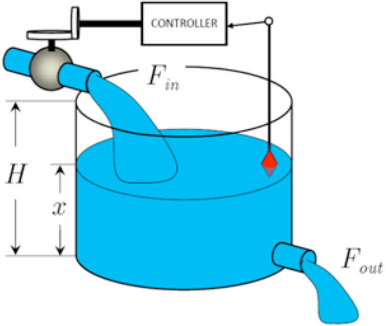
Hybrid Automata: Intuition

A state of an hybrid automaton is a pair (v, r) where v is a discrete location and r is a point in $\mathbb{R}^k$.

Hybrid Automaton = Finite Automaton + Continuous Evolution

Time flows when the automaton stays in a location:

- H evolves from r to s in time t when $Dyn(v)[r, s, t]$;
- in location v, r must satisfy $Inv(v)[r]$;
- H can cross a transition e only if $Act(e)[r]$;
- when H crosses e, $Reset(e)[r, s]$.

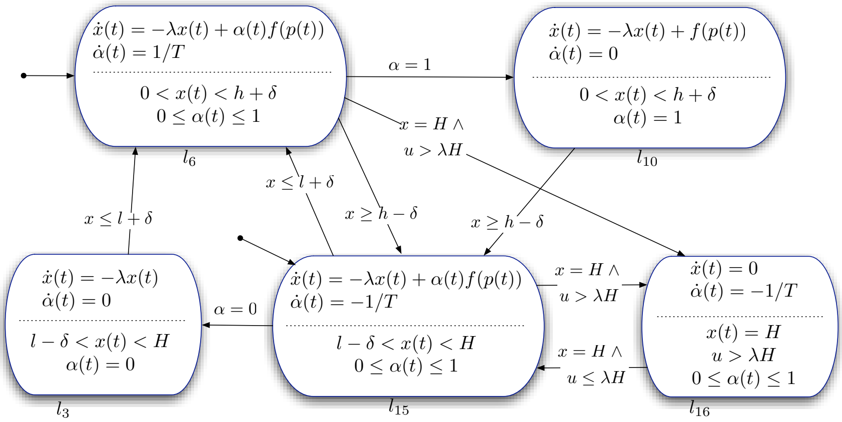


- Outlet flow F_{out} depends on the water level.
- Inlet flow F_{in} is controlled by the valve position.
- The controller senses the water level and sends the appropriate commands to the valve.

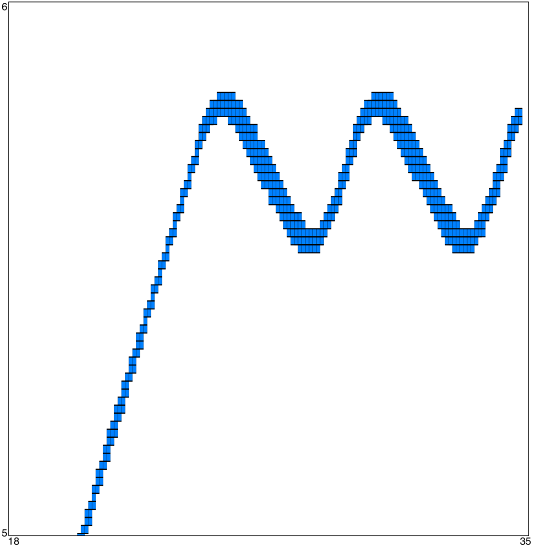
Control Problem

Keep the water level between two given thresholds.

The watertank automaton



Evolution of the watertank



Reachability Problem

Reachability

Given an hybrid automaton H and two sets S and T, is there any $s \in S$ and $t \in T$ such that there exists a trajectory of H from s to t?

The reachability problem for Hybrid Automata is undecidable (Alur et al. 1995).

Can I solve the problem, at least in some cases?

- Restrict to special classes of Hybrid Automata (Timed Automata, Rectangular Automata, . . .)
- Use approximation techniques to obtain an approximation of the reachable set.

Reachability Problem

Reachability

Given an hybrid automaton H and two sets S and T, is there any $s \in S$ and $t \in T$ such that there exists a trajectory of H from s to t?

The reachability problem for Hybrid Automata is undecidable (Alur et al. 1995).

Can I solve the problem, at least in some cases?

- Restrict to special classes of Hybrid Automata (Timed Automata, Rectangular Automata, . . .)
- Use approximation techniques to obtain an approximation of the reachable set.

Reachability Problem

Reachability

Given an hybrid automaton H and two sets S and T, is there any $s \in S$ and $t \in T$ such that there exists a trajectory of H from s to t?

The reachability problem for Hybrid Automata is undecidable (Alur et al. 1995).

Can I solve the problem, at least in some cases?

- Restrict to special classes of Hybrid Automata (Timed Automata, Rectangular Automata, . . .)
- Use approximation techniques to obtain an approximation of the reachable set.

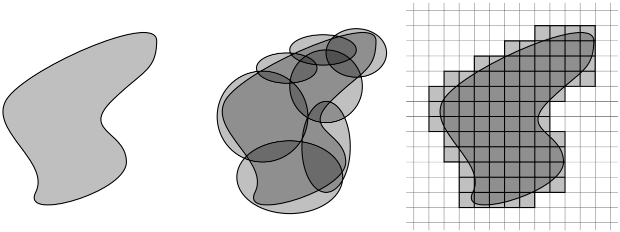
Outline

Introduction to ARIADNE

- Developed by a joint team including CWI, the University of Verona, the University of Udine and the company PARADES (Rome).
- Based on a rigorous mathematical semantics for the numerical analysis of continuous and hybrid systems.
- The computational kernel is written using a mix of generic and polymorphic programming strategies resulting in a highly efficient, modular and extensible framework.
- Released as an open source distribution.

Representing regions of space

- Subsets of $\mathbb{R}^n$ are approximated by finite unions of basic sets:
  - intervals, simplices, cuboids, parallelotopes, zonotopes, polytopes, spheres and ellipsoids
- Finite unions of basic sets of a given type are called *denotable sets*.



Approximating regions

Approximating S with A

1. **Inner approximation:** S strictly contains A.
2. **Outer approximation:** S is strictly contained in A.
3. **ε-lower approximation:** every point of A is at distance less than ε from a point of S.

- Inner approximation is used for specification of systems properties.
- Outer and ε-lower approximation are used for computing evolution.

Approximate Reachability Analysis

Given an hybrid automaton H, an initial set I and a time t, ARIADNE can compute:

- an outer approximation of the states reached by H starting from I up to time t.
- for a given $\varepsilon > 0$, an ε-lower approximation of the states reached by H starting from I up to time t.

Outline

Assume-guarantee system specification

- The system is specified as a set of components
- Every component is annotated with a pair (A, G) of assumptions and guarantees.
- The requirements of the whole system are decomposed into a set of simpler requirements that, if satisfied, guarantees that the overall requirements are satisfied.

Safety checking

Let C be a component of the system, annotated with assumptions A and guarantees G. With ARIADNE we can verify whether the component C respects the guarantees or not (with some limitations).

- Represent the component by an hybrid automata H with inputs and outputs;
- Assumptions A are represented by hybrid automata H_A that specify the possible inputs for H;
- Guarantees G specify the possible outputs Y of the automata;

This is a reachability analysis problem:

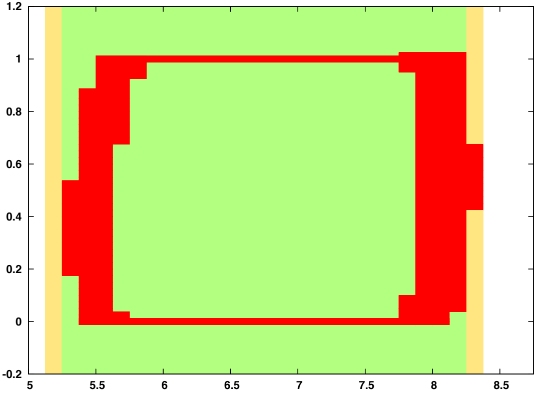
$$Reach(H \| A) \subseteq Sat(G)$$

Safety checking by grid refinement

1. Compute an outer-approximation O of $Reach(H \| H_A)$ using a grid of a given size.
2. If $O \subseteq Sat(G)$, the system is verified to be safe. Exit with success.
3. Otherwise, compute an ε-lower approximation L_ε of $Reach(H \| H_A)$. The value of ε depends on the size of the grid.
4. If there exists at least a point in L_ε that is outside $Sat(G)$ by more than ε, the system is verified to be unsafe. Exit with failure.
5. Otherwise, set the grid to a finer size and restart from point 1.

Verifying the water tank

Safety property: the water level between 5.25 and 8.25 meters.

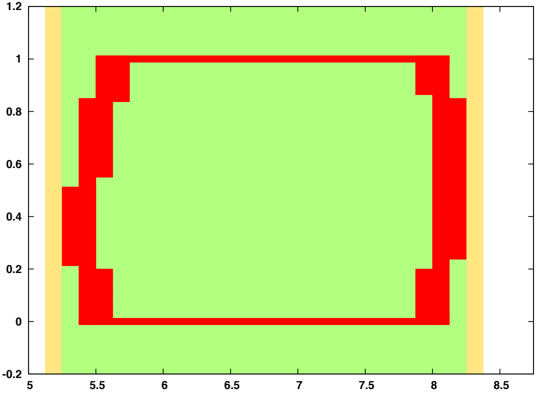


First iteration:
grid $1/8 \times 1/80$.

Outer reach is not safe, try lower reach.

Verifying the water tank

Safety property: the water level between 5.25 and 8.25 meters.

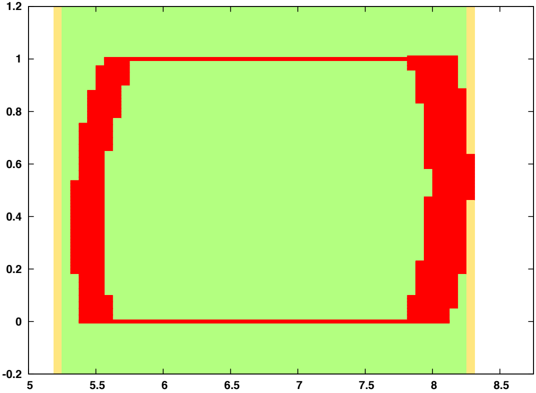


First iteration:
grid $1/8 \times 1/80$.

Lower reach is safe, refine grid.

Verifying the water tank

Safety property: the water level between 5.25 and 8.25 meters.

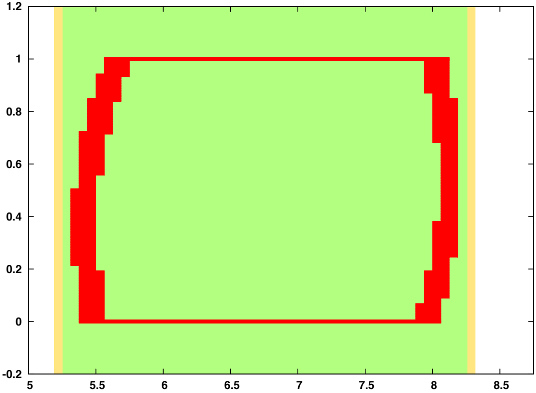


Second iteration: grid $1/16 \times 1/160$.

Outer reach is not safe, try lower reach.

Verifying the water tank

Safety property: the water level between 5.25 and 8.25 meters.

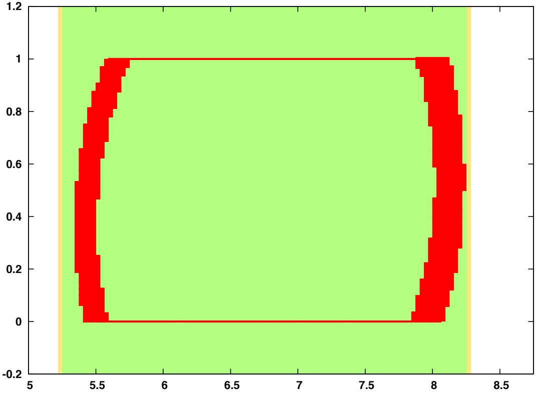


Second iteration: grid $1/16 \times 1/160$.

Lower reach is safe, refine grid.

Verifying the water tank

Safety property: the water level between 5.25 and 8.25 meters.



Third iteration:
grid $1/32 \times 1/320$.

Outer reach is safe, system is proved safe.

Dominance checking

Definition

Given two components C_1 and C_2, with assumptions and guarantees (A_1, G_1) and (A_2, G_2), we say that C_1 dominates C_2 if and only if under weaker assumptions ($A_2 \subseteq A_1$), stronger promises are guaranteed ($G_1 \subseteq G_2$).

If this is the case, the component C_2 can be replaced with C_1 in the system without affecting the whole system behaviour.

Dominance checking by reachability analysis

1. Represent the two components by two hybrid automata H_1 and H_2 with inputs and outputs;
2. Assumptions A_1 and A_2 are represented by hybrid automata H_{A_1} and H_{A_2} that specify the possible inputs U_1, U_2 for the components;
3. Guarantees G_1 and G_2 specify the possible outputs Y_1, Y_2 of the automata;
4. H_1 dominates H_2 if and only if $Y_1 \subseteq Y_2$;

This is a reachability analysis problem:

$$Reach(H_{A_1} \| H_1)|_{Y_1} \subseteq Reach(H_{A_2} \| H_2)|_{Y_2}$$

Dominance checking in ARIADNE

The approximate reachability routines of ARIADNE can be used to test dominance of components:

1. Compute an ε-lower approximation L_2^ε of $Reach(H_{A_2} \| H_2)|_{Y_2}$
2. Remove a border of size ε from L_2^ε
3. Compute an outer approximation O_1 of $Reach(H_{A_1} \| H_1)|_{Y_1}$
4. If $O_1 \subseteq L_2^\varepsilon - \varepsilon$ then $Reach(H_{A_1} \| H_1)|_{Y_1} \subseteq Reach(H_{A_2} \| H_2)|_{Y_2}$ and thus H_1 dominates H_2
5. If not, we cannot say anything about H_1 and H_2, we retry with a finer approximation.

Outline

Conclusions

- ARIADNE can compute approximation of the reachable set of hybrid automata.
- It is currently used to verify complex systems using advanced verification strategies.
- Future improvements:
  - Add support for the analysis of networks of hybrid automata.
  - Provide input support for hybrid automata description languages.
  - Improve the verification and model checking capabilities.